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ON FUZZY SUPPER CONCEPTS IN FUZZY TOPOLOGICAL SPACES

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ABSTRACT

In this paper we define the cocept of fuzzy supper closure, interior and continuous functions. We also define fuzzy supper compact space, ultra normal, almost saupper compact space. Some theorems related to them are established.

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1. INTRODUCTION

Ever since, the introduction of fuzzy set by Zadeh [5] and fuzzy topological space by Chang [1] several authors have tried successfully to generalize numerous pivot concepts of general topology to the fuzzy setting. We intend to introduce the concept of fuzzy supper continuity and which plays a vital role in the theory that we have developed in this paper. We have introduced the notion of fuzzy supper compactness fuzzy ultra normal and fuzzy almost supper compactness.

2. PRELIMINARIES

The concepts of fuzzy topologies are standard by now and can be referred from [1, 3, 5]. We refer to [1] for various definitions of fuzzy topology, fuzzy continuity, fuzzy open map throughout this paper. I denotes X and Y are fuzzy topological space and denotes the unit interval. The set of all fuzzy sets on a non empty set X. O_X and l_x denote the constant fuzzy sets taking values 0 and 1 on x. We denote the fuzzy closure and fuzzy interval as $\overline{A}$ and A^0 respectively. We need the following definitions and results for our subsequent use.

Definition 2.1: A family $\delta \subseteq I^X$ of fuzzy sets is called a fuzzy topology for X if it satisfies the following three axioms:

1. $\overline{0}_X$, $\overline{1}_X$ $\in \delta$ where $\overline{0}_X$ (x) = 0, $\overline{1}_X$ (x) = 1, for every x $\in$ X.
2. $A \wedge B \in \delta$, for every $A, B \in \delta$
3. $\bigvee_{j\in J} A_j \in \delta$ for every $(A_j)_{j\in J} \in \delta$

Example 2.2: Let X = {a, b}. Let A be a fuzzy set on X defined as A(a) = 0.1, A(b) = 0.2. Then $\delta = \{\overline{0}, A, \overline{1}\}$ is a fuzzy topology and (X, δ) is a fuzzy topological space.

Definition 2.3: Let (X, δ) be a fuzzy topological space. A fuzzy point is a fuzzy set in X which takes the value '0' for all y ∈ X except one, say x ∈ X. Denote it by P_x^λ when its value at x is λ ($0 \le \lambda \le 1$).

The point 'x' is called the support of P_x^λ. Let the class of all fuzzy points be denoted by χ.

Definition 2.4: The elements of δ are called fuzzy open sets and a fuzzy set B is fuzzy closed if $B^c \in \delta$.

Definition 2.5: Let closure $\overline{A}$ and interior A^0 of a fuzzy set A of X are defined as

clA = $\overline{A}$ = inf $\{K : A \le K, K^c \in \delta\}$
IntA=A^0 = sup $\{O: O \le A, O \in \delta\}$ respectively.

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3. FUZZY SUPPER CONTINUITY

Definition3.1: Let A be a fuzzy set on (X,δ) the fuzzy topological space.

The **fuzzy supper closure** of A is defined as the set of all fuzzy points P_x^λ in X such that $\overline{U} \wedge A \neq \overline{0}_X$ for every $U \in \delta$ containing P_x^λ. It is denoted as A^+. That is, $A^+ = \{ P_x^\lambda \in \chi : \overline{U} \wedge A \neq \overline{0}_X$ for every $U \in \delta$ containing $P_x^\delta \}$

The fuzzy supper interior of A is defined as the set of all fuzzy points P_x^λ in A such that $\overline{U} \leq A$ for every $U \in \delta$ containing the point P_x^λ.

$A^- = \{ P_x^\lambda \in A: \overline{U} \leq A$ for every $U \in \delta$ containing $P_x^\lambda \}$

The fuzzy set A is fuzzy-supper closed if $A^+ = A$ and fuzzy supper open if $A^- = A$. A is fuzzy supper-open if and only if A is fuzzy supper-closed.

Definition 3.2: A mapping f: X → Y from a fuzzy topological space (X, δ_1) to a fuzzy topological space (Y, δ_2) is said to be fuzzy supper-continuous if the inverse image of every fuzzy closed set in Y is fuzzy supper closed in X.
(ie) $f^{-1}(A)$ is fuzzy supper-closed set of X for every fuzzy closed set A of Y.
(or) inverse of every fuzzy open set of Y is fuzzy supper open in X.
(ie) $f^{-1}(U)$ is fuzzy supper-open set of X for every fuzzy open set U of Y.

