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A RESULT ON CONNECTED GRAPHS

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ABSTRACT

The field of mathematics plays very important role in different fields. One of the important areas in mathematics is graph theory which is used in structural models. This structural preparations of various objects or technologies direct to new inventions and modifications in the existing environment for development in those fields. The field graph theory started its journey from the problem of Konigsberg bridge in 1735.In this paper we discuss Graphs, vertices and edges, connected graph and relation between connected graphs, edges.

1. INTRODUCTION

The origin of graph theory started with the problem of Koinsber bridge, in 1735.

This problem lead to the concept of Eulerian Graph. Euler studied the problem of Koinsberg bridge and constructed a structure to solve the problem called Eulerian graph.

In 1840, A.F Mobius gave the idea of complete graph and bipartite graph and Kuratowski proved that they are planar by means of recreational problems.

The concept of tree, (a connected graph without cycles was implemented by Gustav Kirchhoff in 1845, and he employed graph theoretical ideas in the calculation of currents in electrical networks or circuits. This time is considered as the birth of Graph Theory. Caley studied particular analytical forms from differential calculus to study the trees. This had many implications in theoretical chemistry. This lead to the invention of enumerative graph theory. Any how the term

“Graph” was introduced by Sylvester in 1878 where he drew an analogy between “Quantic invariants” and covariants of algebra and molecular diagrams.

In 1941, Ramsey worked on colorations which lead to the identification of another branch of graph theory called extremel graph theory.

1.1 Definition: A graph – usually denoted G (V, E) or G = (V, E) – consists of set of vertices V together with a set of edges E. The number of vertices in a graph is usually denoted *n* while the number of edges is usually denoted *m*.

1.2 Definition: Vertices are also known as nodes, points and (in social networks) as actors, agents or players.

1.3 Definition: Edges are also known as lines and (in social networks) as ties or links. An edge e = (u, v) is defined by the unordered pair of vertices that serve as its end points.

1.4 Example: The graph depicted in Figure 1 has vertex set V= {a, b, c, d, e. f} and edge set E = {(a, b), (b, c), (c, d), (c, e), (d, e), (e, f)}.

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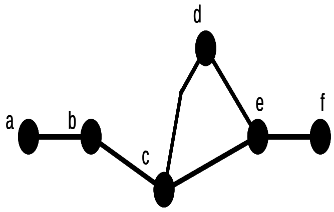

Figure: 1

1. 5 Definition: Two vertices *u* and *v* are *adjacent* if there exists an edge (*u*, *v*) that connects them.

1.6 Definition: An edge (u, v) is said to be *incident* upon nodes u and v.

1.7 Definition: An edge e = (u, u) that links a vertex to itself is known as a *self-loop* or *reflexive* tie.

1.8 Definition: Every graph has associated with it an *adjacency matrix*, which is a binary $n \times n$ matrix A in which $a_{ij} = 1$ and $a_{ji} = 1$ if vertex vi is adjacent to vertex vj, and aij = 0 and aji = 0 otherwise. The natural graphical representation of an adjacency matrix is a table, such as shown below.

	a	b	c	d	e	f
A	0	1	0	0	0	0
B	1	0	1	0	0	0
C	0	1	0	1	1	0
D	0	0	1	0	1	0
E	0	0	1	1	0	1
F	0	0	0	0	1	0

Adjacency matrix for graph in Figure 1.

1.9 Definition: Examining either Figure 1 or given adjacency Matrix, we can see that not every vertex is adjacent to every other. A graph in which all vertices are adjacent to all others is said to be *complete*.

1.10 Definition: A *subgraph* of a graph *G* is a graph whose points and lines are contained in *G*. A complete subgraph of *G* is a section of *G* that is complete

1.11 Definition: While not every vertex in the graph in Figure 1 is adjacent, one can construct a sequence of adjacent vertices from any vertex to any other. Graphs with this property are called *connected*.

1.12 Note: Reachability. Similarly, any pair of vertices in which one vertex can reach the other via a sequence of adjacent vertices is called *reachable*. If we determine reachability for every pair of vertices, we can construct a reachability matrix R such as depicted in Figure 2. The matrix R can be thought of as the result of applying transitive closure to the adjacency matrix A.

