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g**-Quasi – FH-closed spaces and g**-Quasi – CH-closed spaces

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ABSTRACT

*In this paper we have introduced τ - g** boundary, g**-Quasi-FH-closed, g**-FH-closed, g**-Quasi-CH-closed, g**-CH-closed, g**-meager and few results on these new definitions have been established.*

Key Words: *τ - g** boundary, g**-Quasi-FH-closed, g**-FH-closed, g**-Quasi-CH-closed, g**-CH-closed, g**-meager.*

1. INTRODUCTION

Levine **[2]** and M.K.R.S. Veerakumar **[7]** introduced the class of g-closed sets and g*-closed sets in the year 1970 and 1991 respectively. T.R. Hamlett and D. Jankovic**[1]** have introduced the class of Quasi-H-closed and H-closed sets in the year 1990. R.L. Newcomb **[5]** have introduced Quasi-H-closed modulo *I* in the year 1967. We have introduced g**-closed sets**[4]**, separation axioms via g**-closed sets**[5]** and strongly g**-regular and strongly g**-normal spaces**[6]** and investigated many of their properties and in this paper we have introduced g**-Quasi-FH-closed and g**-Quasi-CH-closed and discussed their characteristics.

2. PRELIMINARIES

Definition 2.1: A subset A of a topological space(X, τ) is called

1) *generalized closed* (briefly *g-closed*)**[2]** if cl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
2) *generalized star closed* (briefly *g*-closed*)**[12]** if cl(A) ⊆ U whenever A ⊆ U and U is g- open in (X, τ).
3) *generalized star star closed* (briefly *g**-closed*)**[6]** if cl(A) ⊆ U whenever A ⊆ U and U is g*- open in (X, τ).

Definition 2.2: [9] A topological space (X,τ) is said to be g**-Lindelof if every g**-open cover has a countable sub cover.

Definition 2.3: [7] The topological space (X, τ) is said to be g**-additive if arbitrary union of g**-closed sets is g**-closed. Equivalently arbitrary intersection of g**-open sets is g**-open.

Definition 2.4: [7] A topological space (X,τ) is said to be $g^{**}-multiplicative$ if arbitrary intersection of $g^{**}-closed$ sets is $g^{**}-closed$. Equivalently arbitrary union of $g^{**}-open$ sets is $g^{**}-open$.

Definition 2.5: [9] The topological space (X, τ) is said to be g**- finitely additive if finite union of g**-closed sets is g**-closed.

Definition 2.6:[3] An ideal I on a non empty set X is a collection of subsets of X which satisfies the following properties.(i) $A \in I$, $B \in I \Rightarrow A \cup B \in I$ (ii) $A \in I$, $B \subset A \Rightarrow B \in I$ .A topological space (X,τ) with an ideal I on X is called an ideal topological space and is denoted by (X,τ,I).

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Definition 2.7: [8] An ideal topological space (X,τ,I) is said to be g**-compact modulo *I* if for every g**-open covering $\{U_\alpha\}_{\alpha\in\Delta}$ of X, there exists a finite subset Δ_0 of Δ such that $X-\bigcup_{\alpha\in\Delta_0} U_\alpha \in I.$

Definition 2.8: [10] An ideal topological space (X,τ) is said to be g**-Lindelof modulo I if for every g**-open cover $\{U_\alpha\}_{\alpha\in\Omega}$, there exists a countable subset Ω_0 such that $X-\bigcup_{\alpha\in\Omega_0} U_\alpha \in I.$

Definition 2.9: [7] A topological space (X,τ) is said to be a g**- T_2 space if for every pair of distinct points x, y in X there exists disjoint g**-open sets U and V in X such that $x\in U$ and $y\in V.$

Definition 2.10: [8] A topological space (X,τ) is said to be g**-compact if every g**-open covering of X contains a finite sub collection that also covers X. A subset A of X is said to be g**-compact if every g**-open covering of A contains a finite sub collection that also covers A

Definition 2.11: [7] Let A be a subset of X. A point $x\in X$ is said to be a $g^{**}-$ limit point of A if every $g^{**}-neighbourhood$ of x contains a point of A other than x.

Definition 2.12: [7] Let A be a subset of a topological space (X,τ). $g^{**}cl(A)$ is defined to be the intersection of all $g^{**}-closed$ sets containing A.

If (X,τ) is g**-multiplicative then g**cl(A) is g**-closed.

