International Journal of Mathematical Archive-3(12), 2012, 4866-4871
IJMA Available online through www.ijma.info ISSN 2229 – 5046

Some Properties of Weak Form of b-γ-Open Sets

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(Received on: 05-11-12; Revised & Accepted on: 18-12-12)

ABSTRACT

In this paper, we introduce and explore fundamental properties of maximal b-γ-open sets in topological spaces such as decomposition theorem for maximal b-γ-open set. Basic properties of intersection of maximal b-γ-open sets are established.

Key words. *b-γ-closed (open), b-γ-closure (interior), maximal b-γ-open, pre b-γ-open sets, b-γ-radical.*

1. INTRODUCTION

A proper nonempty b-γ-open subset U of a topological space X is said to be a maximal b-γ-open set if any b-γ-open set which contains U is X or U. In [2], we study minimal b-γ-open sets. Although the definition of the maximal b-γ-open set is obtained by the definition of the minimal b-γ-open set, the properties of them are quite different, as we see in this paper, especially the results in the last two sections. The purpose of this paper is to prove some fundamental properties of maximal b-γ-open sets and establish a part of the foundation of the theory of maximal b-γ-open sets in topological spaces.

In Section 3, we prove some basic results which are necessary for the subsequent arguments. We obtain a relation among maximal b-γ-open sets in Theorem 3.6. At the end of this section, we show that for any proper nonempty cofinite open subset V, there exists, at least, one maximal b-γ-open set U which contains V (Theorem 3.8).

In Section 4, we study some relations among b-γ-closure, b-γ-interior, and maximal b-γ-open sets. As an application, we prove a result about a pre b-γ-open set (Theorem 4.12). In the last two sections, we study various properties of b-γ-radicals.

In Section 5, we prove fundamental properties of b-γ-radicals of maximal b-γ-open sets. We establish a very useful decomposition theorem for a maximal open set in Theorem 5.8. Theorem 5.8 will be applied to prove Theorem 5.9. Theorem 5.10 gives a sufficient condition for the set of all maximal b-γ-open sets. In Section 6, we consider the closure of the b-γ-radicals of maximal b-γ-open sets. We establish "The law of b-γ-radical b-γ-closure" in Theorem 6.3.

2. PRELIMINARIES

Throughout the present paper X denotes the topological space. For a subset A of X, the closure of A and the interior of A will be denoted by cl(A) and int(A), respectively.

Definition 2.1. [3] A subset A of a topological space (X, τ) is said to be b-open if $A \subseteq int(cl(A)) \cup cl(int(A))$. The family of all b-open sets is denoted by BO(X, τ).

The following definitions and results are obtained from [1]. Let γ be a mapping on BO(X) in to P (X) and $\gamma: BO(X) \rightarrow P(X)$ is called an operation on BO(X), such that $V \subseteq \gamma(V)$ for each $V \in BO(X)$.

Definition 2.2. A subset A of a space X is called b-γ-open if for each $x \in A$, there exists a b-open set U such that $x \in U$ and $\gamma(U) \subseteq A$.

Definition 2.3. Let A be a subset of (X, τ) and $\gamma: BO(X) \rightarrow P(X)$ be an operation. Then the b-γ-closure of A is denoted by τ_γ-bcl(A) and defined as follows $\tau_\gamma\text{-bcl}(A) = \cap \{ F: F \text{ is b-}\gamma\text{-closed and } A \subseteq F\}$.

Definition 2.4. An operation γ on BO(X) is said to be b-regular if for every b-open sets U and V of each $x \in X$, there exists a b-open set W of x such that $\gamma(W) \subseteq \gamma(U) \cap \gamma(V)$.

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Proposition 2.5. Let γ be a b-regular operation on BO(X) If A and B are b-γ-open sets in X, then $A \cap B$ is also a b-γ-open set.

3. Maximal b-γ-Open Sets

Definition 3.1. A proper nonempty b-γ-open subset B of X is said to be a maximal b-γ-open set, if any b-γ-open set which contains B is X or B.

