A NEW NOTION OF OPEN SETS IN TOPOLOGICAL SPACES

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ABSTRACT

The determination of this paper is to introduce the new family of sets, namely $S_g^*$-open sets and $S_g^*$-closed sets. Further, we define $S_g^*$-interior and $S_g^*$-closure and discuss its properties. Additionally, we relate $S_g^*$-open sets and $S_g^*$-closed sets with some other sets.

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1. INTRODUCTION

Throughout this paper, we denote a topological space by $(X, \tau)$. For a subset $A$ of $(X, \tau)$, the closure and interior of $A$ with respect to $\tau$ are denoted by $Cl(A)$ and $Int(A)$ respectively. The concept of semi-open sets[3] and generalized closed (briefly g-closed) sets[4] were introduced by Norman Levine in 1963 and 1970 respectively. After the works of Levine, in the year 1987, P. Battacharyya and B.K. Lahiri[1] introduced the concept of semi-generalized closed sets with the help of semi-openness. Recently, A. Robert and S. Pious Missier[6] have introduced the concept of semi*-open sets and studied their properties.

In this direction, we shall introduce a new family of sets called $S_g^*$-open sets, using the semi-generalized closure operator. This definition enables us to obtain some more results. Further, we establish the relationship between this $S_g^*$-open sets and some nearly open sets. Besides, we prove that the class of $S_g^*$-open sets are in between the class of semi-open sets due to Levine and the class of open sets. The notion of $S_g$-closed sets are also introduced. In addition to this, we define $S_g^*$-interior and $S_g^*$-closure of a subset and discuss some of its properties.

2. PRELIMINARIES

Definition 2.1: [3]
(i) A subset $A$ of a topological space $(X, \tau)$ is semi-open if there is an open set $U$ in $X$ such that $U \subseteq A \subseteq Cl(U)$ or equivalently if $A \subseteq Cl(Int(A))$.
(ii) The complement of semi-open set is called semi-closed set.
(iii) The family of all semi-open sets in $(X, \tau)$ is denoted by $SO(X, \tau)$.
(iv) The semi-interior (briefly $sInt(A)$) of $A$ is defined as the union of all semi-open sets of $X$ contained in $A$.
(v) The semi-closure (briefly $sCl(A)$) of $A$ is defined as the intersection of all semi-open sets of $X$ containing $A$.

Definition 2.2:
(i) A subset $A$ of a space $X$ is semi-generalized closed [1] (briefly $Sg$-closed) if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi-open in $X$.
(ii) A subset $A$ of a space $X$ is semi-star generalized closed [2] (briefly $S^*g$-closed) if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi-open in $X$.
(iii) The complement of semi-generalized closed set (resp. semi-star generalized closed) is called semi-generalized open [1] (resp. semi-star generalized open [2]). It is denoted by $Sg$-open (resp. $S^*g$-open).
(iv) The semi-generalized interior [2] (briefly $sInt'(A)$) of $A$ is defined as the union of all $Sg$-open sets of $X$ contained in $A$.
(v) If $A$ is a subset of a space $X$, the semi-generalized closure [5] (briefly $sC\ell'(A)$) of $A$ is defined as the intersection of all $Sg$-closed sets in $X$ containing $A$.

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Let A and B be two subsets of a space (X,τ)
(i) A \subseteq scl'(A) \subseteq cl(A) \subseteq scl(A)
(ii) scl'(ϕ) = ϕ and scl'(X) = X
(iii) scl'(A∪B) \supset scl'(A) \cup scl'(B)
(iv) scl'\left(scl'(A)\right) = scl'(A)

Definition 2.4: [6] A subset A of a topological space (X,τ) is called a semi* -open set (briefly S*-open) if there is an open set U in X such that U \subseteq A \subseteq cl'(U).

3. S* -OPEN SETS
Definition 3.1: A subset A of a topological space (X,τ) is called a S* -open set if there is an open set U in X such that U \subseteq A \subseteq cl'(U). The collection of all S* -open sets in (X, τ) is denoted by S*O(X,τ).

Theorem 3.2: A subset A of a topological space (X,τ) is S* -open set iff A \subseteq scl'(Int(A)).
Proof: Necessity. If A is S* -open, then there exists an open set U such that U \subseteq A \subseteq scl'(U). Let U = Int U \subseteq Int (A). Then scl'(U) = scl'(Int (U)) \subseteq scl'(Int(A)). Hence A \subseteq scl'(Int(A)).

