International Journal of Mathematical Archive-3(11), 2012, 3958-3960
Available online through www.ijma.info ISSN 2229 – 5046

PRIME SEMIRINGS ADMITTING GENERALIZED DERIVATIONS

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(Received on: 01-09-12; Revised & Accepted on: 15-11-12)

ABSTRACT

Let S be a prime semiring. Motivated by some results on derivations in rings, in [2], we defined the notion of derivations and generalized derivations on semirings and investigated some simple and interesting results on the derivations in semirings. In this paper, we discuss the commutativity of S with generalized derivation F associated with the derivation D.

Key-words: *Semiring, Derivation, Generalized derivation, commutativity.*

2000 Mathematics Subject Classification: *Primary 16Y60.*

INTRODUCTION

There has been ongoing interest concerning the relationship between the commutativity of a ring R and the existence of certain specific types of derivations on R. In [1] some results on commutativity of rings with derivations and generalized derivations are discussed. In [2], we have introduced the notion of derivations on semirings and proved some simple but interesting results.

In this concern, in our paper [3], we have established the following theorem:

"Let S be a prime semiring and D, a non-zero derivation on S. If Char(S)$\neq$ 2 and $[D(x), D(y)] = 0, for\ all\ x, y\ \varepsilon\ S$, then S is commutative."

It is natural to ask what can we say about the commutativity of S, if the derivation D is replaced by generalized derivation F. In this paper, we discuss this question and find some interesting results.

2. PRELIMINARIES

Let S be a semiring. For each $x, y\ \varepsilon\ S$, denote the **commutator** $xy - yx$ by $[x, y]$ and the **anti-commutator** $xy + yx$ by $x \circ y$.

We call a semiring S, a **prime semiring** if for $x, y\ \varepsilon\ S$, $xSy = \{0\}\ implies\ x = 0\ or\ y = 0$.

An additive mapping D:S→S is called a **derivation** if $D(xy) = D(x)y + xD(y), forall\ x, y \in S$.

An additive mapping F:S→S is called **generalized derivation** with associated derivation D if

$$F(xy) = F(x)y + xD(y), forall\ x, y \in S.$$

We recall the following results.

Lemma 2.1: (Lemma 4.3, [3])
1. Sum of two generalized derivations on a semiring S is a generalized derivation.
2. If F is a generalized derivation on a semiring S with associated derivation D, then it is of the form $F(x) = cx + D(x)$, c is a fixed constant.

Lemma 2.2: (Lemma 3.4, [4]) Let S be a prime semiring and I, a non-zero right ideal of S. If D is a non-zero derivation on S, then D is non-zero on I.

Lemma 2.3: (Lemma 3.5, [4]) If a prime semiring S contains a non-zero commutative right ideal I, then S is commutative.

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Lemma 2.4: (Theorem4.5, [3]) Let S be a prime semiring. F, a generalized derivation with associated derivation D $\neq 0$ such that $D(F(x)) = 0, forall\ x\ \varepsilon\ S$. Let $c = F(1)$. Then $cD(x) = D(x)c = 0, forall\ x\ \varepsilon\ S$. Moreover, if Char(S)$\neq$ 2, $c^2\ \varepsilon\ Z$ and if $c\ \varepsilon\ Z$ then $c = 0\ and\ F = D$.

Lemma 2.5: (Theorem 4.6, [3]) Let S be a prime semiring. If S admits a non-zero generalized derivation F with associated derivation D $\neq 0$ such that $[F(x), x] = 0,\ for\ all\ x\varepsilon S$, then S is commutative.
We shall make use of the following commutator identities extensively:

$$[xy, z] = x[y, z] + [x, z]y$$
$$[x, yz] = [x, y]z + y[x, z]$$
$$x\circ(yz) = (x\circ y)z - y[x, z] = y(x\circ z) + [x, y]z$$
$$(xy)\circ z = x(y\circ z) - [x, z]y = (x\circ z)y + x[y, z]$$

3. COMMUTATIVITY OF PRIME SEMIRINGS

Theorem 3.1: Let S be a prime semiring and I, a non-zero ideal of S. If S admits a generalized derivation F associated with a non-zero derivation D such that

$$F(x\circ y) = x\circ y, for\ all\ x, y\ \varepsilon\ I, \tag{1}$$

then S is commutative.

Proof: If F = 0, then $x\circ y = 0, forall\ x, y\ \varepsilon\ I$.

Replacing y by yz, (z ε I) , we get $x\circ(yz) = 0$.

$(x\circ y)z + [x, y]z = 0 ==> [x, y]z = 0, forall\ x, y\ \varepsilon\ I\ \ since\ \ x\circ y = 0, forall\ x, y\ \varepsilon\ I.$

$[x, y]I = 0, forall\ x, y\ \varepsilon\ I\ \ ==> [x, y]SI = 0, forall\ x, y\ \varepsilon\ I$

$==>$ $[x, y] = 0, forall\ x, y\ \varepsilon\ I$ due to the primeness of S and I $\neq 0$.

$==>$ I is commutative.

Hence by Lemma (3), S is commutative.

Suppose F $\neq 0$.
(1) $==> F(xy + yx) = xy + yx$
$==> F(x)y + xD(y) + F(y)x + yD(x) = xy + y$ (2)

Replacing y by yx and simplifying using (2),

$(xy + yx)x + xyD(x) + yxD(x) = xyx + yx^2$

implies $(x\circ y)D(x) = 0, forall\ x, y\ \varepsilon\ I.$ (3)

Replacing y by zy in above equation and simplifying we arrive at

$$[x, z]I = 0, forall\ x, z\ \varepsilon\ I\ or\ D(x) = 0, forall\ x\ \varepsilon\ I.$$

By primeness of S, If $D(x) = 0, forall\ x\ \varepsilon\ I$, D = 0, by lemma (2) which is not possible.

Hence [x, z]I = 0, for all x, z ε I which [x, z] = 0, for all x, z ε I since I $\neq 0$.

Hereby we get that I is commutative implying S is commutative from Lemma (3).

Following the same lines as above with necessary variations, we can prove the following:

Theorem 3.2: Let S be a prime semiring and I, a non-zero ideal of S. If S admits a generalized derivation F associated with a non-zero derivation D such that

$$F(x\circ y) + x\circ y = 0, for\ all\ x, y\ \varepsilon\ I,$$

then S is commutative.

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Source of support: Nil, Conflict of interest: None Declared