Example 3.2. Let X = {a, b, c}, τ = {∅, X, {a}, {a, b}} be a topology on X. Define an operation γ: BO(X) → P(X) by $\gamma(A) = cl(A)$ if $a \notin A$ and $\gamma(A) = A$ if $a \in A$. Calculation gives that b-γ-open sets are {a}, {a, b}, {a, c}, X, ∅. Clearly {a, b}, {a, c} and X are maximal b-γ-open sets.

Lemma 3.3. (1) Let U be a maximal b-γ-open set and W a b-γ-open set. Then $U \cup W = X$ or $W \subseteq U$.
(2) Let U and V be maximal b-γ-open sets. Then $U \cup V = X$ or $U = V$.

Proof. (1) Let W be a b-γ-open set such that $U \cup W \neq X$. Since U is a maximal b-γ-open set and $U \subset U \cup W$, we have $U \cup W = U$. Therefore, $W \subset U$.
(2) If $U \cup V \neq X$, then $U \subset V$ and $V \subset U$ by (1). Therefore $U = V$.

Proposition 3.4. Let U be a maximal b-γ-open set. If $x \in U$, then $W \cup U = X$ or $W \subseteq U$, for any b-γ-open neighborhood W of x.

Proof. By Lemma 2.3 (1), we have the result.

Theorem 3.5. Let U_α, U_β and U_γ be maximal b-γ-open sets such that $U_\alpha \neq U_\beta$. If $U_\alpha \cap U_\beta \subseteq U_\gamma$, then $U_\alpha = U_\gamma$ or $U_\beta = U_\gamma$.

Proof. We see that

$$\begin{aligned} U_\alpha \cap U_\gamma &= U_\alpha \cap (U_\gamma \cap X) \\ &= U_\alpha \cap (U_\gamma \cap (U_\alpha \cup U_\beta)) \text{ (by Lemma 3.3 (2))} \\ &= U_\alpha \cap ((U_\gamma \cap U_\alpha) \cup (U_\gamma \cap U_\beta)) \\ &= (U_\alpha \cap U_\gamma) \cup (U_\gamma \cap U_\alpha \cap U_\beta) \\ &= (U_\alpha \cap U_\gamma) \cup (U_\alpha \cap U_\beta) \text{ (by } U_\alpha \cap U_\beta \subseteq U_\gamma) \\ &= U_\alpha \cap (U_\gamma \cup U_\beta) \end{aligned}$$

Hence we have $U_\alpha \cap U_\gamma = U_\alpha \cap (U_\gamma \cup U_\beta)$. If $U_\gamma \neq U_\beta$, then $U_\gamma \cup U_\beta = X$ and hence $U_\alpha \cap U_\gamma = U_\alpha$ implies $U_\alpha \subseteq U_\gamma$.

Since U_α and U_γ are maximal b-γ-open sets, we have $U_\alpha = U_\gamma$.

The following Theorem gives a relationship among maximal b-γ-open sets:

Theorem 3.6. Let U_1, U_2 and U_3 be disjoint maximal b-γ-open sets. Then, $U_1 \cap U_2$ is not a subset of $U_1 \cap U_3$.

Proof. If $U_1 \cap U_2 \subseteq U_1 \cap U_3$, then we see that $(U_1 \cap U_2) \cup (U_2 \cap U_3) \subseteq (U_1 \cap U_3) \cup (U_2 \cap U_3)$. Therefore, $U_2 \cap (U_1 \cup U_3) \subseteq (U_1 \cup U_2) \cap U_3$. Since $U_1 \cup U_3 = X = U_1 \cup U_2$, we have $U_2 \subseteq U_3$. It follows that $U_2 = U_3$, which contradicts our assumption.

Proposition 3.7. Let U be a maximal b-γ-open sets and $x \in U$. Then $U = \cup\{W$: W is a b-γ-open neighborhood of x such that $W \cup U \neq X\}$, where γ is a b-regular operation.

Proof. This follows from Proposition 3.4 and the fact that U is a b-γ-open neighborhood of x, we have $U \subset \cup\{W$: W is a b-γ-open neighborhood of x such that $W \cup U \neq X\} \subset U$. Therefore, we have the result.

