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A NEW NOTION OF OPEN SETS IN TOPOLOGICAL SPACES

S. Pious Missier[1] & J. Arul Jesti[2*]
[1]PG and Research Department of Mathematics, V.O. Chidambaram College, Thoothukudi, India
[2]Research Scholar, PG and Research Department of Mathematics,
V.O. Chidambaram College, Thoothukudi, India

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ABSTRACT

The determination of this paper is to introduce the new family of sets, namely S_g^*-open sets and S_g^*-closed sets. Further we define S_g^*-interior and S_g^*-closure and discuss its properties. Additionally we relate S_g^*-open sets and S_g^*-closed sets with some other sets.

Mathematics Subject Classification: *54A05.*

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1. INTRODUCTION

Throughout this paper, we denote a topological space by (X,τ). For a subset A of (X,τ),the closure and interior of A with respect to τ are denoted by *Cl*(A) and *Int*(A) respectively. The concept of semi-open sets[3]and generalized closed(briefly g-closed) sets[4] were introduced by Norman Levine in 1963 and 1970 respectively. After the works of Levine , in the year 1987, P. Battacharyya and B.K. Lahiri[1] introduced the concept of semi-generalized closed sets with the help of semi-openness. Recently, A.Robert and S. Pious Missier[6] have introduced the concept of semi*-open sets and studied their properties.

In this direction, we shall introduce a new family of sets called S_g^*-open sets, using the semi-generalized closure operator. This definition enables us to obtain some more results. Further we establish the relationship between this S_g^*-open sets and some nearly open sets. Besides we prove the class of S_g^*-open sets are inbetween the class of semi-open sets due to Levine and the class of open sets. The notion of S_g^*-closed sets are also introduced. In addition to this, we define S_g^*-interior and S_g^*-closure of a subset and discuss some of its properties.

2. PRELIMINARIES

Definition 2.1: [3]

(i) A subset A of a topological space (X, τ) is ***semi-open*** if there is an open set U in X such that U⊆A⊆*Cl* (U) or equivalently if A⊆*Cl*(*Int*(A)).
(ii) The complement of semi-open set is called ***semi-closed set.***
(iii)The family of all semi-open sets in (X, τ) is denoted by SO(X, τ).
(iv)The ***semi-interior*** (briefly *sInt*(A)) of A is defined as the union of all semi-open sets of X contained in A.
(v) The ***semi-closure*** (briefly *sCl*(A)) of A is defined as the intersection of all semi-open sets of X containing A.

Definition 2.2:

(i) A subset A of a space X is ***semi-generalized closed*** [1](briefly Sg-closed)if s*Cl*(A)⊆U whenever A⊆U and U is semi-open in X.
(ii) A subset A of a space X is ***semi-star generalized closed*** [2](briefly S^*g-closed)) if *Cl*(A)⊆U whenever A⊆U and U is semi-open in X.
(ii) The complement of semi-generalized closed set(resp. semi-star generalized closed) is called ***semi-generalized open***[1](resp. semi star-generalized open[2]). It is denoted by Sg-open (resp. S^*g-open).
(iv) The ***semi-generalized interior***[2](briefly *sInt**(A)) of A is defined as the union of all Sg -open sets of X contained in A.
(iii) If A is a subset of a space X, the ***semi-generalized closure*** [5] (briefly sCl^*(A)) of A is defined as the intersection of all Sg-closed sets in X containing A.

Corresponding author: *J. Arul Jesti[2*],*
Research Scholar, PG and Research Department of Mathematics, V.O. Chidambaram College, Thoothukudi, India

Results 2.3:[5] Let A and B be two subsets of a space (X,τ)
(i)$A \subseteq sCl^*(A) \subseteq sCl(A) \subseteq Cl(A)$
(ii)$sCl^*(\phi) = \phi$ and $sCl^*(X) = X$
(iii) $sCl^*(A \cup B) \supset sCl^*(A) \cup sCl^*(B)$
(iv) $sCl^*\big(sCl^*(A)\big) = sCl^*(A)$

Definition 2.4: [6] A subset A of a topological space (X,τ) is called a ***semi*-open set*** (briefly S^*-open) if there is an open set U in X such that U⊆A⊆ Cl^*(U).