Definition 2.13: [9] A subset A of a space (X,τ) is said to be g**-dense in X, if g**cl(A) = X.

Definition 2.14: [7] Let (X,τ) be a topological space and A be a subset of X. A point $x\in A$ is said to be g^{**} -interior point of A if there exists $g^{**}-open$ set U such that $x\in U\subseteq A$.

Definition 2.15: [7] Let A be a subset of a topological space (X,τ). $g^{**}\mathrm{int}(A)$ is defined to be the union of all $g^{**}-open$ sets contained in A.

If (X,τ) is g**-multiplicative then g**int(A) is g**-open.

Definition 2.16: [11] A topological space (X,τ) is said to be a g**-space if (X,τ) is g**-finitely additive and g**-multiplicative.

Definition 2.17: [1] A topological space (X,τ) is said to be Quasi-H-closed or QHC if every open cover of a space contains a finite sub collection whose closures cover the space X.

Definition 2.18: [5] Let (X,τ) be a topological space and *I* be an ideal on X. Then X is said to be Quasi-H-closed modulo *I* if for every open cover $U=\{U_\alpha / \alpha\in\Omega\}$ of X there exists a finite sub family $\{U_{\alpha_i} / i-1,2,.......n\}$ of U such that $X-\bigcup_{i=1}^{n} cl(U_{\alpha_i})\in I.$

Definition 2.19: [1] A T_2 space which is QHC is said to be H-closed.

Definition 2.20: [5] A space (X,τ,I) is called τ-boundary if $\tau\cap I=\{\varphi\}.$

Example 2.21: [5] Let X be an infinite set, $\tau=\{A\subseteq X / A^c$ is finite}. $I_F=\{$ finite subsets of X}. Then $\tau\cap I_F=\varphi$.

3. g-Quasi – FH-closed spaces and g**-Quasi – CH-closed spaces**

Definition 3.1 Given an ideal topological space (X,τ,I), *I* is called g**-codense in X if $G^{**}O(X)\cap I=\{\varphi\}$

Example 3.2: Let X be an infinite set with cofinite topology and $I_F = \{$ finite subsets of X}. Then $G^{**}O(X) \cap I_F = \{\varphi\}$

Definition 3.3: A space (X,τ) is said to be g**-Quasi-FH-closed (briefly g**-QFHC) if every g**-open cover of X contains a finite sub collection whose g**-closures cover X.

Definition 3.4: A g**-T_2 space which is g**-QFHC is said to be g**-FH-closed.

Definition 3.5: A space (X,τ) is said to be g**-Quasi-CH-closed (briefly g**-QCHC) if every g**-open cover of X contains a countable sub collection whose g**-closures cover X.

Definition 3.6: A g**-T_2 space which is g**-QCHC is said to be g**-CH-closed.

Theorem 3.7: Every g**-compact space is g**-QFHC.

Proof: Let $\{U_\alpha / \alpha \in \Omega\}$ be a g**-open cover for X. Then there exists $\alpha_1, \alpha_2, \ldots\ldots\ldots$ such that $X = \bigcup_{i=1}^{n} U_{\alpha_i}$ and this implies $X = \bigcup_{i=1}^{n} \overline{U_{\alpha_i}}$. Therefore X is g**-QFHC.

Note: Any finite space is g**-compact and hence g**-QFHC.

Theorem 3.8: Every g**-Lindelof space is g**-QCHC.

Proof: similar to the above.

Example 3.9: An infinite cofinite topological space (X,τ) is g**-compact and hence g**-QFHC.

Example 3.10: In infinite indiscrete topological space (X,τ), all subsets are g**-open and g**-closed. $\{\{x\}/ x \in X\}$ is a g**-open cover which has no finite sub cover such that g**-closures can cover X and hence (X,τ) is g**-T_2 but not g**-QFHC and hence not g**-FH.

Example 3.11: A finite indiscrete topological space (X,τ) is g**-T_2 and g**-QFHC and hence it is g**-FH-closed.

Example 3.12: An infinite, cofinite topological space is g**-QFHC but not g**-T_2 and hence it is not g**-FH-closed.

Remark 3.13: Any g**-QFHC space is g**-QCHC space and g**-FH-closed space is g**-CH-closed. Converse need not be true as seen in the following example.

Example 3.14: A countably infinite indiscrete topological space is g**-QCHC and g**-CH-closed but not g**-QFHC and g**-FH-closed.