Recall that the complement of any finite subset is a cofinite subset. The following theorem shows the existence of maximal b-γ-open sets for special cases.

Theorem 3.8. Let V be a proper nonempty cofinite b-γ-open sets subset. Then, there exists, at least, one (cofinite) maximal b-γ-open set U such that $V \subseteq U$.

Proof. If V is a maximal b-γ-open sets, we may set U = V. If V is not a maximal b-γ-open set, then there exists (cofinite) b-γ-open set V_1 such that $V \subseteq V_1 \neq X$. If V_1 is a maximal b-γ-open set, we may set $U = V_1$. If V_1 is not a maximal b-γ-open set, then there exists a (cofinite) b-γ-open set V_2 such that $V \subseteq V_1 \subseteq V_2 \neq X$. Continuing this process, we have a sequence of b-γ-open set

$$V \subseteq V_1 \subseteq V_2 \cdots \subseteq V_k \subseteq \cdots$$

Since V is a cofinite set, this process ends in a finite number of steps. Finally, we get a maximal b-γ-open set $U = V_n$, for some positive integer n.

4. Applications of b-γ-closure, b-γ-interior in maximal b-γ-open sets

Now, we study some relationship among b-γ-closure, b-γ-interior and a maximal b-γ-open set. As an application, we prove Theorem 4.12 about pre b-γ-open sets:

Theorem 4.1. Let U be a maximal b-γ-open set and $x \in (X - U)$. Then, $(X - U) \subseteq W$, for any b-γ-open neighborhood W of x.

Proof. Since $x \in (X - U)$, we have W is not subset of U, for any b-γ-open neighborhood W of x. Then $W \cup U = X$, by Lemma 3.3 (1). Therefore, $(X - U) \subseteq W$.

Corollary 4.2. Let U be a maximal b-γ-open set. Then, either of the following holds:
(1) W = X, for each $x \in (X - U)$ and each b-γ-open neighborhood W of x.
(2) There exists a b-γ-open set W such that $(X - U) \subseteq W$ and $W \subseteq X$.

Proof. If (1) does not hold, then there exists an element $x \in (X - U)$ and a b-γ-open neighborhood W of x such that $W \subseteq X$. By Theorem 4.1, we have $(X - U) \subseteq W$.

Corollary 4.3. Let U be a maximal b-γ-open sets. Then, either of the following holds:
(1) $(X - U) \subseteq W$, for each $x \in (X - U)$ and each b-γ-open neighborhood W of x.
(2) There exists a b-γ-open set W such that $(X - U) = W \neq X$.

Proof. Assume that (2) does not hold. Then, by Theorem 4.1, we have $(X - U) \subseteq W$, for each $x \in (X - U)$ and each b-γ-open neighborhood W of x. Hence, we have $(X - U) \subseteq W$.

Theorem 4.4. Let U be a maximal b-γ-open set. Then τ_γ-bcl(U) = X or τ_γ-bcl(U) = U.

Proof. Since U is a maximal b-γ-open set, only the following cases (1) and (2) occur by Corollary 4.3:

(1) for each $x \in (X - U)$ and each b-γ-open neighborhood W of x, we have $(X - U) \subseteq W$. Since $(X - U) \neq W$, we have $W \cap U \neq \varnothing$, for any b-γ-open neighborhood W of x. Hence, $(X - U) \subseteq \tau_\gamma$-bcl(U). Since $X = U \cup (X - U) \subseteq U \cup \tau_\gamma$-bcl(U) = τ_γ-bcl(U) $\subseteq X$, we have τ_γ-bcl(U) = X.

(2) There exists a b-γ-open set W such that $(X - U) = W \neq X$, since $(X - U) = W$ is a b-γ-open set, U is a b-γ-closed set.

Therefore, U = τ_γ-bcl(U).

Let A be a subset of (X, τ) and γ: BO(X)→ P(X) be an operation. Then the b -γ-interior of A is denoted b τ_γ-bint(A) and defined as follows τ_γ-bint(A) = ∪ { U: U is b-γ-open and $U \subseteq A$}.