3.S_g^*-OPEN SETS

Definition 3.1: A subset A of a topological space (X,τ) is called a ***S_g^*-open set*** if there is an open set U in X such that U⊆A⊆ sCl^*(U). The collection of all S_g^*-open sets in (X, τ) is denoted by S_g^*O(X,τ).

Theorem 3.2: A subset A of a topological space (X,τ) is S_g^*-open set iff A ⊆ sCl^*(*Int(*A)).

Proof: Necessity. If A is S_g^*-open, then there exists an open set U⊆A⊆ sCl^*(U). Let U=*Int* U⊆*Int (*A). Then sCl^*U=sCl^*(*Int* (U))⊆ sCl^*(*Int(*A)). Hence A⊆ sCl^*(*Int(*A)).

Sufficiency. Let A⊆ sCl^*(*Int(*A)). Then for U= *Int(*A), we get U is an open set in X such that U⊆A⊆ sCl^*(U).

Theorem 3.3: Let $\{A_\alpha\}$ be a collection of S_g^*-open sets in a topological space X. Then $\cup A_\alpha$ is S_g^*-open.

Proof: A_α is S_g^*-open implies for each α, there is an open set U_α in X such that $U_\alpha \subseteq A_\alpha \subseteq sCl^*(U_\alpha)$. Then
$\cup U_\alpha \subseteq \cup A_\alpha \subseteq \cup scl^*(U_\alpha) \subseteq sCl^*(\cup U_\alpha)$. Since $\cup U_\alpha$ is open, $\cup A_\alpha$ is S_g^*-open.

Remark 3.4: The problem of determining the intersection of S_g^*-open sets is S_g^*-open remains open.

Theorem 3.5: Every open set is S_g^*-open

Proof: Let A be an open set in X. Then Int(A)=A. Hence A⊆ sCl^*(A)= sCl^*(*Int*(A)).Therefore by the necessary and sufficient condition of S_g^*-open, A is S_g^*-open.

Remark 3.6: The converse of Theorem 3.5 need not be true as can be seen from the following example.

Example 3.7: Let X={a,b,c} and τ ={X,ϕ,{a}}. In the topological space(X, τ), the subsets {a, b} and {a, c} are S_g^*-open but not open.

Theorem 3.8: If a subset A is S_g^*-open and B is open, then so is A⋃B.

Proof: Follows from Theorem 3.5 and Theorem 3.3.

Theorem 3.9: If A is S_g^*-open in X and B is open in X, then A⋂B is S_g^*-open.

Proof: Since A is S_g^*-open in X, there exists an open set U in X such that U⊆A⊆ sCl^*(U). Now B is open implies U⋂B ⊆A⋂B⊆ sCl^*(U)⋂B⊆ sCl^*(U⋂B). Therefore A⋂B is S_g^*-open.

Theorem 3.10: Let A⊆ $B \subseteq sCl^*$(A). If A is S_g^*-open in the topological space X, then B is S_g^*-open.

Proof: A is S_g^*-open in X⟹ There exists an open set U in X such that U⊆A⊆ sCl^*(U). Therefore
U ⊆ $B \Longrightarrow sCl^*$(U) ⊆ sCl^*(B).

Hence U⊆B⊆ sCl^*(U) and B is S_g^*-open.

Theorem 3.11: A subset A of X is S_g^*-open if and only if A contains a S_g^*-open set about each of its points.

Proof: ***Necessity:*** Obvious.

***Sufficiency*:** Let x∈A. Then there is a S_g^*-open set U_x containing x such that U_x ⊆A. Then we have ∪{U_x: x∈A}=A. By Theorem 3.3, A is S_g^*-open.