Definition 3.3: A mapping f: X → Y is fuzzy strongly continuous if f(cl A) ≤ f(A) for every fuzzy subset A of X.

Definition 3.4: A mapping f: X → Y is fuzzy closure continuous if for every P_x^λ and fuzzy open set V in Y, such that $f(P_x^\lambda) \in V$ there exists a fuzzy open set U in X containing P_x^λ and satisfies f (cl U) ≤ cl V.

REMARK

Supper continuous mapping have been studied in topological space by Talal Ali. Al-Haway [4]. Here we generalize this concept to the fuzzy settings.

Theorem 3.5: For f: X → Y where X and Y are fuzzy spaces,
(i) f is fuzzy strongly continuous implies f is fuzzy continuous.
(ii) f is surjective fuzzy strongly continuous implies f is fuzzy supper continuous.
(iii) f is fuzzy supper continuous implies f is fuzzy closure continuous.

Definition 3.6: A fuzzy space X is said to be
(i) fuzzy supper compact if every fuzzy supper open cover has a finite subcover.
(ii) Fuzzy almost supper compact if every fuzzy supper opencover of X has a finite subfamily the closures of whose members cover X.

Theorem3.7: A surjective fuzzy supper continuous image of a fuzzy almost supper compact space is fuzzy compact.

Proof: Let f : X → Y be fuzzy supper continuous and X be the fuzzy almost supper compact space. Let $\{U_i\}_{i\in I}$ be the fuzzy open cover of Y.

Since f is fuzzy supper continuous, it follows that, $\{f^{-1}(U_i)\}_{i\in I}$ is a fuzzy supper open cover for X. Since the space X is fuzzy almost supper compact, there exists a finite K of I such that,

$$\bigvee_{i\in K} f^{-1}(U_i) = 1_X$$

Since f is surjective, we have

$$f\left(\bigvee_{i\in K} f^{-1}(U_i)\right) = 1_Y$$

(ie) $\bigvee_{i\in K} f(f^{-1}(U_i)) = 1_Y$

(ie) $\bigvee_{i\in K} U_i = 1_Y$.

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NOTE
Every fuzzy supper continuous map is both fuzzy continuous and fuzzy semi-continuous.

Corollary 3.8:
(i) A surjective fuzzy continuous image of a fuzzy supper almost compact space is fuzzy compact.
(ii) A surjective fuzzy semi-continuous image of fuzzy supper almost compact space is fuzzy compact.

Definition3.9: A space X is fuzzy regular every open set U can be written in form U = $\bigvee_{i\in I} U_i$ where U_i are fuzzy open sets with $clU_i \le U$.

Definition3.10: A space X is fuzzy supper regular if every open set U can be written in the form U = $\bigvee_{i\in I} U_i$ where U_i are fuzzy supper open sets with (supper closure U_i) ≤ U (ie) $U_i^+ \le U$.

Theorem3.11: Let f: X → Y be a fuzzy supper continuous and fuzzy closed injection then X is fuzzy supper regular if Y is fuzzy regular.

Proof: Let A be a fuzzy open set of X. Then f (A′) is fuzzy closed set of Y. Y being fuzzy regular we have f (A′)′ = $\bigvee_{i\in J} U_i$ where U_i are fuzzy open sets of Y with cl $U_i \le f(A')'$.

Since f is fuzzy supper continuous and injective,

$$A = f^{-1}(f(A')') = f^{-1}\left(\bigvee_{i\in J} U_i\right) = \bigvee_{i\in J} f^{-1}U_i$$

where $f^{-1}(U_i)$ are fuzzy supper open sets of X with

$$[f^{-1}(U_i)]^+ \le f^{-1}(clU_i) \le A$$

Thus X is fuzzy supper regular.

Definition 3.12: A space X is fuzzy normal if K and U are fuzzy closed and fuzzy open sets of X such that K ≤ U then f (K) ⊆ f (U′)′.

Definition 3.13: A space X is fuzzy supper normal if for any closed set K and fuzzy open set U such that K ≤ U, there exists a fuzzy set V such that

$$K \le A^- \le A^+ \le U.$$

Theorem 3.14: Let f: X → Y be a fuzzy supper continuous map and fuzzy closed injection then X is fuzzy supper normal if Y is fuzzy normal.

