

SEMI SIMPLE NEAR FIELDS GENERATING FROM ALGEBRAIC K-THEORY (SS-NF-G-F-AK-T)

N V Nagendram^{1*}

Assistant Professor (Mathematics), Department of Science & Humanities,
Lakireddy Balireddy College of Engineering, L B Reddy Nagar, Mylavaram 521 230
Andhra Pradesh INDIA.

Dr. T V Pradeep Kumar²

Asst. Professor (Mathematics)
Acharya Nagarjuna University college of Engg.,
522510 Guntur District, Nagarjuna Nagar 522510
Andhra Pradesh, INDIA.

Dr. Y Venkateswara Reddy³

Professor (Mathematics) [Retd.]
ANU College of Engineering, Nagarjuna Nagar
Nagarjuna Nagar 522510, Guntur District
Andhra Pradesh, INDIA.

(Received on: 15-11-12; Revised & Accepted on: 17-12-12)

ABSTRACT

In this paper we discussed about semi simple near fields generating efficiently from algebraic K-theory by defining required fundamental definitions, algorithms wherever necessary from near rings concept explained clearly by Gunter Pilz. Here we extended the concepts and derived some results on and applicable to semi simple near fields, simple near field algebras, near field invariants and near field centralizers keeping precisely under section 1 to section 5.

Key words: near field, simple near field, semi simple near field, semi simple near field algebras, invariants, centralizers.

Subject classification Code: AMS 2000 16D50, 16P20; 16P60.

SECTION 1: INTRODUCTION - SEMI SIMPLE NEAR FIELDS

Definition 1.1: A near field N with 1 is a semi simple, or Left semi simple to be precise. If the free left N -module underlying N is a sum of simple N -module.

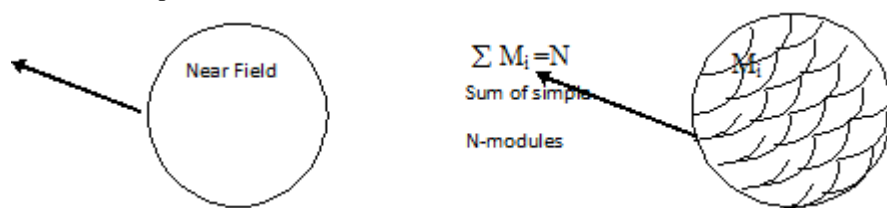


Fig. 1

Definition 1.2: A near field N with 1 is simple or left simple to be precise, if N is semi simple and any two simple left ideals (i.e., any two simple left sub near fields of N) are isomorphic.

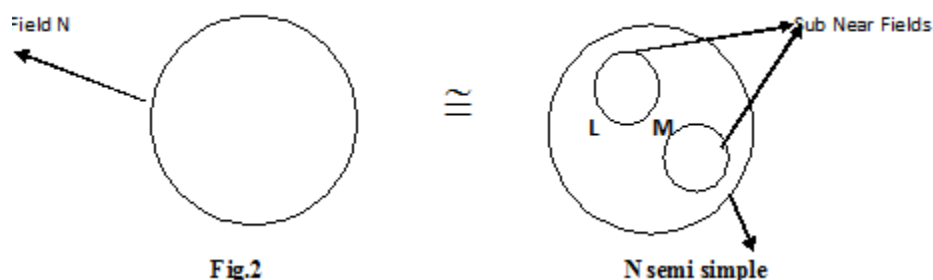


Fig.2

Corresponding author: N V Nagendram^{1*}

Assistant Professor (Mathematics), Department of Science & Humanities,
Lakireddy Balireddy College of Engineering, L B Reddy Nagar, Mylavaram 521 230, (A.P.), India

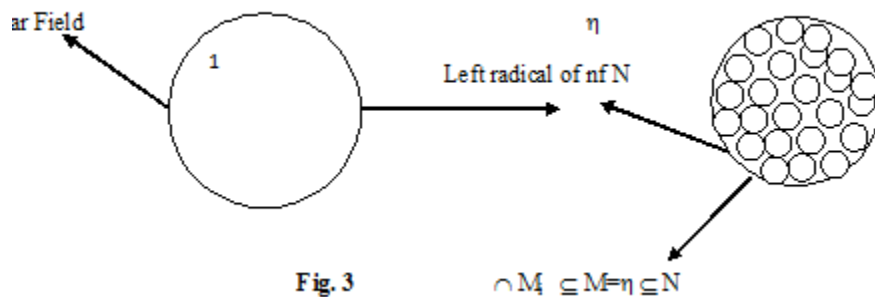
Note 1.3: A near field N is semi simple if and only if there exists a near field S and semi simple S -module M of finite length such that $N \cong \text{End}_S(M)$.