Sufficiency. Let A \subseteq scl'(Int(A)). Then for U = Int(A), we get U is an open set in X such that U \subseteq A \subseteq scl'(U).

Theorem 3.3: Let \{A_a\} be a collection of S* -open sets in a topological space X. Then \bigcup A_a is S* -open.
Proof: A_a is S* -open implies for each a, there is an open set U_a in X such that U_a \subseteq A_a \subseteq scl'(U_a). Then \bigcup U_a \subseteq \bigcup A_a \subseteq U \subseteq scl'(U_a) \subseteq scl'(\bigcup U_a). Since \bigcup U_a is open, \bigcup A_a is S* -open.

Remark 3.4: The problem of determining the intersection of S* -open sets is S* -open remains open.

Theorem 3.5: Every open set is S* -open
Proof: Let A be an open set in X. Then Int(A) = A. Hence A \subseteq scl'(Int(A)). Therefore by the necessary and sufficient condition of S* -open, A is S* -open.

Remark 3.6: The converse of Theorem 3.5 need not be true as can be seen from the following example.

Example 3.7: Let X = \{a,b,c\} and τ = \{X,ϕ,\{a\}\}. In the topological space(X, τ), the subsets \{a, b\} and \{a, c\} are S* -open but not open.

Theorem 3.8: If a subset A is S* -open and B is open, then so is A∪B.
Proof: Follows from Theorem 3.5 and Theorem 3.3.

Theorem 3.9: If A is S* -open in X and B is open in X, then A∩B is S* -open.
Proof: Since A is S* -open in X, there exists an open set U in X such that U \subseteq A \subseteq scl'(U). Now B is open implies U∩B \subseteq A∩B \subseteq scl'(U)\cap B \subseteq scl'(U∩B). Therefore A∩B is S* -open.

Theorem 3.10: Let A \subseteq B \subseteq scl'(A). If A is S* -open in the topological space X, then B is S* -open.
Proof: A is S* -open in X \Rightarrow There exists an open set U in X such that U \subseteq A \subseteq scl'(U). Therefore U \subseteq B \Rightarrow scl'(U) \subseteq scl'(B).
Hence U \subseteq B \subseteq scl'(U) and B is S* -open.

Theorem 3.11: A subset A of X is S* -open if and only if A contains a S* -open set about each of its points.

Sufficiency: Let x \in A. Then there is a S* -open set U_x containing x such that U_x \subseteq A. Then we have U \{U_x: x \in A\} = A. By Theorem 3.3, A is S* -open.
Theorem 3.12: Every $S_g^*$-open set is semi-open.

Proof: Let A be a $S_g^*$-open set. Then there exists an open set U in X such that $U \subseteq A \subseteq sCl^*(U)$. Note that $sCl^*(U) \subseteq Cl(U)$. Therefore $U \subseteq A \subseteq Cl(U)$. Hence A is semi-open.

Remark 3.13: Converse of the above Theorem 3.12 is not true as can be seen from the following example.

Example 3.14: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a, b\}\}$. In this space, $SO(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $S_g^*O(X, \tau) = \{X, \phi, \{a, b\}\}$. Here the subsets $\{a\}$ and $\{b\}$ are semi-open but not $S_g^*$-open.

Theorem 3.15: Every $S_g^*$-open set is semi-generalised open.

Proof: It follows from two facts that every $S_g^*$-open set is semi-open and every semi open set is semi-generalised open [1].

Remark 3.16: Converse of the above Theorem 3.15 is not true as can be seen from the following example.

Example 3.17: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a, b\}\}$. In this topological space $(X, \tau)$, $SgO(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $S_g^*O(X, \tau) = \{X, \phi, \{a, b\}\}$. It shows that the subsets $\{a\}$ and $\{b\}$ are semi-generalised open but not $S_g^*$-open.

Theorem 3.18: For any topological space $(X, \tau)$, $\tau \subseteq S_g^*O(X, \tau) \subseteq SO(X, \tau) \subseteq SgO(X, \tau)$

Proof: It follows from Theorem 3.5, Theorem 3.12 and Theorem 3.15.

Remark 3.19: $S_g^*$-open sets and $S^*g$-open sets are independent as shown by the following examples.