Definition 3.15: A subset A of (X,τ) is said to be nowhere g**-dense if g**int[g**cl(A)] = φ.

Theorem 3.16: If A is g**-closed then A is nowhere g**-dense if and only if A^c is g**-dense.

Proof: Let $x \in X$ and $g^{**}cl(A) = A. \therefore g^{**}\text{int}[g^{**}cl(A)] = g^{**}\text{int}[A] = \varphi.$ Therefore every g**-open set containing x should intersect A^c. $\therefore A^c$ is g**-dense in X. Let A^c be g**-dense in X. $\therefore g^{**}cl(A^c) = X$. For $x \in X$, every g**-open set containing x should intersect A^c. $\therefore g^{**}\text{int}(A) = \varphi.$ Therefore A is nowhere g**-dense.

Theorem 3.17: Let (X,τ) be a g**-space. Then $g^{**}(I_n) = \{$ nowhere g**-dense subsets of X} is an ideal in X.

Proof: Let $A \in g^{**}(I_n)$ and $B \subseteq A$ then $g^{**}\text{int}[g^{**}cl(B)] \subseteq g^{**}\text{int}[g^{**}cl(A)] = \varphi. \therefore B \in g^{**}(I_n).$ Let $A, B \in g^{**}(I_n).$

Under the given hypothesis, $g^{**}cl(A\cup B)=g^{**}cl(A)\cup g^{**}cl(B)$.

$$\therefore g^{**}\mathrm{int}[g^{**}cl(A\cup B) = g^{**}\mathrm{int}[g^{**}cl(A)\cup g^{**}cl(B)]$$
$$= g^{**}\mathrm{int}[g^{**}cl(A)]\cup[g^{**}\mathrm{int}[g^{**}cl(B)] = \varphi .$$

$\therefore A\cup B\in g^{**}(I_n)$. Therefore $g^{**}(I_n)$ is an ideal.

Theorem 3.18: If an ideal topological space (X,τ,I) is g**-multiplicative, g**-compact modulo I and $G^{**}O(X)\cap I=\{\varphi\}$ then (X,τ) is g**-QFHC.

Proof: Let $\{U_\alpha / \alpha\in\Omega\}$ be a g**-open cover for X. Therefore there exists $U_{\alpha_1}, U_{\alpha_2}, ...U_{\alpha_n}$ such that $X-\bigcup_{i=1}^{n} U_{\alpha_i}\in I$.

Case (1): If $X-\bigcup_{i=1}^{n} U_{\alpha_i}=\varphi$, then $X=\bigcup_{i=1}^{n} U_{\alpha_i}$. $\therefore X=\bigcup_{i=1}^{n} g^{**}cl(U_{\alpha_i})$ Therefore (X,τ) is g**-QFHC.

Case (2): If $X-\bigcup_{i=1}^{n} U_{\alpha_i}\neq\varphi$, then $X-\bigcup_{i=1}^{n} U_{\alpha_i}=\bigcup_{\alpha\neq\alpha_i} U_\alpha\in I$. But $\bigcup_{\alpha\neq\alpha_i} U_\alpha\in G^{**}O(X)$ (since X is g**-multiplicative). $\therefore \bigcup_{\alpha\neq\alpha_i} U_\alpha\in G^{**}O(X)\cap I=\{\varphi\}$. $\therefore X=\bigcup_{i=1}^{n} U_{\alpha_i}$ and this implies $X=\bigcup_{i=1}^{n} g^{**}cl(U_{\alpha_i})$.

Theorem 3.19: Let (X,τ) be a g**-multiplicative space. If (X,τ,I) is g**-Lindeloff modulo I and $G^{**}O(X)\cap I=\{\varphi\})$ then (X,τ) is g**-QCHC.

Proof: Similar to the above proof.

Theorem 3.20: Let (X,τ) be a g**-space.

(1) Then (X,τ) is g**-compact modulo g**(I_n) if and only if (X,τ) is g**-QFHC.

(2) If (X,τ) is g** T_2 then (X,τ) is g**-compact modulo g**(I_n) if and only if (X,τ) is g**-FH- closed.