Theorem 3.12: Every S_g^*-open set is semi-open.

Proof: Let A be a S_g^*-open set. Then there exists an open set U in X such that U⊆A⊆ sCl^*(U). Note that sCl^*(U) ⊆ sCl(U) ⊆ Cl(U). Therefore U⊆A⊆ Cl(U). Hence A is semi-open.

Remark 3.13: Converse of the above Theorem 3.12 is not true as can be seen from the following example.

Example 3.14: Let X= {a, b, c} and τ ={X,ϕ,{a},{c},{a,c}}. In this space, SO(X, τ)={X, ϕ,{a},{c},{a,c},{a,b},{b,c}} and $S_g^*O(X, \tau) = \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$. Here the subsets {a, b} and {b, c} are semi-open but not S_g^*-open.

Theorem 3.15: Every S_g^*-open set is semi-generalised open.

Proof: It follows from two facts that every S_g^*-open set is semi-open and every semi open set is semi-generalised open [1].

Remark 3.16: Converse of the above Theorem 3.15 is not true as can be seen from the following example.

Example 3.17: Let X= {a, b, c} and τ ={X,ϕ, {a, b}}. In this topological space (X, τ), SgO(X, τ)={X, ϕ,{a},{b},{a, b}} and $S_g^*O(X, \tau) = \{X, \phi, \{a, b\}\}$. It shows that the subsets {a} and {b} are semi-generalised open but not S_g^*-open.

Theorem 3.18: For any topological space (X, τ), $\tau \subseteq S_g^*O(X, \tau) \subseteq SO(X, \tau) \subseteq SgO(X, \tau)$

Proof: It follows from Theorem 3.5, Theorem 3.12 and Theorem 3.15.

Remark 3.19: S_g^*-open sets and S^*g-open sets are independent as shown by the following examples.

Example 3.20: Let X= {a, b, c, d, e } and τ ={X, ϕ,{a}}. S^*gO(X, τ) = {X, ϕ,{a}}. S_g^*O(X, τ) = {X, ϕ,{a},{a, b},{a, c},{a, d},{a, e}, {a,b,c},{a,b,d},{a,b,e},{a,c,d},{a,c,e},{a,d,e},{a,b,c,d},{a,b,d,e}, {a, c, d, e},{a, b, c, e}}. Here the subsets {a, b},{a, c},{a, d},{a, e}, {a, b, c},{a, b, d},{a, b, e},{a, c, d}, {a, c, e},{a, d, e},{a, b, c, d},{a, b, d, e},{a, c, d, e} and {a, b, c, e}} are S_g^*-open but not S^*g -open.

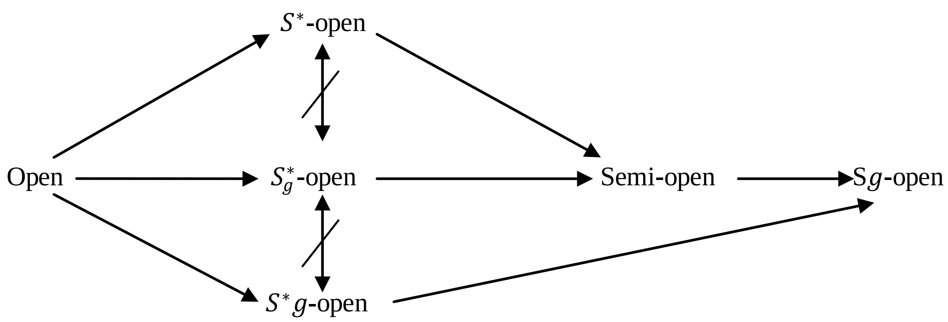
Example 3.21: Let X= {a, b, c} and τ ={X, ϕ, {a,b}}. S^*gO(X, τ) = {X, ϕ,{a},{b},{a,b}}. S_g^*O(X, τ) = {X, ϕ,{a,b}}. Here the subsets {a} and {b} are S^*g -open but not S_g^*-open.