Example 3.20: Let $X = \{a, b, c, d, e\}$ and $\tau = \{X, \phi, \{a\}\}$. $S^*gO(X, \tau) = \{X, \phi, \{a\}\}$. $S_g^*O(X, \tau) = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, e\}, \{a, b, c\}, \{a, b, d\}, \{a, b, e\}, \{a, c, d\}, \{a, c, e\}, \{a, d, e\}, \{a, b, c, d\}, \{a, b, c, e\}, \{a, b, d, e\}, \{a, b, c, d, e\}\}$. Here the subsets $\{a, b\}$, $\{a, c\}$, $\{a, b, c\}$, $\{a, b, d\}$, $\{a, b, e\}$, $\{a, c, d\}$, $\{a, c, e\}$, $\{a, d, e\}$, $\{a, b, c, d\}$, $\{a, b, c, e\}$, $\{a, b, d, e\}$, $\{a, b, c, d, e\}$ and $\{a, b, c, e, d\}$ are $S_g^*$-open but not $S^*g$-open.

Example 3.21: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a, b\}\}$. $S^*gO(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. $S_g^*O(X, \tau) = \{X, \phi, \{a, b\}\}$. Here the subsets $\{a\}$ and $\{b\}$ are $S^*g$-open but not $S_g^*$-open.

Remark 3.22: $S_g^*$-open sets and $S^*$-open sets are independent as shown by the following examples.

Example 3.23: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}\}$. $S^*O(X, \tau) = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$. The subsets $\{a, b\}$ and $\{a, c\}$ are $S_g^*$-open but not $S^*$-open.

Example 3.24: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a, c\}\}$. $S^*O(X, \tau) = \{X, \phi, \{a\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. The subsets $\{a, b\}$ and $\{b, c\}$ are $S^*$-open but not $S_g^*$-open.

Remark 3.25: From the above Theorems and Remarks, we have the following diagram.

![Diagram](image)

Definition 3.26: The $S_g^*$-interior of A is defined as the union of all $S_g^*$-open sets of X contained in A. It is denoted by $s_g^*Int(A)$.

Definition 3.27: Let A be a subset of X. A point $x \in X$ is called $S_g^*$-interior point of A if A contains a $S_g^*$-open set containing x.
Theorem 3.32: If A is any subset of X, $S^*_g \text{Int}(A)$ is $S^*_g$-open. In fact $S^*_g \text{Int}(A)$ is the largest $S^*_g$-open set contained in A.

Proof: Follows from Definition 3.26 and Theorem 3.3.

Theorem 3.29: A subset A of X is $S^*_g$-open if and only if $S^*_g \text{Int}(A)=A$.

Proof: If A is $S^*_g$-open then $S^*_g \text{Int}(A)=A$ is obvious. Conversely, let $S^*_g \text{Int}(A)=A$. By Theorem 3.28, $S^*_g \text{Int}(A)$ is $S^*_g$-open and hence A is $S^*_g$-open.

Theorem 3.30: If A is a subset of X, then $S^*_g \text{Int}(A)$ is the set of all $S^*_g$-interior points of A.

Proof: If $x \in S^*_g \text{Int}(A)$, then x belongs to some $S^*_g$-open subset U of A. That is, x is a $S^*_g$-interior point of A.

Corollary 3.31: A subset A of X is $S^*_g$-open if and only if every point of A is a $S^*_g$-interior point of A.

Proof: It follows from Theorem 3.29 and Theorem 3.30.

Theorem 3.32: If A and B are subsets of a topological space $(X, \tau)$, then the following results hold:
(i) $S^*_g \text{Int}(\emptyset)=\emptyset$.
(ii) $S^*_g \text{Int}(X)=X$.
(iii) $S^*_g \text{Int}(A)\subseteq A$.
(iv) $A \subseteq B \implies S^*_g \text{Int}(A) \subseteq S^*_g \text{Int}(B)$.
(v) $\text{Int}(A) \subseteq S^*_g \text{Int}(A) \subseteq \text{Int}(A) \subseteq S^*_g \text{Int}(A) \subseteq \text{Int}(A)$.
(vi) $S^*_g \text{Int}(A \cup B) \supseteq S^*_g \text{Int}(A) \cup S^*_g \text{Int}(B)$.
(vii) $S^*_g \text{Int}(A \cap B) \supseteq S^*_g \text{Int}(A) \cap S^*_g \text{Int}(B)$.
(viii) $S^*_g \text{Int}(S^*_g \text{Int}(A))=S^*_g \text{Int}(A)$.
(ix) $\text{Int}(S^*_g \text{Int}(A))= S^*_g \text{Int}(A)$.
(x) $S^*_g \text{Int}(\text{Int}(A))=\text{Int}(A)$.