Lemma 1.4: Every semi simple near field is Artinian.

Proposition 1.5: Let N be a semi simple near field. Then N is isomorphic to a finite direct product $\prod N_i$ for all $i=1, 2, \dots, n$ where each N_i is a simple near field.

Proposition 1.6: Let N be a simple near field. Then there exists a division field D and a positive integer n such that $N \cong M_n(D)$.

Definition 1.7: Let N be a near field with 1. Define radical of N to be the intersection of all maximal left ideals of N . The above defined definition uses left N -modules emphasized by me that η is the left radical of near field N .



Proposition 1.8: The radical of a semi simple near field is zero.

Proposition 1.9: Let N be a simple near field. Then N has no non-trivial two sided ideals and its radical is zero.

Proposition 1.10: Let N be an Artinian near field whose radical is zero. Then N is semi simple near field in particular, if N has no non-trivial two sided ideal, Then N is simple near field.

Note 1.11: The standard definition for a near field to be a semi simple is that “Its radical is zero” i.e., $\eta = 0$ Implies $\cap M_i = 0$, where η is radical near field.

Example 1.12: Let Z is not a semi simple near field and the radical of Z is zero, Let N be a simple near field and then N has no non-trivial two sided ideal is true in N .

SECTION 2: MAIN RESULTS ON SEMI SIMPLE AND SIMPLE NEAR FIELD ALGEBRAS

In this section, I studied about Algebras of simple near fields and derived some results in the form of propositions and theorems of course with the help of proving lemmas and fundamental definitions.

Proposition 2.1: Let K be a field. Let P be a central simple near field algebras over K and Let Q be simple near field K -algebra. Then $P \otimes_K Q$ is a simple near field K -algebra. Moreover $Z(P \otimes_K Q) = Z(Q)$. i.e., every element of the center of $P \otimes_K Q$ has the form $1 \otimes q$ for a unique element $q \in Z(Q)$. In particular, $P \otimes_K Q$ is a central simple near field algebra over K if both P and Q are.

Proof: Let us assume for simplicity of exposition that $\dim_K(Q) < \infty$.

The proof works for the element $x = \sum p_i \otimes q_i \in P \otimes_K Q$ for $i=1, 2, \dots$ let $p_i \in P$ and $q_i \in Q$. Let I be any ideal or sub near field in $P \otimes_K Q$. let x be a non-zero element of I of minimal length. After re-labelling the q_i s we may and do assume that x has the form $x = 1 \otimes b_1 + \sum a_i$ for $i=2, 3, \dots, r$.

Let us consider $[p \otimes 1, x] \in I$ with $p \in P$ whose length is less than the length of x .

So $[p \otimes 1, x] = 0$ for all $p \in P$. i.e., $[p, p_i] = 0$ for all $p \in P$ and $i=2, 3, \dots, r$. In other words, $p_i \in K$ for all $i=2, 3, \dots, r$.

Write $p_i = \lambda_i \in K$ and $x = 1 \otimes q \in I$ where $q = q_1 + \lambda_2 q_2 + \dots + \lambda_r q_r \in Q$, $q_1 = 1$.

Hence, $1 \otimes q \in I$. since, Q is simple near field, we have $QqQ = Q$ and so $I = P \otimes_K Q$. we have shown that $P \otimes_K Q$ is simple near field algebras.

Let $x = \sum p_i \otimes q_i \in Z(P \otimes_K Q)$ with $p_1, p_2, \dots, p_r \in P$ we have $U = [p \otimes 1, x] = \sum [p, p_i] \otimes q_i$ for all $p \in P$.

Hence $p_i \in Z(P) = K$ for $i=1, 2, \dots, r$ and $x = 1 \otimes q$ for some $q \in Q$. The condition that $0 = [1 \otimes y, x]$ for all $y \in Q$ implies that $y \in Z(Q)$ and hence $x \in 1 \otimes Z(Q)$. This completes the Proof of the proposition.