Proof:

(1) **Necessity:** (1) Let (X,τ) be g**-compact modulo g**(I_n) and $\{U_\alpha\}_{\alpha\in\Omega}$ be a g**-open cover for X. Then there exists $U_{\alpha_1}, U_{\alpha_2}, ...U_{\alpha_n}$ such that $X-\bigcup_{i=1}^{n} U_{\alpha_i}\in g^{**}(I_n)$. $\therefore X-\bigcup_{i=1}^{n} U_{\alpha_i}$ is nowhere g**-dense in X and it is g**-closed. Therefore $\left[X-\bigcup_{i=1}^{n} U_{\alpha_i}\right]^c$ is g**-dense in X. Hence $g^{**}cl(\bigcup_{i=1}^{n} U_{\alpha_i})=X$. In a g**-space, $g^{**}cl(\bigcup_{i=1}^{n} U_{\alpha_i})=\bigcup_{i=1}^{n} g^{**}cl(U_{\alpha_i})$. $\therefore X=\bigcup_{i=1}^{n} g^{**}cl(U_{\alpha_i})$. Therefore (X,τ) is g**-QFHC.

Sufficiency: Let (X,τ) be g**-QFHC.

Since (X,τ) is g**-space, g**(I_n) is an ideal in X. Let $\{U_\alpha / \alpha\in\Omega\}$ be a g**-open cover. Then there exists $\alpha_1,\alpha_2 ....\alpha_n$ such that $X=\bigcup_{i=1}^{n} g^{**}cl(U_{\alpha_i})=g^{**}cl\left[\bigcup_{i=1}^{n} U_{\alpha_i}\right]$. $\therefore \bigcup_{i=1}^{n} U_{\alpha_i}$ is g**-dense in X. $\therefore \left[\bigcup_{i=1}^{n} U_{\alpha_i}\right]^c$ is nowhere g**-dense in X by theorem 3.16 and so $X-\bigcup_{i=1}^{n} U_{\alpha_i}\in g^{**}(I_n)$.

(2) By (1), (X,τ) is g**-compact modulo g** (I_n) if and only if (X,τ) is g**-QFHC. By definition (3.4), (X,τ) is g**-compact modulo g** (I_n) if and only if it is g** FH-closed.

Theorem 3.21: Let (X,τ) be g**-multiplicative, g**-countably additive space. Then
1) (X,τ) is g**-QCHC space if and only if (X,τ) is g**-Lindeloff modulo g**(I_n).
2) If (X,τ) is g**-T_2 then (X,τ) is g**-Lindeloff modulo g**- (I_n) if and only if (X,τ) is g**-CH-closed.

Proof: Since (X,τ) is g**-multiplicative and g**-countably additive, $\overset{\infty}{\underset{i=1}{\cup}} g^{**}cl(U_{\alpha_i}) = g^{**}cl\left[\overset{\infty}{\underset{i=1}{\cup}} U_{\alpha_i}\right]$

The rest of the proof is similar to the proof of theorem (3.20)

Definition 3.24: A space (X,τ) is said to be g**-meager or g**- first category if it is a countable union of nowhere g**-dense sets.

Theorem 3.25: Let (X,τ) be a g**-space and let $g^{**}(I_m) = \{$ g**-meager subsets of X}. Then $g^{**}(I_m)$ is an ideal in X.

Proof: Let $A \in g^{**}(I_m)$ and $B \subseteq A$. Then $A = \overset{\infty}{\underset{i=1}{\cup}} G_i$ where each G_i is nowhere g**-dense subsets. Now

$B = \bigcup_{i=1}^{\infty}(B \cap G_i).$ and $g^{**}cl(B \cap G_i) \subseteq g^{**}cl(B) \cap g^{**}cl(G_i).$

Therefore

$$g^{**}\mathrm{int}[g^{**}cl(B \cap G_i)] \subseteq g^{**}\mathrm{int}[g^{**}cl(B) \cap g^{**}cl(G_i)] = g^{**}\mathrm{int}[g^{**}cl(B)] \cap g^{**}\mathrm{int}[g^{**}cl(G_i),$$

Since X is a g**-space. Therefore $B \cap G_i$ is nowhere g**-dense for all i and so $B \in g^{**}(I_m)$. Obviously $A, B \in g^{**}(I_m) \Rightarrow A \cup B \in g^{**}(I_m)$. Therefore $g^{**}(I_m)$ is an ideal in X.

Definition 3.26: A topological space (X,τ) is said to be of g**-second category if it is not of g**-first category.