Proof: (i), (ii), (iii) and (iv) follows from Definition 3.26.
(v) follows from Theorem 3.18.
(vi) and (vii) follow from (iv) above.
(viii) and (ix) follows from Theorem 3.28 and Theorem 3.29.
(x) $\text{Int}(A)$ is open which implies $\text{Int}(A)$ is $S^*_g$-open. Now by Theorem 3.29, $S^*_g \text{Int}(\text{Int}(A))=\text{Int}(A)$.

4. $S^*_g$-CLOSED SETS

Definition 4.1: A subset A of a topological space $(X, \tau)$ is called a $S^*_g$-closed set if $X \setminus A$ is $S^*_g$-open. The collection of all $S^*_g$-closed sets in $(X, \tau)$ is denoted by $S^*_g \text{C}(X, \tau)$.

Theorem 4.2: A subset A of a topological space $(X, \tau)$ is $S^*_g$-closed set if and only if there exists a closed set F in X such that $s\text{Int}^*(F) \subseteq A \subseteq F$.

Proof: Necessity. A is $S^*_g$-closed $\implies X \setminus A$ is $S^*_g$-open. By Definition 3.1, there exists an open set U in X such that $U \subseteq X \setminus A \subseteq s\text{Cl}^*(U)$. This implies $X \setminus U \supseteq A \supseteq X \setminus s\text{Cl}^*(U)$. Since $X \setminus s\text{Cl}^*(U) = s\text{Int}^*(X \setminus U)$, $s\text{Int}^*(X \setminus U) \subseteq A \subseteq X \setminus U$ where $X \setminus U$ is a closed set in X.

Sufficiency. Suppose there exists a closed set F in X such that $s\text{Int}^*(F) \subseteq A \subseteq F$. Then $X \setminus s\text{Int}^*(F) \supseteq X \setminus A \supseteq X \setminus F$. Note that $X \setminus s\text{Int}^*(F) = s\text{Cl}^*(X \setminus F)$. Hence $X \setminus F \subseteq X \setminus A \subseteq s\text{Cl}^*(X \setminus F)$ where $X \setminus F$ is an open set in X. This implies $X \setminus A$ is a $S^*_g$-open set and A is a $S^*_g$-closed set.

Theorem 4.3: A subset A of a topological space $(X, \tau)$ is $S^*_g$-closed set iff $s\text{Int}^*(\text{Cl}(A)) \subseteq A$.

Proof: Necessity. If A is a $S^*_g$-closed set, then by Theorem 4.2 there exists a closed set F such that $s\text{Int}^*(F) \subseteq A \subseteq F$. Take $F = \text{Cl}(F)$. Then $\text{Cl}(A) \subseteq \text{Cl}(F) \implies s\text{Int}^*(\text{Cl}(A)) \subseteq s\text{Int}^*(\text{Cl}(F)) \subseteq s\text{Int}^*(F)$. Hence $s\text{Int}^*(\text{Cl}(A)) \subseteq A$.

Sufficiency. Let $s\text{Int}^*(\text{Cl}(A)) \subseteq A$. Then for $F = \text{Cl}(A)$, we get F is a closed set in X such that $s\text{Int}^*(F) \subseteq A \subseteq F$. Hence by Theorem 4.2, A is $S^*_g$-closed in X.

Theorem 4.4: If $\{A_n\}$ is a collection of $S^*_g$-closed sets in a topological space X, then $\bigcap A_n$ is $S^*_g$-closed.
Proof: $A_d$ is $S^*_g$-closed in $X$ if $X \setminus A_d$ is $S^*_g$-open in $X$. By Theorem 3.3, $\bigcup X \setminus A_d$ is $S^*_g$-open in $X$. Hence $X \setminus A_d$ is $S^*_g$-closed.

Remark 4.5: The problem of determining the union of $S^*_g$-closed sets is $S^*_g$-closed remains open.