Corollary 2.2: Let P be a finite dimensional simple near field algebras over a field K , and let $n = \dim_K(P)$. If P is a central simple near field algebras over K , then $P \otimes_K P^{opp} \rightarrow \text{End}_K(P) \approx M_n(K)$. Conversely, if $P \otimes_K P^{opp} \rightarrow \text{End}_K(P)$, then P is a central simple near field algebras over K .

Proof: Suppose that P is a central simple near field algebras over K . By proposition 2.1, $P \otimes_K P^{opp}$ is a central simple near field algebras over K . Let us consider a mapping, $\alpha : P \otimes_K P^{opp} \rightarrow \text{End}_K(P)$ which sends $x \otimes y$ to the element $u \rightarrow x u y \in \text{End}_K(P)$. The source of α is simple by known proposition 2.1, so α is injective mapping because it is clearly non-trivial. Hence it is an isomorphism because the source and the target have the same dimension over field K .

Conversely, suppose that $P \otimes_K P^{opp} \rightarrow \text{End}_K(P)$ and I is a proper ideal of A . Then the image of $I \otimes_K P^{opp}$ in $\text{End}_K(P)$ is a sub near filed and ideal of $\text{End}_K(P)$ which does not contain Id_P . So P is a simple near field K -Algebra. Let us define $L := Z(P)$, then the image of the canonical mapping $P \otimes_K P^{opp}$ in $\text{End}_K(P)$ lies in the sub-algebra $\text{End}_L(P)$, hence $L = K$. This completes the Proof of the corollary.

Lemma 2.3: Let D be a finite dimensional central simple near filed over an algebraically closed near field K then $D = K$.

Corollary 2.4: The dimension of any central simple near field algebra over a near filed K is a perfect square.

Lemma 2.5: Let P be a finite dimensional central simple near field over a near filed K . Let $F \subseteq P$ be an “over-field” of K contained in P . then $[F : K] \mid [P : K]^{1/2}$. In particular if $[F : K]^2 = [P : K]$, then F is a maximal sub - near field of P .

Proof: Write $[P : K] = n^2$, $[F : K] = d$. Multiplication on the left defines an embedding $P \otimes_K F \rightarrow \text{End}_F(P)$.

So, $n^2 = [P \otimes_K F]$ divides $[\text{End}_F(P) : F] = (n^2/d)^2$, i.e., $d^2 \mid n^2$. so d divides n . This completes the Proof of the lemma.

SECTION 3: MAIN RESULTS ON SEMI SIMPLE AND SIMPLE NEAR FIELD ALGEBRAS

Lemma 3.1: Let P be a finite dimensional central simple near filed algebra over a near filed K . If F is a sub- near filed of P containing K , and $[F : K]^2 = [P : K]$, then F is a maximal sub near field of K and $P \otimes_K F \cong M_n(F)$, where $n = [P : K]^{1/2}$.

Proof: Since F is a maximal sub near field of P . consider the natural map $\alpha : P \otimes_K F \rightarrow \text{End}_K(P)$, which is injective because $P \otimes_K F$ is simple near field and α is non trivial. Since the dimension of the source and the target of α are both equal to n^2 , α is an isomorphism.

Proposition 3.2: Let P be a central semi simple near field algebra over a near field K . Then there exists a finite separable semi simple near filed extension F / K such that $P \otimes_K F \cong M_n(N)$, where $n = [P : K]^{1/2}$.

Proof: It suffices to show that $\otimes_K F \cong M_n(K^{sep})$. Changing notation, we may assume that $K = K^{sep}$. By Wedderburn's theorem, we know that $P \cong M_n(D)$, where D is a central division algebra over $K = K^{sep}$. Write $n = md$ and $[D : K] = d^2$, $d \in \mathbb{N}$. Suppose that $D \neq K$, i.e., $d > 1$. The char $(K) = p > 0$, and every element of D is purely inseparable over K . There exists a power q of p such that $x^q \in K$ for every element x in D . Then for the central semi simple near filed algebra $B := D \otimes_K (K^{alg}) \cong M_n(K^{alg})$, we have y^q belongs to K^{alg} for every element $y \in B \cong M_d(K^{alg})$. The last statement is clearly false, since $d > 1$. This completes the Proof of the proposition.

Theorem 3.3: Let Q be a finite dimensional central semi simple near - fields algebra over a near field K . Let P_1, P_2 be simple K – sub-algebras of Q . Let $\theta : P_1 \xrightarrow{\sim} P_2$ be a K – linear isomorphism of K -algebra. Then there exists an element $t \in Q^x$ such that $\theta(y) = t^{-1} y t$ for $y \in P_1$.