Remark 3.22: S_g^*-open sets and S^*-open sets are independent as shown by the following examples.

Example 3.23: Let X= {a, b, c} and τ ={X, ϕ, {a}}=S^*O(X, τ). S_g^*O(X, τ) ={X, ϕ,{a},{a,b}, {a,c}}. The subsets {a,b} and {a, c} are S_g^*-open but not S^*-open.

Example 3.24: Let X={a, b, c} and τ ={X, ϕ,{a},{c},{a,c}}=S_g^*O(X, τ). S^*O(X, τ) = X, ϕ,{a},{c},{a,b},{b,c},{a,c}}. The subsets {a, b} and {b, c} are S^*-open but not S_g^*-open

Remark 3.25: From the above Theorems and Remarks, we have the following diagram.



Definition 3.26: The ***S_g^*-interior*** of A is defined as the union of all S_g^*-open sets of X contained in A. It is denoted by s_g^*Int(A).

Definition 3.27: Let A be a subset of X. A point x∈X is called ***S_g^*-interior point*** of A if A contains a S_g^*-open set containing x.

Theorem 3.28: If A is any subset of X, S_g^*Int(A) is S_g^*-open. In fact s_g^*Int(A) is the largest S_g^*-open set contained in A.

Proof: Follows from Definition 3.26 and Theorem 3.3.

Theorem 3.29: A subset A of X is S_g^*-open if and only if s_g^*Int(A)=A.

Proof: If A is S_g^*-open then s_g^*Int(A)=A is obvious. Conversely, let s_g^*Int(A)=A. By Theorem 3.28, s_g^*Int(A) is S_g^*-open and hence A is S_g^*-open.

Theorem 3.30: If A is a subset of X, then s_g^*Int (A) is the set of all S_g^*-interior points of A.

Proof: If x∈S_g^*Int (A), then x belongs to some S_g^*-open subset U of A. That is, x is a S_g^*-interior point of A.

Corollary 3.31: A subset A of X is S_g^*-open if and only if every point of A is a S_g^*- interior point of A.

Proof: It follows from Theorem 3.29 and Theorem 3.30.

Theorem 3.32: If A and B are subsets of a topological space (X,τ), then the following results hold:
(i) s_g^*Int(ϕ)=ϕ.
(ii) s_g^*Int(X)=X.
(iii) s_g^*Int(A)⊆A.
(iv) A⊆B ⟹ s_g^*Int (A)⊆ s_g^*Int(B).
(v) Int(A)⊆ s_g^*Int(A)⊆ $sInt$(A)⊆$sInt^*$(A) ⊆A.
(vi) s_g^*Int(A∪B)⊇ s_g^*Int(A)∪ s_g^*Int(B).
(vii) s_g^*Int(A∩B)⊆ s_g^*Int(A)∩ s_g^*Int(B).
(viii) s_g^*Int(s_g^*Int(A))= s_g^*Int(A).
(ix) Int(s_g^*Int(A))=Int(A).
(x) s_g^*Int(Int(A))=Int(A).

Proof: (i), (ii), (iii) and (iv) follows from Definition 3.26.
(v) follows from Theorem 3.18.
(vi) and (vii) follow from (iv) above.
(viii) and (ix) follows from Theorem 3.28 and Theorem 3.29.
(x) Int(A) is open which implies Int(A) is S_g^*-open. Now by Theorem 3.29, s_g^*Int(Int(A))=Int(A).

4. S_g^*-CLOSED SETS

Definition 4.1: A subset A of a topological space (X,τ) is called a ***S_g^*-closed set*** if X\A is S_g^*-open. The collection of all S_g^*-closed sets in (X, τ) is denoted by S_g^*C(X,τ).

Theorem 4.2: A subset A of a topological space (X,τ) is S_g^*-closed set if and only if there exists a closed set F in X such that $sInt^*$(F)⊆A⊆F.