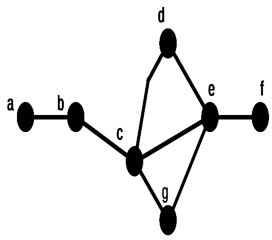

Figure: 2

1.13 Definition: A *component* of a graph is defined as a maximal subgraph in which a path exists from every node to every other (i.e., they are mutually reachable). The size of a component is defined as the number of nodes it contains. A connected graph has only one component.

1.14 Definition: A sequence of adjacent vertices $v_0, v_1, \ldots, v_n$ is known as a *walk*. In Figure 3, the sequence *a, b, c, b, c, g* is a walk. A walk can also be seen as a sequence of *incident* edges, where two edges are said to be incident if they share exactly one vertex.

1.15 Definition: A walk is closed if $v_o = v_n$.

1.16 Definition: A walk in which no vertex occurs more than once is known as a *path*. In Figure 3, the sequence *a, b, c, d, e, f* is a path.

1.17 Definition: A walk in which no edge occurs more than once is known as a *trail*. In Figure 3, the sequence *a, b, c, e, d, c, g* is a trail but not a path. Every path is a trail, and every trail is a walk.

1.18 Definition: A *cycle* can be defined as a closed path in which *n* >= 3. The sequence *c, e, d* in Figure 3 is a cycle.

1.19 Definition: A *tree* is a connected graph that contains no cycles. In a tree, every pair of points is connected by a unique path. That is, there is only one way to get from A to B.

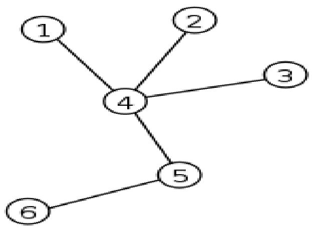

Figure 3: A labeled tree with 6 vertices and 5 edges

1.20 Definition: A *spanning tree* for a graph *G* is a sub-graph of *G* which is a tree that includes every vertex of *G*.

1.21 Definition: The length of a walk (and therefore a path or trail) is defined as the number of edges it contains. For example, in Figure 3, the path *a*, *b*, *c*, *d*, *e* has length 4.

1.22 Definition: The number of vertices adjacent to a given vertex is called the *degree* of the vertex and is denoted d(v).

2. In this section mainly proved A connected graph with 'n' vertices and 'k' components can have atmost (n – k) (n- k +2)/2 edges.

2.1 Result: If a connected graph has exactly two vertices of odd degree, there must be a path joining these two vertices.

Proof: Let G be a graph with all even vertices v_1 and v_2 , which are odd degree.

We know that by the definitions of graph, vertices and edges it holds for every graph and therefore for every component of a disconnected graph, i.e. no graph can have an odd number of odd vertices.

Therefore, in graph G, v_1 and v_2 must belong to the same component, and hence must have a path between them.

2.2 Theorem: A connected graph with 'n' vertices and 'k' components can have atmost (n – k) (n- k +2)/2 edges.

Proof: Let the number of vertices in each of the k components of a graph G be $n_{1,}$ $n_{2,}$ $n_{3,}$ $n_{4,\ldots..}$ $n_{k.}$

Thus we have n_{1+} n_{2+} n_{3+} $n_{4+\ldots\ldots..+}$ $n_{k=}$ n, $n_i \geq 1$.

The proof of the theorem depends on an algebraic inequality

$$\sum_{i=1}^{k} n_i^2 \leq n^2 - (k-1)(2n-k)$$

Now the maximum number of edges in the ' I' th component of G (which is a connected graph) is $\frac{1}{2} n_i (n_i - 1)$.

Therefore, the maximum number of edges in G is

$$\frac{1}{2}\sum_{i=1}^{k}(n_i - 1)n_i = \frac{1}{2}(\sum_{i=1}^{k} n_i^2) - \frac{n}{2} \leq \frac{1}{2}(n^2 - (k-1)(2n-1)) - \frac{n}{2} = \frac{1}{2}(n-k)(n-k+1).$$

CONCLUSION

It may be noted that this theorem is a generalization of the many Problems.

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