Proof: Consider the semi simple near field K -algebra $N := Q \otimes_K P_1^{opp}$, and two N -module structures on the K -vector space V underlying B : an element $u \otimes a$ with $u \in Q$ and $a \in P_1^{opp}$ operates either as $b \mapsto uba$ for all $b \in V$, or as $b \mapsto ub\phi(a)$ for all $b \in V$. Hence there exists a $\psi \in GL_K(V)$ such that $\psi(uba) = u \psi(b) \phi(a)$ for all $u, b \in B$ and all $a \in P_1$. One can check easily that $\psi(1) \in Q^x$: if $u \in Q$ and $u \bullet \psi(1)$ for every $a \in P_1$. This completes the Proof of the theorem.

Theorem 3.4: Let Q be a K -algebra and let P be a finite dimensional central simple K -sub-algebra of Q . Then the natural homomorphism $\alpha : P \otimes_K Z_B(A) \rightarrow Q$ is an isomorphism.

Proof: Passing from K to K^{alg} , we may add do assume that $P \cong M_n(K)$, and we fix an isomorphism $A \xrightarrow{\sim} M_n(K)$.

Firstly, we show that α is surjective. Given an element $b \in Q$, define elements $b_{ij} \in Q$ for $1 \leq i, j \leq n$ by $b_{ij} :=$

$\sum_{k=1}^n e_k i b e_j k$, where e_{ki} belongs to $M_n(K)$ is the $n \times n$ matrix whose (k, i) -entry is equal to 1 and all other entries equal

to 0. One can check that each b_{ij} commutes with all elements of $P = M_n(K)$. The following computation $\sum_{i,j=1}^n b_{ij} e_{ij} =$

$\sum_{i,j,k} e_k i b e_j k e_{ij} = \sum_{i,j=1}^n e_i i b e_j i = b$ shows that α is surjective.

Suppose that $0 = \sum_{i,j=1}^n b_{ij} e_{ij}$, $b_{ij} \in Z_Q(P)$ for all $1 \leq i, j \leq n$. Then $0 = \sum_{k=1}^n e_k i \left(\sum_{i,j=1}^n b_{ij} e_{ij} \right) e_{mk} = b_{lm} \forall 0 \leq l, m \leq n$.

Hence α is injective. This completes the Proof of the theorem.

Theorem 3.5: Let Q be finite dimensional central semi simple near-field algebra over a field K , and let P be a semi simple near-field K -sub-algebra of Q . Then $Z_Q(P)$ is semi simple near-field and $Z_Q(Z_Q(P)) = P$.

Proof: Let $C = \text{End}_K(P) \cong M_n(K)$, where $n = [P:K]$. Inside the central semi simple near-field K -algebra $Q \otimes_K C$ we have two semi simple near-field K -sub-algebras, $P \otimes_K K$ and $K \otimes_K P$. here the right factor of $K \otimes_K P$ is the image of P in $C = \text{End}_K(P)$ under left multiplication. Clearly these two semi simple near-field K -sub-algebras $Q \otimes_K C$ isomorphic, since both are isomorphic to P as a K -algebra. By Noether-Skolem these two sub-algebras are conjugate in $Q \otimes_K C$ by a suitable element of $(Q \otimes_K C)^{\times}$, therefore their centralizers i.e., respective double centralizers in $Q \otimes_K C$ are conjugate, hence isomorphic.

Let's compute the centralizers first: $Z_{Q \otimes_K C}(P \otimes_K K) = Z_Q(P) \otimes_K C$, While $Z_{Q \otimes_K C}(K \otimes_K P) = Q \otimes_K P^{\text{opp}}$.

Since $Q \otimes_K P^{\text{opp}}$ is central semi simple near-field over K , so is $Z_Q(P) \otimes_K C$. Hence $Z_Q(P)$ is semi simple near-field.

We compute the double centralizers: $Z_{Q \otimes_K C}(Z_{Q \otimes_K C}(P \otimes_K K)) = Z_{Q \otimes_K C}(Z_Q(P) \otimes_K C) = Z_Q(Z_Q(P)) \otimes_K C$,

While $Z_{Q \otimes_K C}(Z_{Q \otimes_K C}(K \otimes_K P)) = Z_{Q \otimes_K C}(Q \otimes_K P^{\text{opp}}) = K \otimes_K P$

So $Z_Q(Z_Q(P))$ is isomorphic to P as K -algebras. Since $P \subseteq Z_Q(Z_Q(P))$, the inclusion is an equality.