Definition 3.27: A g**-space (X,τ) is said to be a g**-Baire space if $G^{**}O(X) \cap g^{**}(I_m) = \{\varphi\}$.
Theorem 3.28: Let (X,τ) be a g**-baire space. Then

(1) (X,τ) is g**-compact modulo $g^{**}(I_m)$ if and only if (X,τ) is g**-QFHC.

(2) In addition if (X,τ) is g**-T_2 then (X,τ) is g**-compact modulo $g^{**}(I_m)$ if and only if (X,τ) is g**-FH-closed.

Proof: (1) Let (X,τ) be g**-Baire space and g**-compact modulo $g^{**}(I_m)$ .Let $\{U_\alpha\}$ be a g**-open cover for X. Then there exists $U_{\alpha_1}, U_{\alpha_2}, ...U_{\alpha_n}$ such that $X - \overset{n}{\underset{i=1}{\cup}} U_{\alpha_i} \in g^{**}(I_m)$..

Case (i): $X - \overset{n}{\underset{i=1}{\cup}} U_{\alpha_i} = \varphi$ then $X = \overset{n}{\underset{i=1}{\cup}} g^{**}cl(U_{\alpha_i})$ and so it is g**-QFHC.

Case (ii): $X - \overset{n}{\underset{i=1}{\cup}} U_{\alpha_i} \neq \varphi$ then $X - \overset{n}{\underset{i=1}{\cup}} U_{\alpha_i} = \underset{\alpha \neq \alpha_i}{\cup} U_{\alpha_i} \in G^{**}O(X)$ (since (X,τ) is a g**-space).

$\therefore X - \overset{n}{\underset{i=1}{\cup}} U_{\alpha_i} \in G^{**}O(X) \cap g^{**}(I_m) = \varphi.$ $\therefore X = \overset{n}{\underset{i=1}{\cup}} g^{**}cl(U_{\alpha_i}).$ (by case(i)). Therefore X is g**-QFHC.

Conversely, let (X,τ) be g**-QFHC. By theorem (3.20), (X,τ) is g**-compact modulo $g^{**}(I_n)$.

This implies that (X,τ) is g**-compact modulo $g^{**}(I_m)$, since $g^{**}(I_n) \subseteq g^{**}(I_m)$. (2)Follows from (1) and definition of g**-FH-closed space.

Theorem 3.29: Let (X,τ) be a g**-baire space which is g**-countably additive. Then

(1) (X,τ) is g**-Lindeloff modulo $g^{**}(I_m)$ if and only if (X,τ) isg**-QCHC.

(2) In addition if (X,τ) is g**-T$_2$ then (X,τ) is g**-Lindeloff modulo $g^{**}(I_m)$ if and only if (X,τ) is g**-CH-closed.

Proof: Similar to the above proof.

4. g-Quasi-H-closed modulo an ideal**

Definition 4.1: An ideal topological space (X,τ,I) is said to be g**-QFHC modulo an ideal if for every g**-open cover $\{U_\alpha / \alpha \in \Omega\}$ of X there exists a finite sub family $\{U_{\alpha_i} / i = 1,2,....n\}$ such that $X - \bigcup_{i=1}^{n} g^{**}cl(U_\alpha) \in I$. Such a sub family is said to be proximate g**-sub cover modulo I .

Definition 4.2: A subset A of (X,τ) is said to be g**pre-open if $A \subseteq g^{**}(g^{**}\text{int}(A))$. The collection of all g**-preopen sets is denoted by G**PO(X).

Definition 4.3: An ideal I in (X,τ) is said to be completely g**-codense if $I \cap G^{**}PO(X) = \varphi$.

Note: (1) $G^{**}O(X) \subseteq G^{**}PO(X)$.

(2) $I \cap G^{**}PO(X) = \varphi \Rightarrow I \cap G^{**}O(X) = \varphi$.

Therefore every completely g**-codense ideal is g**-codense. But the converse is not true as seen in the following example.

Example 4.4: Consider R with cofinite topology. A subset is g**-closed if and only if it is finite. Let I_c be the ideal of all countable subsets. Then $G^{**}O(R) \cap I_c = \varphi$ and so I_c is g**-codense.

g**cl(Q) = R. g**int(g**cl(Q)) = g**int(R) = R. $\therefore Q \in G^{**}PO(R)$. $\therefore Q \in I_c \cap G^{**}PO(R)$.

Therefore I_c is not completely g**-codense in this space.

Theorem 4.5: For a space (X,τ), the following statements are equivalent.