Theorem 4.5. Let U be a maximal b-γ-open set. Then τ_γ-bint(X − U) = (X − U) or τ_γ-bint(X − U) = ∅.

Proof. By Theorem 4.4, we have either (1) τ_γ-bint(X − U) = ∅ or (2) τ_γ-bint(X − U) = X − U.

Theorem 4.6. Let U be a maximal b-γ-open set and $\varnothing \neq S \subseteq (X - U)$. Then τ_γ-bcl(S) = (X − U).

Proof. Since $\varnothing \neq S \subseteq (X - U)$, then by Theorem4.1, we have $W \cap S \neq \varnothing$, for any $x \in (X - U)$ and any b-γ-open neighborhood W of x. Then $(X - U) \subseteq \tau_\gamma$-bcl(S). Since X − U is a b-γ-closed set and $S \subseteq (X - U)$, we see that τ_γ-bcl(S) $\subseteq \tau_\gamma$-bcl(X − U) = X − U. Therefore, X − U = τ_γ-bcl(S).

Corollary 4.7. Let U be a maximal b-γ-open set and $M \subseteq X$ with $U \subseteq M$. Then τ_γ-bcl(M) = X. Where γ is a b-regular operation.

Proof. Since $U \subseteq M \subseteq X$, there exists a $\varnothing \neq S \subseteq (X - U)$ such that $M = U \cup S$. By b-regularity of operation γ and Theorem 4.6, we have τ_γ-bcl(M) = τ_γ-bcl($S \cup U$) = τ_γ-bcl(S) $\cup$ τ_γ-bcl(U) $\supseteq (X - U) \cup U = X$. Therefore, τ_γ-bcl(M) = X.

Theorem 4.8. Let U be a maximal b-γ-open set and assume that $(X - U)$ has at least two elements. Then, for any element $a \in (X - U)$, τ_γ-bcl($X - \{a\}$) = X, where γ is a b-regular operation.

Proof. Since $U \subseteq X - \{a\}$ by our assumption, we have the result by Corollary 3.7.

Theorem 4.9. Let U be a maximal b-γ-open set and $U \subseteq N \subseteq X$. Then, τ_γ-bint(N) = U.

Proof. If N = U, then τ_γ-bint(N) = τ_γ-bint(U) = U. Otherwise $N \neq U$, and hence $U \subseteq N$. It follows that $U \subseteq \tau_\gamma$-bint(N). Since U is a maximal b-γ-open set, we have also τ_γ-bint(N)$\subseteq$U. Therefore, τ_γ-bint(N) = U.

The following Theorem follows from Theorems 4.6 and 4.9:

Theorem 4.10. Let U be a maximal b-γ-open set and $\varnothing \neq S \subseteq (X - U)$. Then, $X - \tau_\gamma$-bcl(S) = τ_γ-bint($X - S$) = U.

Definition 4.11 [2]. A subset M of X is said to be pre b-γ-open-set, if $M \subseteq \tau_\gamma$-bint(τ_γ-bcl(M)).

Theorem 4.12. Let U be a maximal b-γ-open set and $U \subseteq M \subseteq X$. Then, M is a pre b-$\gamma$-open set, where γ is a b-regular operation.

Proof. If M = U, then M is a b-γ-open set. Therefore, M is a pre b-γ-open set [2].
Otherwise $U \subseteq M$, then by Corollary 4.7, τ_γ-bint(τ_γ-bcl(M)) = τ_γ-bint(X) = X $\supseteq$ M. Therefore M is a pre b-γ-open set.

The following Corollary directly follows from Theorem 4.12:

Corollary 4.13. Let U be a maximal b-γ-open set. Then, $X - \{a\}$ is a pre b-γ-open set, for any $a \in (X - U)$, where γ is a b-regular operation.

5. Basic properties of b-γ-radical

In this section, we prove fundamental properties of radical of maximal b-γ-open sets. We establish a very useful decomposition theorem for a maximal b-γ-open set in Theorem 5.7.