This completes the Proof of the theorem.

SECTION 4: SOME INVARIANTS ON SEMI SIMPLE AND SIMPLE NEAR FIELD ALGEBRAS

In this section, I studied invariants on semi simple near field algebras and simple near filed algebras.

Lemma 4.1: Let K be a semi simple near field and let P be a finite dimensional semi simple K -algebra. Let M be an (P, P) -bi-module. Then M is free as a left P -module.

Definition 4.2: Let K be a semi simple near field, Q be a K -algebra, and let P be a finite dimensional semi simple K -sub-algebra of Q . Then Q is a free left P -module by lemma 4.1. We define the rank of Q over P , denoted $[Q : P]$, to be the rank of Q as a free left P -module. Clearly $[Q : A] = \dim_K(Q) / \dim_K(P)$ if $\dim_K(P) < \infty$.

Definition 4.3: Let K be a semi simple near field. Let Q be a finite dimensional semi simple K -alg K -algebra, and let P be a simple K -sub-algebra of P . Let N be a left simple Q -module, and let M be a left simple P -module.

Define $i(Q, P) := \text{length}_Q(Q \oplus_P M)$, is called the index of P in Q .

Define $h(Q, P) := \text{length}_P(N)$, called the height of Q over P .

Recall the $[Q: P]$ is the P - rank of Q_s , where Q_s is free left P -module underlying is free left P -module underlying Q .

Note 4.4: Let $P \subset Q \subset R$ be inclusion of semi simple near field algebras over a near field K . Then

$$I(R, P) = i(R, Q) i(Q, A), h(R, P) = h(R, Q) h(Q, P) \text{ and } [R: P] = [R: Q] [Q: A].$$

Lemma 4.5: Let K be an algebraically closed semi simple near field. Let Q be a finite dimensional semi simple K -algebra, and let P be a semi simple K -sub-algebra of Q . Let M be a semi simple P -module, and let N be a semi simple Q -module.

- (i) N contains M as a left P -module.
- (ii) the following equalities hold good.

$$\begin{aligned} \dim_K(\text{Hom}_Q(Q \otimes_P M, N)) &= \dim_K(\text{Hom}_P(M, N)) \\ &= \dim_K(\text{Hom}_P(N, M)) \\ &= \dim_K(\text{Hom}_Q(N, \text{Hom}_P(Q, M))) \end{aligned}$$

- (iii) Assume in addition that P is simple. Then $i(Q, P) = h(Q, P)$.

Proof: Statements (i) and (ii) are easy and the statement (iii) follows from the first equality in (ii). This completes the proof.

Lemma 4.6: Let P be a semi simple near field algebra over a field K . Let M be a non-trivial finitely generated left P -module, and Let $P' := \text{End}_P(M)$. Then $\text{length}_P(M) = \text{length}_{P'}(P_s')$, where P_s' is the left P_s' - module underlying P' .

Proof: Write $M \cong U^n$, where U is a semi simple P -module. Then $P' \cong M_n(D)$, where $D := \text{End}_P(U)$ is division algebra. So $\text{length}_{P'}(P_s') = n = \text{length}_P(M)$. This completes the proof of lemma.

SECTION 5: SOME CENTRALIZERS ON SEMI SIMPLE AND SIMPLE NEAR FIELD ALGEBRAS

In this section, I studied Centralizers on semi simple near field algebras and simple near field algebras.

Theorem 5.1: Let K be a semi simple near field. Let Q be a finite dimensional central semi simple near field algebra over K . Let P be a semi simple near field K -sub-algebra of Q , and let $P' := Z_Q(P)$. Let $L = Z(P) = Z(P')$. Then the following holds good.

- [i] P' is a semi simple near field K -algebra
- [ii] $P = Z_Q(Z_Q(P))$.
- [iii] $[Q: P'] = [K: K]$, $[Q: P] = [P': K]$, $[Q: K] = [P: K] [P': K]$
- [iv] P and P' are linearly disjoint over L and [v] If P is a central semi simple near field algebras over K , then $P \otimes P' \xrightarrow{\sim} Q$.

Proof: Is obvious.