Proof: Necessity. A is S_g^*-closed⟹X\A is S_g^*-open. By Definition 3.1, there exists an open set U in X such that U⊆X\A⊆ sCl^*(U). This implies X\U⊇A⊇X\$sCl^*$(U). Since X\$sCl^*$(U)= $sInt^*$(X\U), $sInt^*$(X\U) ⊆A⊆X\U where X\U is a closed set in X.

Sufficiency. Suppose there exists a closed set F in X such that $sInt^*$(F)⊆A⊆F. Then X\$sInt^*$(F)⊇X\A⊇X\F. Note that X\$sInt^*$(F)= sCl^*(X\F). Hence X\F⊆X\A⊆ sCl^*(X\F) where X\F is an open set in X. This implies X\A is a S_g^*-open set and A is a S_g^*-closed set.

Theorem 4.3: A subset A of a topological space (X,τ) is S_g^*-closed set iff $sInt^*$(Cl(A)) ⊆A.

Proof: Necessity. If A is a S_g^*-closed set, then by Theorem 4.2 there exists a closed set F such that $sInt^*$(F)⊆A⊆F. Take F=Cl(F). Then Cl(A)⊆Cl(F)⟹ $sInt^*$(Cl(A))⊆ $sInt^*$(Cl(F))= $sInt^*$(F). Hence $sInt^*$(Cl(A)) ⊆A.

Sufficiency. Let $sInt^*$(Cl(A)) ⊆A. Then for F= Cl(A), we get F is a closed set in X such that $sInt^*$(F)⊆A⊆F. Hence by Theorem 4.2, A is S_g^*-closed in X.

Theorem 4.4: If $\{A_\alpha\}$ is a collection of S_g^*-closed sets in a topological space X, then $\cap A_\alpha$ is S_g^*-closed.

Proof: A_α is S_g^*-closed in X⟹ X\$A_\alpha$ is S_g^*-open in X⟹ By Theorem 3.3,⋃X\$A_\alpha$ is S_g^*-open in X⟹ X\⋂A_α is S_g^*-open in X⟹⋂A_α is S_g^*-closed.

Remark 4.5: The problem of determining the union of S_g^*-closed sets is S_g^*-closed remains open.

Theorem 4.6: Suppose A is S_g^*-closed in X and $sInt^*$(A)⊆B⊆A. Then B is S_g^*-closed in X.

Proof: A is S_g^*-closed in X implies X\A is S_g^*-open in X. $sInt^*$(A)⊆B⊆A⟹X\$sInt^*$(A)⊇X\B⊇ X\A⟹ X\A⊆X\B⊆ sCl^*(X\A). Now by Theorem 3.10, X\B is S_g^*-open in X. Hence B is S_g^*-closed in X.

Theorem 4.7: Every closed set is S_g^*-closed.

Proof: Let A be a closed set in X. Then X\A is open in X. By Theorem 3.5, X\A is S_g^*-open. Hence A is S_g^*-closed.

Remark 4.8: The converse of Theorem 4.7 need not be true as shown in the following example.

Example 4.9: Let X= {a, b, c, d, e} and $\mathcal{F}$ ={X,ϕ,{c, d, e}}. S_g^*C(X,τ)={X, ϕ,{c},{d},{e},{c,d},{c,e},{d,e},{c,d,e}}. Here the subsets {c},{d},{e},{c, d},{c, e} and {d, e} are S_g^*-closed set but not closed.

Theorem 4.10: If a subset A is S_g^*-closed and B is closed, then A⋂B is S_g^*-closed.

Proof: This follows from Theorem 4.7 and Theorem 4.4.

Theorem 4.11: If A is S_g^*-closed in X and B is closed in X, then A⋃B is S_g^*-closed.

Proof: A is S_g^*-closed in X implies X\A is S_g^*-open in X. Also B is closed in X implies X\B is -open in X. Hence by Theorem 3.9, X\A⋂X\B is S_g^*-open in X. Note X\A⋂X\B=X\(A⋃B). Therefore X\(A⋃B) is S_g^*-open and so A⋃B is S_g^*-closed.