(1) (X,τ) is g**-QFHC.

(2) (X,τ) is g**-QFHC modulo φ.

(3) (X,τ) is g**-QFHC modulo I_F

If (X,τ) is a g**-space then these are equivalent to

(4) (X,τ) is g**-QFHC modulo I for every g**-codense ideal I.

Proof: $(1) \Leftrightarrow (2)$ is obvious. $(2) \Rightarrow (3)$ is obvious.

$(3) \Rightarrow (1)$: Let $\{U_\alpha\}$ be a g**-open cover for X. Then there exists $U_{\alpha_1}, U_{\alpha_2}, ...U_{\alpha_n}$ such that $X - \bigcup_{i=1}^{n} g^{**}cl(U_{\alpha_i}) \in I_F$. Let $X - \bigcup_{i=1}^{n} g^{**}cl(U_{\alpha_i}) = \{x_1, x_2, ...x_k\}$. Choose U_{β_i} such that $x_i \in U_{\beta_i}$. Let $\Delta_0 = \{\alpha_1, ...\alpha_n, \beta_1, ...\beta_k\}$. Then Δ_0 is finite and $X = \bigcup_{\alpha \in \Delta_0} g^{**}cl(U_\alpha)$.

Therefore X is g**-QFHC.

$(1) \Rightarrow (4)$ is obvious.

$(4) \Rightarrow (1)$: Let I be g**-codense ideal. Let $\{U_\alpha\}_{\alpha\in\Omega}$ be a g**-open cover in X. Then there exists $\{U_{\alpha_i} / i = 1,2.....n\}$ such that $X - \bigcup_{i=1}^{n} g^{**}cl(U_{\alpha_i}) \in I$. But $X - \bigcup_{i=1}^{n} g^{**}cl(U_{\alpha_i}) \in G^{**}O(X)$, since X is a g**-space. But $G^{**}O(X) \cap I = \varphi$. $\therefore X = \bigcup_{i=1}^{n} g^{**}cl(U_{\alpha_i})$. $\therefore (X,\tau)$ is g**-QFHC.

Theorem 4.6: For a topological space (X,τ) the following statements are equivalent.

(1) (X,τ) is g**-QCHC.

(2) (X,τ) is g**-QCHC modulo {φ}.

(3) (X,τ) is g**-QCHC modulo $\{I_c\}$.

If (X,τ) is a g**-multiplicative and g**-countably additive then these are equivalent to
(4) (X,τ) is g**-QCHC modulo *I* for every g**-codense ideal *I*.

Proof: Similar to the above proof, since (X,τ) is g**-countably additive implies $\bigcup_{i=1}^{\infty} g^{**}cl(U_{\alpha_i})$ is g**-closed.

Remark 4.7: In theorem (4.6) the condition *I* is g**-codense is necessary as seen in the following example.

Example 4.8: Consider R with indiscrete topology τ.. Let $A=\{\alpha_1,\alpha_2,.........\alpha_n\}$ be a finite subset of R. then $I=\wp(R-A)$ is an ideal in R. Since all subsets are g**-open, $G^{**}O(X)\cap I\neq\varphi.$

Therefore *I* is not g**-codense. Let $\{U_\alpha/\alpha\in\Omega\}$ be a g**-open cover for X. Choose U_{α_i} such that $a_i\in U_{\alpha_i}$. Then $R-\bigcup_{i=1}^{n}U_{\alpha_i}\in I.$ But $R-\bigcup_{i=1}^{n}g^{**}cl(U_{\alpha_i})\subseteq R-\bigcup_{i=1}^{n}U_{\alpha_i}$. $\therefore R-\bigcup_{i=1}^{n}g^{**}cl(U_{\alpha_i})\in I.$

Therefore R is g**-QFHC modulo *I*. $\{\{x\}/x\in R\}$ is a g**-open cover which has no finite sub cover whose g**-closures cover X. Therefore R is not g**-QFHC.

Remark 4.9: In theorem (4.7) the condition *I* is g**-codense is necessary as seen in the following example.

Example 4.10: In example(4.8), (R,τ) is g**-QCHC modulo *I* but not g**-QCHC.
The following theorem contains a number of characterizations of g**-QFHC modulo *I* spaces.

Theorem 4.11: For a topological space (X,τ) and an ideal *I* on X, the following statements are equivalent.

(1) (X,τ) is G**-QFHC modulo *I*.