Proposition 5.2: Let P be a finite dimensional central semi simple near field algebra over K . Let F be an extension semi simple near field of K such that $[F: K] = n = [P: K]^{1/2}$. Then there exists a K -linear near field homomorphism $F \mapsto P$ if and only if $P \otimes_K F \cong M_n(F)$.

Proof: Is obvious.

Theorem 5.3: Let K be a semi simple near field and let Q be a finite dimensional central semi simple near field algebra over L . Let N be a non-trivial Q -module of finite length. Let P be a semi simple near field K -sub-algebra of Q . Let $P' := Z_Q(A)$ be the centralizer of P in Q . then we have a natural isomorphism say $\text{End}_Q(N) \otimes_K P' \xrightarrow{\sim} \text{End}_P(N)$.

Proof: We know that P' is a semi simple near field K -algebra, and $H := \text{End}_Q(N)$ is a central semi simple near field K -algebra. So $H \otimes_K P'$ is a semi simple near field K -algebra. Let J be the image of $H \otimes_K P'$ in $\text{End}_P(N)$; clearly we have $H \otimes_K P' \xrightarrow{\sim} J$. Let $S := \text{End}_K(N)$; Let $J' := \text{End}_J(N)$. Further, we have

$J' = \text{End}_H(N) \cap \text{End}_{P'}(N) = Q \cap Z_S(P') = Z_B(P') = P$. the second and fourth equality follows from the double centralizer theorem. Hence $J = \text{End}_P(N)$. This completes the proof.

Note 5.4: Notation as in prop. 5.3. Let $L := Z(P) = Z(P')$. Then $[P \otimes_L Z_Q(P)]$ and $[Q \otimes_Q L]$ are equal as elements of $\text{Br}(L)$.

REFERENCES

- [01] A. Dress, on the decomposition of modules, BZ & Anaer. & fath. Sot. 75 (1969), 984-986.
- [02] Bass I, II, K-theory and stable algebra, Pub. Math. I.H.E., 8. No.22 (1964), 5-m.
- [03] Bass T-I, Modules which support non-singular forms, j. Algebra 13(1969), 246-252.
- [04] Bass II, "algebraic K-theory ", Benjamin, Menlo Park, Cal., 1968.
- [05] D Eisenbud and E G Evans, Every Algebraic set in N-space is the intersection of N Hyper surfaces, invent. Math. 19(1973), 107-112.
- [06] A Note on Asymptotic value of the Maximal size of a Graph with rainbow connection number $2*(AVM-SGR-CN2^*)$ IJAA, Jordan @ Research India Publications, Rohini, New Delhi, ISSN 0973-6964 Volume 5, Number 2 (2012), pp. 103-112.
- [07] Polynomials over Euclidean Domain in Noetherian Regular Delta Near Ring Some Problems related to Near Fields of Mappings(PED-NR-Delta-NR & SPR-NF) IJMA, An International Peer Review Journal for Mathematical, Science & Computing Professionals ISSN: 2229-5046, Vol.3(8), pp no.2998 – 3002 August, 2012
- [08] Near Left almost near-fields (N-LA-NF) Conference: IJMSA@ mindreader publications, New Delhi on 23-04-2012 also for publication.
- [09] A Generalized near fields and (m, n) Bi-Ideals over Noetherian regular Delta-near rings (GNF-(m, n) BI-NR-delta-NR) TMA, Greece, Athens, dated 08-04-2012.
- [10] Applications of Linear Programming on optimization of cool freezers (ALP-on-OCF) IJPAM-2012-17-670 ISSN-1314-0744 Vol-75 No-3(2011).
- [11] A Note on Algebra to spatial objects and Data Models(ASO-DM) AJMMS, USA, Copyright @ Mind Reader Publications, Rohini, New Delhi, ISSN. 2250-3102, Vol.1, No.2 (Dec. 2012), pp. 233 – 247.
- [12] A Note on Pi-Regularity and Pi-S-Unitality over Noetherian Regular Delta near rings (PI-R-PI-S-U-NR-Delta-NR) IeJPAM-2012-17-669 ISSN-1314-0744 Vol-75 No-4(2011).
- [13] Ideal Comparability over Noetherian Regular Delta near rings(IC-NR-Delta-NR) IJAA, Jordan, @ Research India publications, Rohini, New Delhi, ISSN 0973-6964 Vol:5, NO:1(2012), pp.43-53.
- [14] A NOTE ON SUFFICIENT CONDITION OF HAMILTONIAN PATH TO COMPLETE GRPHS (SC-HPCG)", IJMA-2(11), 2011, pp.1-6.
- [15] On Noetherian Regular Delta near rings and their Extensions (NR-delta-NR)" IJCMS, Bulgaria, IJCMS-5-8-2011, Vol.6, 2011, No.6, 255-262.
- [16] On Semi Noetherian Regular Matrix Delta near rings and their Extensions (SNRM-delta-NR) IJAA. Jordan© Research India Publications ISSN 0973-6964 Volume 4, Number 1 (2011), pp.51-55.
- [17] On Boolean Noetherian Regular Delta near ring(BNR-delta-NR) IJCM Int. J. of Contemporary Mathematics, Vol. 2, No. 1-2, Jan-Dec 2011, Mind Reader Publications, ISSN No: 0973-6298, pp. 23-27.
- [18] On Bounded Matrix over a Noetherian Regular Delta near rings(BMNR-delta-NR) IJCM, Vol. 2, No. 1-2, Jan-Dec 2011, Copyright @ Mind Reader Publications, ISSN No: 0973-6298, pp.11-16.
- [19] On Strongly Semi Prime over Noetherian Regular Delta near rings and their Extensions (SSPNR-delta-NR) IJCM, Copyright@ Mind Reader Publications, Vol. 2, No. 1, Jan-Dec 2011, ISSN No: 0973-6298, pp.69-74.
- [20] On IFP Ideals on Noetherian Regular Delta near rings(IFPINR-delta-NR) IJCM, Copyright@Mind Reader Publications, Vol. 2, No. 1-2, Jan-Dec 2011, ISSN No: 0973-6298, pp.43-46.
- [21] On Structure Theory and Planar of Noetherian Regular delta-near-rings (STPLNR-delta-NR) IJSMA, New Delhi December 18, 2011, ISSN, Vol., No., pp:79-83.