Theorem 4.12: Every S_g^*-closed set is semi-closed.

Proof: Let A be a S_g^*-closed set. Then X\A is a S_g^*-open set. Now by Theorem 3.12, X\A is semi-open which implies A is semi-closed.

Remark 4.13: Converse of the above Theorem 4.12 is not true as can be seen from the following example.

Example 4.14: Let X= {a, b, c, d} and τ ={X,ϕ,{a},{b},{c},{a, b},{a, c},{b, c},{a, b, c}}. In this space, SC(X, τ)={X, ϕ,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,d},{a,c,d},{b,c,d}}andS_g^*C(X, τ)={X, ϕ,{d}, {a, d}, {b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}}. Here the subsets {a},{b},{c},{a, b},{a, c} and {b, c} are semi-closed but not S_g^*-closed.

Theorem 4.15: Every S_g^*-closed set is semi-generalised closed.

Proof: It follows from two facts that every S_g^*-closed set is semi-closed and every semi-closed set is semi-generalised closed [1].

Remark 4.16: Converse of the above Theorem 4.15 is false as shown in the following example.

Example 4.17: Let X={a, b, c, d} and τ ={X,ϕ,{b},{c},{b, c}}. In this space, SgC(X, τ)={X, ϕ,{a},{b},{c},{d},{a, b},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, d},{a, c, d}} and S_g^*C(X, τ)={X, ϕ,{a},{d},{a, d},{a, b, d},{a, c, d}}. It shows that the subsets {b},{c},{a, b},{a, c},{b, c},{b, d} and {c, d} are semi-generalized closed but not S_g^*-closed.

Remark 4.17: The concept of S_g^*-closed sets and S^*g-closed sets are independent as shown by the following examples.

Example 4.18: Let X= {a, b, c, d, e} and τ ={X, ϕ,{a},{b},{a, b},{b, c, d, e}}. S^*gC(X, τ) = {X, ϕ,{a},{c, d, e},{b, c, d, e},{a, c, d, e}}. S_g^*C(X, τ) = {X, ϕ,{a},{c},{d},{e},{a, c},{a, d},{a, e},{c, d},{c, e},{d, e},{a, c, d}, {a, c, e} ,{a, d, e},{c, d, e}, {a, c, d, e},{b, c, d, e}}. Here the subsets {c},{d},{e},{a, c},{a, d},{a, e},{c, d},{c, e},{d, e},{a, c, d},{a, c, e} and {a, d, e}are S_g^*-closed but not S^* g-closed.

Example 4.19: Let X= {a, b, c} and τ ={X, ϕ, {a, b}}. S^*gC(X, τ)={X, ϕ,{c},{b, c},{a, c}}. S_g^*O(X, τ) = {X, ϕ,{c}}. Here the subsets {b, c} and {a, c} are S^*g-closed but not S_g^*-closed.

Remark 4.20: S_g^*-closed sets and S^*-closed sets are independent as shown by the following examples.

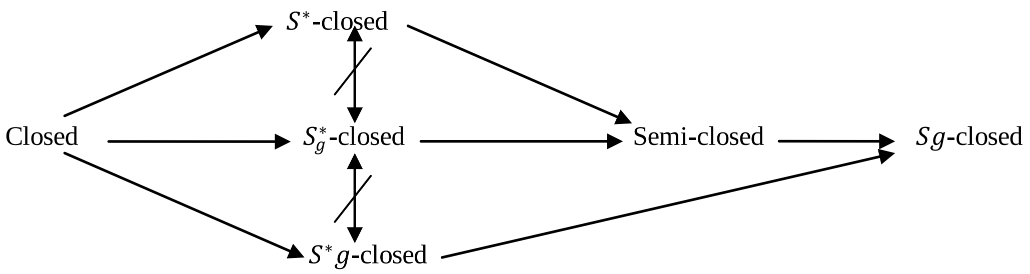
Example 4.21: Let X= {a, b, c, d, e} and τ ={X, ϕ,{a}}. S^*C(X, τ)={X, ϕ,{b,c,d,e}}. S_g^*O(X, τ) = {X, ϕ, {b},{c},{d},{e},{b, c}, {b, d},{b, e},{c, d},{c, e},{d, e},{b, c, d},{b, c, e},{b, d, e},{c, d, e},{b, c, d, e}}. Here the subsets {b},{c},{d},{e},{b, c}, {b, d},{b, e}, {c, d},{c, e},{d, e},{b, c, d},{b, c, e},{b, d, e} and {c, d, e} are S_g^*-closed but not S^*-closed.