Definition 5.1. Let $U = \{U_\lambda : \lambda \in I\}$ be a class of maximal b-γ-open sets. Then, $\cap U = \cap_{\lambda \in I} U_\lambda$ is called the b-γ-radical of U.

Example 5.2. Let X = {a, b, c}, τ = {$\varnothing$, X, {b}} be a topology on X. Define an operation γ: BO(X) $\to$ P(X) by $\gamma(A)$ = A for all A $\in$ BO(X) Then, , the b-γ-radical is the set {b}.

The intersection of all maximal ideals of a ring R is called the (Jacobson) radical of R [4, 5]. Following this terminology in the theory of rings, we use the terminology "radical" for the intersection of maximal b-γ-open sets.

The Symbol $I \setminus \Gamma$ means difference of index sets; namely, $I \setminus \Gamma = I - \Gamma$, and the cardinality of a set I is denoted by $|I|$ in the following arguments:

Theorem 5.3. Suppose that $|I| \geq 2$. Let U_λ be a maximal b-γ-open set for any $\lambda \in I$ and $U_\lambda \neq U_\mu$, for any $\lambda, \mu \in I$ with $\lambda \neq \mu$.
(1) Let μ be any element of I. Then, $X - (\cap_{\lambda \in I-\{\mu\}} U_\lambda) \subseteq U_\mu$.
(2) Let μ be any element of I. Then, $\cap_{\lambda \in I-\{\mu\}} U_\lambda \neq \varnothing$.

Proof. (1) By Lemma 3.3 (2), we have $(X - U_\mu) \subseteq U_\lambda$ for any $\lambda \in I$ with $\lambda \neq \mu$. Then, $(X - U_\mu) \subseteq \cap_{\lambda \in I-\{\mu\}} U_\lambda$. Therefore, we have $X - \cap_{\lambda \in I-\{\mu\}} U_\lambda \subseteq U_\mu$.
(2) If $\cap_{\lambda \in I-\{\mu\}} U_\lambda = \varnothing$. By (1), we have $X = U_\mu$. This is contradiction to our supposition that U_λ is a maximal b-γ-open set. Therefore, we have $\cap_{\lambda \in I-\{\mu\}} U_\lambda \neq \varnothing$.

Corollary 5.4. Let U_λ be a maximal b-γ-open set, for any $\lambda \in I$ and $U_\lambda \neq U_\mu$, for any $\lambda, \mu \in I$ with $\lambda \neq \mu$. If $|I| \geq 3$, then $U_\lambda \cap U_\mu \neq \varnothing$, for any $\lambda, \mu \in I$ with $\lambda \neq \mu$.

Proof. By Theorem 5.3 (2), we have the result.

Theorem 5.5. Let U_λ be a maximal b-γ-open set for any $\lambda \in I$ and $U_\lambda \neq U_\mu$, for any $\lambda, \mu \in I$ with $\lambda \neq \mu$. Assume that $|I| \geq 2$. Let μ be any element of *I*. Then, $\cap_{\lambda\in I-\{\mu\}} U_\lambda \not\subset U_\mu \not\subset \cap_{\lambda\in I-\{\mu\}} U_\lambda$.

Proof. If $\cap_{\lambda\in I-\{\mu\}} U_\lambda \subseteq U_\mu$. Then by Theorem5.3 (2), we have $X = (X- \cap_{\lambda\in I-\{\mu\}} U_\lambda) \cup (\cap_{\lambda\in I-\{\mu\}} U_\lambda) \subseteq U_\mu$. This is contradiction to our assumption. If $U_\mu \subseteq (\cap_{\lambda\in I-\{\mu\}} U_\lambda$, then we have $U_\mu \subseteq U_\lambda$ and hence $U_\mu = U_\lambda$ for any element $\lambda \in (I - \{\mu\})$. This contradicts our assumption that $U_\mu \neq U_\lambda$ when $\lambda \neq \mu$.