- [22] On Matrix's Maps over Planar of Noetherian Regular delta-near-rings (MMPLNR-delta-NR) IJSMA, New Delhi December 18, 2011 ISSN. Vol., No., pp. 203-211.
- [23] Some Fundamental Results on P- Regular delta-near-rings and their extensions (PNR-delta-NR) IJCM, Jan-Dec 2011, Copyright @ Mind Reader Publications, ISSN: 0973-6298, vol.2, No.1-2, PP.81-85.
- [24] A Generalized ideal based-zero divisor graphs of Noetherian regular Delta-near rings (GIBDNR- d-NR) MA, Greece, Athens, ISSN: 1792- 9687 vol.1, no.1, 2011, 59-71, 1792-9709 (online), International Scientific Press, 2011.
- [25] Inversive Localization of Noetherian regular Delta-near rings (ILNR- Delta-NR) IJPAM-ISSN.1314-0744 vol-75 No-3, pp No. SOFIA, 2012-17-668, Bulgaria.
- [26] A note on Relations between Barnette and Sparse Graphs IJMA, An International Peer Review Journal for Mathematical, Science & Computing Professionals, ISSN 2229 – 5046 Vol. 2, No.12, pp no.2538-2542, 2011.
- [27] On Semi Modules over Artinian Regular Delta near rings(S Modules-AR-Delta-NR) IJMA", An International Peer Review Journal for Mathematical, Science & Computing Professionals ISSN: 2229-5046, Vol.3 (8), pp No.2991 - 2997, August, 2012.
- [28] Optical Near field Mapping of Plasmonic Nano Prisms over Noetherian Regular Delta near fields (ONFMPN-NR-Delta-NR) Conference: IJMSA @ mind reader publications, New Delhi going to conduct on 15 – 16 th December 2012 also for publication.
- [29] A Note on Present Trends on Yoga Apart From Medicine Usage and Its Applications (PTYAFMUIA). International Association of Journal of Yoga Thrapy, IAYT, Houston U.S.A. September issue, 2012.
- [30] A Note on Finite Pseudo Artinian Regular Delta near fields (FP AR-Delta-NF) IJAA, Jordan communicated on 22 nd August 2012.
- [31] Generating near fields Efficiently: Theorems From Algebraic K-Theory (G-NF-E-TFA-K-T)" communicated to and provisionally accepted by an International Journal of Mathematical Archive (IJMA)", An International Peer Review Journal for Mathematical, Science & Computing Professionals on 20th September, 2012.

Source of support: Nil, Conflict of interest: None Declared