Theorem 4.6: Suppose $A$ is $S^*_g$-closed in $X$ and $s\text{Int}^*(A) \subseteq B \subseteq A$. Then $B$ is $S^*_g$-closed in $X$.

Proof: $A$ is $S^*_g$-closed in $X$ implies $X \setminus A$ is $S^*_g$-open in $X$. $s\text{Int}^*(A) \subseteq B \subseteq A$ implies $X \setminus X \setminus B \supseteq X \setminus A \supseteq X \setminus B$. By Theorem 3.10, $X \setminus B$ is $S^*_g$-open in $X$. Hence $B$ is $S^*_g$-closed in $X$.

Theorem 4.7: Every closed set is $S^*_g$-closed.

Proof: Let $A$ be a closed set in $X$. Then $X \setminus A$ is open in $X$. By Theorem 3.5, $X \setminus A$ is $S^*_g$-open. Hence $A$ is $S^*_g$-closed.

Remark 4.8: The converse of Theorem 4.7 need not be true as shown in the following example.

Example 4.9: Let $X = \{a, b, c, d, e\}$ and $F = \{X, d, \{c, d, e\},\{d, e\},\{c, d, e\}\}$. Here the subsets $\{c\}, \{d\}, \{e\}, \{c, d\}, \{c, e\}, \{d, e\}$ are $S^*_g$-closed but not $S^*_g$-closed.

Theorem 4.10: If a subset $A$ is $S^*_g$-closed and $B$ is closed, then $A \cap B$ is $S^*_g$-closed.

Proof: This follows from Theorem 4.7 and Theorem 4.4.

Theorem 4.11: If $A$ is $S^*_g$-closed in $X$ and $B$ is closed in $X$, then $A \cup B$ is $S^*_g$-closed.

Proof: $A$ is $S^*_g$-closed in $X$ implies $X \setminus A$ is $S^*_g$-open in $X$. Also $B$ is closed in $X$ implies $X \setminus B$ is open in $X$. By Theorem 3.9, $X \setminus A \cup X \setminus B$ is $S^*_g$-open in $X$. Note $X \setminus A \cap X \setminus B = (X \setminus A) \setminus (B \setminus A)$. Therefore $X \setminus (A \cup B)$ is $S^*_g$-open and so $A \cup B$ is $S^*_g$-closed.

Theorem 4.12: Every $S^*_g$-closed set is semi-closed.

Proof: Let $A$ be a $S^*_g$-closed set. Then $X \setminus A$ is $S^*_g$-open. Now by Theorem 3.12, $X \setminus A$ is semi-open which implies $A$ is semi-closed.

Remark 4.13: Converse of the above Theorem 4.12 is not true as can be seen from the following example.

Example 4.14: Let $X = \{a, b, c, d\}$ and $\tau = \{X, \{a\}, \{b, c\}, \{a, b, c\}\}$. In this space, $S(C(X, r)) = \{X, \emptyset, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, c, d\}\}$. Here the subsets $\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}$ and $\{b, c\}$ are semi-closed but not $S^*_g$-closed.

Theorem 4.15: Every $S^*_g$-closed set is semi-generalised closed.

Proof: It follows from two facts that every $S^*_g$-closed set is semi-closed and every semi-closed set is semi-generalised closed [1].

Remark 4.16: Converse of the above Theorem 4.15 is false as shown in the following example.

Example 4.17: Let $X = \{a, b, c, d\}$ and $\tau = \{X, \{a\}, \{b, c\}, \{a, b, c\}\}$. In this space, $Sg(C(X, r)) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}\}$. It shows that the subsets $\{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}$ and $\{d\}$ are semi-generalized closed but not $S^*_g$-closed.

Remark 4.17: The concept of $S^*_g$-closed sets and $S^*g$-closed sets are independent as shown by the following examples.

Example 4.18: Let $X = \{a, b, c, d, e\}$ and $\tau = \{X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, c, d\}\}$. Here the subsets $\{c\}, \{d\}, \{a, c\}, \{a, d\}, \{a, c, d\}, \{a, c, e\}$ and $\{a, d, e\}$ are $S^*_g$-closed but not $S^*g$-closed.
Example 4.19: Let X = {a, b, c} and \( \tau = \{ X, \phi, \{ a, b \} \} \). \( S^*gC(X, \tau) = \{ X, \phi, \{ c \}, \{ a, c \} \} \). Here the subsets \{b, c\} and \{a, c\} are \( S^*g \)-closed but not \( S^g \)-closed.