Corollary 5.6. Let U_λ be a maximal b-γ-open set, for any $\lambda \in I$ and $U_\lambda \neq U_\mu$ for any $\lambda, \mu \in I$, with $\lambda \neq \mu$. If $\varnothing \neq \delta \subseteq I$, then $\cap_{\lambda\in I-\{\delta\}} U_\lambda \not\subset \cap_{i\in\delta} U_i \not\subset \cap_{\lambda\in I-\{\delta\}} U_\lambda$.

Proof. Let $i \in \delta$. By Theorem 5.5,

$$\cap_{\lambda\in I-\{\delta\}} U_\lambda = \cap_{\lambda\in(I-(\{\delta\}\cup\{i\}))} U_\lambda \not\subset U_i.$$

Therefore, we see $\cap_{\lambda \in I-\{\delta\}} U_\lambda \not\subset \cap_{i\in\delta} U_i$. On the other hand, since $\cap_{i\in\delta} U_i = \cap_{i\in((I-(I-\{\delta\})))} U_i \not\subset \cap_{\lambda \in I-\{\delta\}} U_\lambda$, we have $\cap_{i\in\delta} U_i \not\subset \cap_{\lambda \in I-\{\delta\}} U_\lambda$.

Theorem 5.7. Let U_λ be a maximal b-γ-open set, for any $\lambda \in I$ and $U_\lambda \neq U_\mu$ for any $\lambda, \mu \in I$, with $\lambda\neq\mu$. If $\varnothing \neq \delta \subseteq I$, then $\cap_{\lambda \in I} U_\lambda \subseteq \cap_{i\in\delta} U_i$.

Proof. By Corollary 5.6, we have $\cap_{\lambda \in I} U_\lambda = (\cap_{\lambda \in I-\{\delta\}} U_\lambda) \cap (\cap_{i\in\delta} U_i) \subseteq \cap_{i\in I} U_i$.

Theorem 5.8. (Decomposition Theorem for Maximal b-γ-Open Set). Let $|I| \geq 2$. Let U_λ be a maximal b-γ-open set for any $\lambda \in I$ and $U_\lambda \neq U_\mu$, for any $\lambda, \mu \in I$ with $\lambda \neq \mu$. Then, for any $\mu \in I$,

$$U_\mu = (\cap_{\lambda \in I} U_\lambda) \cup (X - \cap_{\lambda \in I-\{\mu\}} U_\lambda).$$

Proof. Let $\mu \in I$. By Theorem 5.3 (1), we have

$$\begin{aligned}(\cap_{\lambda \in I} U_\lambda) \cup (X - \cap_{\lambda \in I-\{\mu\}} U_\lambda) &= ((\cap_{\lambda \in I-\{\mu\}} U_\lambda) \cap U_\mu) \cup (X - \cap_{\lambda \in I-\{\mu\}} U_\lambda) \\ &= ((\cap_{\lambda \in I-\{\mu\}} U_\lambda) \cup (X - \cap_{\lambda \in I-\{\mu\}} U_\lambda)) \cap (U_\mu \cup ((X - \cap_{\lambda \in I-\{\mu\}} U_\lambda))) \\ &= U_\mu \cup (X - \cap_{\lambda \in I-\{\mu\}} U_\lambda) = U_\mu.\end{aligned}$$

Therefore, we have $U_\mu = (\cap_{\lambda \in I} U_\lambda) \cup (X - \cap_{\lambda \in (I-(\{\mu\}))} U_\lambda)$.

Theorem 5.9. Let I be a finite set and U_λ be a maximal b-γ-open set for any $\lambda \in I$ and $U_\lambda \neq U_\mu$, for any $\lambda, \mu \in I$ with $\lambda \neq \mu$. If $\cap_{\lambda\in I} U_\lambda$ is a b-γ-closed set, then U_μ is a b-γ-closed set, for any $\lambda \in I$, where γ is a b-regular operation.

Proof. By Theorem 5.8, we have $U_\mu = (\cap_{\lambda \in I} U_\lambda) \cup (X - \cap_{\lambda \in I-\{\mu\}} U_\lambda) = (\cap_{\lambda \in I} U_\lambda) \cup (\cup_{\lambda \in I-\{\mu\}} (X - U_\lambda))$. Since *I* is a finite set and γ is b-regular so, $\cup_{\lambda \in I-\{\mu\}} (X - U_\lambda)$ is a b-γ-closed set. Hence, U_μ is a b-γ-closed set by our assumption.