(2) For each family $A'=\{A_\alpha/\alpha\in\Omega\}$ of g**-closed sets having empty intersection there exists a finite sub family $\{A_{\alpha_1},A_{\alpha_2},.........A_{\alpha_n}\}$ such that $\bigcap_{i=1}^{n}g^{**}\mathrm{int}(A_{\alpha_i})\in I.$

(3) For each family $A'=\{A_\alpha/\alpha\in\Omega\}$ of g**-closed sets such that $\{g^{**}\mathrm{int}(A_\alpha)/\alpha\in\Omega\}$ with FIP modulo *I* one has, $\bigcap_{A\in A'}A\neq\varphi.$

Proof: $(1)\Rightarrow(2)$ Let $A'=\{A_\alpha/\alpha\in\Omega\}$ be a family of g**-closed sets such that $\bigcap_{\alpha\in\Omega}A_\alpha=\varphi.$ Then $X=\bigcup_{\alpha\in\Omega}(G_\alpha)$ where $G_\alpha=X-A_\alpha$ is g**-open. By (1), there exists $\{G_{\alpha_1},G_{\alpha_2},.............G_{\alpha_n}\}$ such that $X-\bigcup_{i=1}^{n}g^{**}cl(G_{\alpha_i})\in I.$ (ie) $\bigcap_{i=1}^{n}[g^{**}cl(G_{\alpha_i})]^c\in I.$ But $[g^{**}cl(G_{\alpha_i})]^c=X-g^{**}cl(G_{\alpha_i})$ which is equal to $g^{**}\mathrm{int}(X-G_{\alpha_i})=g^{**}\mathrm{int}(A_{\alpha_i}).$ $\therefore\bigcap_{i=1}^{n}g^{**}\mathrm{int}(A_{\alpha_i})\in I.$

$(2)\Rightarrow(3)$: Let A' be a family of g**-closed sets such that $\{g^{**}\mathrm{int}(A_\alpha)/\alpha\in\Omega\}$ has FIP modulo *I*.

Suppose $\bigcap_{\alpha\in\Omega}A_\alpha=\varphi.$ Then by (2) there exists a finite sub family $\{A_{\alpha_1},A_{\alpha_2},...............A_{\alpha_n}\}$ such that $\bigcap_{i=1}^{n}g^{**}\mathrm{int}(A_{\alpha_i})\in I$ which is a contradiction. $\therefore\bigcap_{\alpha\in\Omega}A_\alpha\neq\varphi.$

$(3)\Rightarrow(1)$: Let $\{U_\alpha/\alpha\in\Omega\}$ be a g**-open cover for X. To prove that there exists a finite sub family $\{U_{\alpha_i}/i=1,2.........n\}$ such that $X-\bigcup_{i=1}^{n}g^{**}cl(U_{\alpha_i})\in I.$ If not there is no finite sub family with this property.
Now $\{X-U_\alpha/\alpha\in\Omega\}$ is a family of g**-closed sets. For any finite sub family $U_{\alpha_1},U_{\alpha_2},............U_{\alpha_n}$, $X-\bigcup_{i=1}^{n}g^{**}cl(U_{\alpha_i})\notin I.$ (ie) $\bigcap_{i=1}^{n}g^{**}\mathrm{int}(X-U_{\alpha_i})\notin I.$ By (3), $\bigcap_{\alpha\in\Omega}(X-U_\alpha)\neq\varphi.$ $\therefore\bigcup_{\alpha\in\Omega}(U_\alpha)\neq X$ which is a contradiction. Therefore (X,τ) is g**-QFHC.

Theorem 4.12: For a topological space (X,τ) and an ideal *I* on X, the following statements are equivalent.

(1) (X,τ) is G**-QCHC modulo *I*.

(2) For each family $A' = \{A_\alpha / \alpha \in \Omega\}$ of g**-closed sets having empty intersection there exists a countable sub family $\{A_{\alpha_1}, A_{\alpha_2}, ...\}$ such that $\bigcap_{i=1}^{\infty} g^{**}\mathrm{int}(A_{\alpha_i}) \in I$.

(3) For each family $A' = \{A_\alpha / \alpha \in \Omega\}$ of g**-closed sets such that $\{g^{**}\mathrm{int}(A_\alpha)/\alpha \in \Omega\}$ with CIP modulo *I* one has $\bigcap_{A\in A'} A \neq \varphi$.

Proof: Similar to the above proof.

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