Remark 4.20: \( S^g \)-closed sets and \( S^* \)-closed sets are independent as shown by the following examples.

Example 4.21: Let X = {a, b, c, d, e} and \( \tau = \{ X, \phi, \{ a \} \} \). \( S^*g C(X, \tau) = \{ X, \phi, \{ a, b, c, d, e \} \} \). Here the subsets \{b, c\}, \{d\}, \{e\}, \{a, b, c\}, \{a, d\}, \{b, c, d\}, \{a, b, c, d\}, \{a, b, d\}, \{b, c, d\}, \{a, b, c, d, e\} are \( S^g \)-closed but not \( S^* \)-closed.

Example 4.22: Let X = {a, b, c, d} and \( \tau = \{ X, \phi, \{ a, b, c, d \} \} \). In this space, \( S^*g C(X, \tau) = \{ X, \phi, \{ a, b, c, d \} \} \) and \( S^g C(X, \tau) = \{ X, \phi, \{ a, b, c, d \} \} \). Here the subsets \{a, b, c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\} are \( S^g \)-closed but not \( S^* \)-closed.

Theorem 4.23: Let \((X, \tau)\) be any topological space and \( \mathcal{F} \) be the family of all closed sets, then \( \mathcal{F} \subseteq S^g C(X, \tau) \subseteq SC(X, \tau) \subseteq S^g C(X, \tau) \).

Proof: It follows from Theorem 4.7, Theorem 4.12 and Theorem 4.15.

Remark 4.24: From the above discussions we have the following figure.

\[ \text{S}^* \text{-closed} \quad \text{Closed} \quad S^g \text{-closed} \]

\[ \text{S}^g \text{-closed} \quad \text{Semi-closed} \quad \text{S}^* \text{-closed} \]

Definition 4.25: The \( S^g \)-closure of A is defined as the intersection of all \( S^g \)-closed sets of X containing A. It is denoted by \( s^g Cl(A) \).

Theorem 4.26: If A is any subset of X, \( s^g Cl(A) \) is \( S^g \)-closed. In fact \( s^g Cl(A) \) is the smallest \( S^g \)-closed set in X containing A.

Proof: Follows from Definition 4.25 and Theorem 4.4.

Theorem 4.27: A subset A of X is \( S^g \)-closed iff \( s^g Cl(A) = A \).

Proof: If A is \( S^g \)-closed then \( s^g Cl(A) = A \) is obvious. Conversely, suppose \( s^g Cl(A) = A \). By Theorem 4.26, \( s^g Cl(A) \) is \( S^* \)-closed and hence A is \( S^g \)-closed.

Theorem 4.28: If A and B be subsets of a topological space \((X, \tau)\), then the following results hold:

(i) \( s^g Cl(\phi) = \phi \).

(ii) \( s^g Cl(X) = X \).

(iii) \( A \subseteq s^g Cl(A) \).

(iv) \( A \subseteq B \Rightarrow s^g Cl(A) \subseteq s^g Cl(B) \).

(v) \( A \subseteq Cl'(A) \subseteq Cl(A) \subseteq s^g Cl(A) \subseteq Cl(A) \).

(vi) \( s^g Cl(A \cup B) = s^g Cl(A) \cup s^g Cl(B) \).

(vii) \( s^g Cl(A \cap B) = s^g Cl(A) \cap s^g Cl(B) \).

(viii) \( s^g Cl(s^g Cl(A)) = s^g Cl(A) \).

(ix) \( Cl(s^g Cl(A)) = Cl(A) \).

(x) \( s^g Cl(Cl(A)) = Cl(A) \).

Proof: (i), (ii), (iii) and (iv) follows from Definition 4.25.

(v) Follows from Result 2.3, Theorem 4.7, Theorem 4.12 and Theorem 4.15.
(vi) and (vii) follow from (iv) above.
(viii) and (ix) follows from Theorem 4.26 and Theorem 4.27.
(x) Since \( Cl(A) \) being closed, \( Cl(A) \) is \( S^*_g \)-closed. Hence by Theorem 4.27, \( s^*_g Cl(Cl(A)) = Cl(A) \).

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