The following Theorem gives a sufficient condition for the set of all maximal b-γ-open sets:

Theorem 5.10. Assume that $|I| \geq 2$. Let U_λ be a maximal b-γ-open set, for any $\lambda \in I$ and $U_\lambda \neq U_\mu$, for any $\lambda, \mu \in I$ with $\lambda \neq \mu$. If $\cap_{\lambda \in I} U_\lambda = \varnothing$, then $\{U_\lambda : \lambda \in I\}$ is the set of all maximal b-γ-open sets of X.

Proof. If there exists another maximal b-γ-open set U_υ of X, which is not equal to U_λ, for any $\lambda \in I$, then $\varnothing = \cap_{\lambda \in I} U_\lambda = \cap_{\lambda \in (I\cup\{\upsilon\})-\{\upsilon\}} U_\lambda$. By Theorem 5.3 (2), we see that $\cap_{\lambda \in (I\cup\{\upsilon\})-\{\upsilon\}} U_\lambda \neq \varnothing$. This contradicts our assumption.

Example 5.11. If each point {x} is b-γ-closed of a space X, then $X - \{a\}$ is a maximal b-γ-open set for any $a \in X$. Moreover, we see that $\{X - \{a\}: a \in X\}$ is the set of all maximal b-γ-open sets of X by Theorem 5.10, since $\cap_{a\in X}(X - \{a\}) U_\lambda = \varnothing$.

6. More about b-γ-radical of maximal b-γ-open sets

In this section, we study the b-γ-closure of b-γ-radical of maximal b-γ-open sets, we begin with a proposition.

Proposition 6.1. Let U_λ be a set, for any $\lambda \in I$. If $\tau_\gamma\text{-bcl}(\cap_{\lambda\in I} U_\lambda) = X$, then $\tau_\gamma\text{-bcl}(U_\lambda) = X$, for any $\lambda \in I$.

Proof. We see that $X = \tau_\gamma\text{-bcl}(\cap_{\lambda\in I} U_\lambda) \subseteq \tau_\gamma\text{-bcl}(U_\lambda)$. It follows that $\tau_\gamma\text{-bcl}(U_\lambda) = X$, for any $\lambda \in I$.

Theorem 6.2. Let I be afinite set and U_λ be a maximal b-γ-open set for any $\lambda \in I$. If τ_γ-bcl$(\cap_{\lambda\in I}U_\lambda) \neq X$, then there exists $\lambda \in I$ such that τ_γ-bcl$(U_\lambda) = U_\lambda$, where γ is a b-regular operation.

Proof. Suppose that τ_γ-bcl$(U_\lambda) = X$ for any $\lambda \in I$. let $\mu \in I$. Since γ is b-regular, so $\cap_{\lambda\in I-\{\mu\}} U_\lambda$ is a b-γ-open set. Also b-regularity of operation γ implies that τ_γ-bcl$(\cap_{\lambda\in I}U_\lambda) = \tau_\gamma$-bcl$((\cap_{\lambda\in I-\{\mu\}}U_\lambda) \cap U_\mu) = \tau_\gamma$-bcl$(\cap_{\lambda\in I-\{\mu\}}U_\lambda) \cap \tau_\gamma$-bcl$(U_\mu) \supseteq (\cap_{\lambda\in I-\{\mu\}}U_\lambda) \cap \tau_\gamma$-bcl$(U_\mu) = (\cap_{\lambda\in I-\{\mu\}}U_\lambda) \cap X = (\cap_{\lambda\in I-\{\mu\}}U_\lambda)$. Therefore, τ_γ-bcl$(\cap_{\lambda\in I-\{\mu\}}U_\lambda) \subseteq \tau_\gamma$-bcl$(\tau_\gamma$-bcl$(\cap_{\lambda\in I}U_\lambda)) = \tau_\gamma$-bcl$(\cap_{\lambda\in I}U_\lambda)$. On the other hand, we see that $\cap_{\lambda\in I}U_\lambda \subseteq \cap_{\lambda\in I-\{\mu\}}U_\lambda$ and hence τ_γ-bcl$(\cap_{\lambda\in I}U_\lambda) = \tau_\gamma$-bcl$(\cap_{\lambda\in I-\{\mu\}}U_\lambda)$.Then, by induction on the element of I, we see that τ_γ-bcl$(\cap_{\lambda\in I}U_\lambda) = \tau_\gamma$-bcl$(U_\lambda) = X$, for any $\lambda \in I$. This contradicts our assumption that τ_γ-bcl$(\cap_{\lambda\in I}U_\lambda) \neq X$. Therefore, we see that there exists $\lambda \in I$ such that τ_γ-bcl$(U_\lambda) = U_\lambda$.