Example 4.22: Let X= {a, b, c, d} and τ ={X,ϕ,{a},{b},{c},{a, b},{a, c},{b, c},{a, b, c}}. In this space, S^*C(X, τ)={X, ϕ,{a},{b},{c},{d},{a, b},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}} and S_g^*C(X, τ)={X, ϕ,{d},{a,d},{b,d},{c,d},{a,b,d},{a,c,d},{b,c,d}}. Here the subsets {a},{b},{c},{a,b},{a,c} and {b,c} are S^*-closed but not S_g^*-closed.

Theorem 4.23: Let (X, τ) be any topological space and $\mathcal{F}$ be the family of all closed sets, then $\mathcal{F} \subseteq S_g^*$C(X , τ) $\subseteq$ SC(X, τ) $\subseteq$ SgC(X, τ)

Proof: It follows from Theorem 4.7, Theorem 4.12 and Theorem 4.15.

Remark 4.24: From the above discussions we have the following figure.



Definition 4.25: The ***S_g^*-closure*** of A is defined as the intersection of all S_g^*-closed sets of X containing A. It is denoted by s_g^*Cl(A).

Theorem 4.26: If A is any subset of X, s_g^*Cl(A) is S_g^*-closed. In fact s_g^*Cl(A) is the smallest S_g^*-closed set in X containing A.

Proof: Follows from Definition 4.25 and Theorem 4.4.

Theorem 4.27: A subset A of X is S_g^*-closed iff s_g^*Cl(A)=A.

Proof: If A is S_g^*-closed then s_g^*Cl(A)=A is obvious. Conversely, suppose s_g^*Cl(A)=A. By Theorem 4.26, s_g^*Cl(A) is S_g^*-closed and hence A is S_g^*-closed.

Theorem 4.28: If A and B be subsets of a topological space (X,τ), then the following results hold:
(i) s_g^*Cl(ϕ)=ϕ.
(ii) s_g^*Cl(X)=X.
(iii) A⊆s_g^*Cl(A).
(iv) A⊆B⟹s_g^*Cl(A)⊆ s_g^*Cl(B).
(v)A⊆ sCl^*(A)⊆sCl(A) ⊆s_g^*Cl(A)⊆Cl(A).
(vi) s_g^*Cl(A∪B)⊇ s_g^*Cl(A)∪ s_g^*Cl(B).
(vii) s_g^*Cl(A∩B)⊆s_g^*Cl(A)∩s_g^*Cl(B).
(viii) s_g^*Cl(s_g^*Cl(A))= s_g^*Cl(A).
(ix) Cl(s_g^*Cl(A))=Cl(A).
(x) s_g^*Cl(Cl(A))=Cl(A).

Proof: (i), (ii), (iii) and (iv) follows from Definition 4.25.
(v) Follows from Result 2.3, Theorem 4.7, Theorem 4.12 and Theorem 4.15.

(vi) and (vii) follow from (iv) above.
(viii) and (ix) follows from Theorem 4.26 and Theorem 4.27.
(x) Since Cl(A) being closed, Cl(A) is S_g^*-closed. Hence by Theorem 4.27, $s_g^*Cl(Cl(\mathrm{A}))=Cl(\mathrm{A})$.

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