Proof: Let Y be fuzzy normal. Let K and U be, fuzzy closed and fuzzy open sets of X such that K ≤ U. Thus f (K) ⊆ f (U′)′ Y being fuzzy regular, there exists a fuzzy set V of Y such that

$$f(K) \le \text{int } V \le clV \le f(U')'.$$

Since f is fuzzy supper continuous and injective, we have

$$K = f^{-1}(f(K)) \le f^{-1}(\text{int } V) \le \text{int } f^{-1}(v) \le (f^{-1}(V))^+$$

$$\le f^{-1}(V) \le f^{-1}(V)^- \le \text{cl } f^{-1}(V) \le f^{-1}(\text{cl } V) \le U$$

$$\therefore \quad K \le (f^{-1}(V))^+ \le (f^{-1}(V))^- \le U.$$

4. FUZZY SEMI SUPPER CONTINUOUS FUNCTION

Definition4.1: A mapping f: X → Y from a space X to a fuzzy space Y is said to be a fuzzy semi supper continuous function if for every fuzzy semi open set U of Y, $f^{-1}(U)$ is fuzzy supper open in X.

Theorem4.2: A fuzzy semi-supper continuous image of a fuzzy supper compact space is fuzzy semi compact.

Proof: Let f: X → Y be a fuzzy supper semi continuous mapping of a fuzzy supper compact space X to a fuzzy space Y.

Suppose $\{U_i\}_{i\in I}$ be a fuzzy semi open cover of Y. Since f is fuzzy supper semi-continuous $\{f^{-1}(U_i)\}_{i\in I}$ is a fuzzy supper open cover of X.

Since X is fuzzy supper compact, there exists a finite subset K of I such that,

$$\bigvee_{i\in K} f^{-1}(U_i) = 1_X.$$

By surjectivity of f, we have

$$f\left(\bigvee_{i\in K} f^{-1}(U_i)\right) = \bigvee_{i\in K} f^{-1}(f(U_i)) = \bigvee_{i\in K} U_i = 1_Y$$

Thus Y is fuzzy semi compact.

Definition4.3: A fuzzy space X is said to be fuzzy semi T_0, if every fuzzy set U of X can be written in the form $U = \bigvee_{i\in J} \bigwedge_{j\in J} U_{ij}$ where U_{ij} are fuzzy semi open or fuzzy semi closed sets.

Definition 4.4: A fuzzy space X is said to be supper fuzzy T_0 if every fuzzy set U of X can be written in the form $U = \bigvee_{i\in J} \bigwedge_{j\in J} U_{ij}$ where U_{ij} are fuzzy supper open sets of X.

Theorem4.5: Let f: X → Y be an injective fuzzy semi-supper continuous image of a fuzzy space X to a fuzzy space Y. If Y is fuzzy semi T_0 then X is supper fuzzy T_0.

Proof: Let A be a fuzzy set of X. Then f (A) being a fuzzy set of a fuzzy semi T_0 space Y, can be written in the form $f(A) = \bigvee_{i\in I} \bigwedge_{j\in J_i} U_{ij}$ where U_{ij} are fuzzy semi open or fuzzy semi closed sets of Y. Since f is fuzzy semi-supper continuous, injection we have

$$\begin{aligned} A &= f^{-1}(f(A)) \\ &= f^{-1}\Big(\bigvee_{i\in I} \bigwedge_{j\in J_i} U_{ij}\Big) \\ &= \bigvee_{i\in I} \bigwedge_{j\in J_i} f^{-1}(U_{ij}) \end{aligned}$$

where $f^{-1}(U_{ij})$ are fuzzy supper open sets of X.

Thus X is supper fuzzy T_0.

Definition4.6: A mapping f: X → Y from a fuzzy space X to fuzzy space Y is said to be fuzzy supper clopen mapping. If f(U) is fuzzy supper clopen set of Y for each fuzzy supper clopen set (both fuzzy supper closed & fuzzy supper open) of X.

Definition4.7: A space X is said to be fuzzy supper ultra normal if for any fuzzy closed set A and fuzzy open set B such that A ≤ B there exists a fuzzy supper clopen set U such that A ≤ U ≤ B.

Theorem 4.8: Let f: X → Y be a fuzzy open, fuzzy supper clopen mapping from a fuzzy space X to a fuzzy space Y. If X is fuzzy supper ultral normal then so is Y.

Proof: Let A, B be fuzzy closed and fuzzy open sets of Y such that A ≤ B. since f is fuzzy open, $f^{-1}(A)$ and $f^{-1}(B)$ are fuzzy closed and fuzzy open sets of X such that $f^{-1}(A) \le f^{-1}(B)$.

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Since X is a fuzzy supper ultra normal there exists a fuzzy supper clopen set U such that

$f^{-1}(A) \leq U \leq f^{-1}(B)$

By surjectivity, $A = f(f^{-1}(A)) \leq f(U) \leq f(f^{-1}(B)) = B$

Since f is fuzzy supper clopen, f (U) is fuzzy supper clopen in Y. Thus Y is fuzzy supper ultra-normal.

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