The b-γ-radical of maximal b-γ-open sets have the following outstanding property:

Theorem 6.3. (The b-γ-Closure Law of b-γ-Radical). Let I be finite and U_λ is a maximal b-γ-open set for each $\lambda \in I$. Let $\Gamma \subseteq I$ such that τ_γ-bcl$(U_\lambda) = U_\lambda$ for any $\lambda \in \Gamma$ and τ_γ-bcl$(U_\lambda) = X$ for any $\lambda \in I - \Gamma$. Then, τ_γ-bcl$(\cap_{\lambda\in I}U_\lambda) = \cap_{\lambda\in\Gamma}U_\lambda$ (=X, if $\Gamma = \varnothing$), where γ is a b-regular operation.

Proof. If $\Gamma = \varnothing$, then we have the result by Theorem 6.2. Otherwise $\Gamma \neq \varnothing$, and hence we see that
τ_γ-bcl$(\cap_{\lambda\in I}U_\lambda) = \tau_\gamma$-bcl$((\cap_{\lambda\in\Gamma}U_\lambda) \cap (\cap_{\lambda\in I-\Gamma}U_\lambda)) = \tau_\gamma$-bcl$((\cap_{\lambda\in\Gamma}U_\lambda)) \cap \tau_\gamma$-bcl$((\cap_{\lambda\in I-\Gamma}U_\lambda)) \supseteq (\cap_{\lambda\in\Gamma}U_\lambda) \cap \tau_\gamma$-bcl$(\cap_{\lambda\in I-\Gamma}U_\lambda) = (\cap_{\lambda\in\Gamma}U_\lambda) \cap X = \cap_{\lambda\in\Gamma}U_\lambda$. By Theorem 6.2 and the fact that $(\cap_{\lambda\in\Gamma}U_\lambda)$ is a b-γ-open set. Since γ is b-regular, it follows that τ_γ-bcl$(\cap_{\lambda\in I}U_\lambda) = \tau_\gamma$-bcl$(\tau_\gamma$-bcl$(\cap_{\lambda\in I}U_\lambda)) \supseteq (\tau_\gamma$-bcl$(\cap_{\lambda\in\Gamma}U_\lambda)$. On the other hand, we see that $\cap_{\lambda\in I} U_\lambda \subseteq \cap_{\lambda\in\Gamma} U_\lambda$, and hence τ_γ-bcl$(\cap_{\lambda\in I}U_\lambda) \subseteq \tau_\gamma$-bcl$(\cap_{\lambda\in\Gamma}U_\lambda)$. It follows that τ_γ-bcl$(\cap_{\lambda\in I} U_\lambda)) = \tau_\gamma$-bcl$(\cap_{\lambda\in\Gamma}U_\lambda)$. The b-γ-radical $\cap_{\lambda\in\Gamma}U_\lambda$ is a b-γ-closed set since U_λ is a b-γ-closed set for any $\lambda \in \Gamma$ by our assumption. Therefore, we see that τ_γ-bcl$(\cap_{\lambda\in I} U_\lambda) = \cap_{\lambda\in\Gamma}U_\lambda$.

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Source of support: Nil, Conflict of interest: None Declared