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EFFECT OF CHEMICAL REACTION AND RADIATION ABSORPTION ON THE UNSTEADY CONVECTIVE HEAT AND MASS TRANSFER FLOW OF A VISCOUS FLUID IN A VERTICAL WAVY CHANNEL WITH OSCILLATORY FLUX AND HEAT SOURCES

Dr. G. Viswanatha Reddy^{1*} & S. Venkatarami Reddy²

¹Associate Professor, Department of Mathematic, S. V. University, Tirupathi, A. P, India ²Department of Mathematics, S. V. University, Anantapur, A.P, India

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ABSTRACT

We analyse the effect of chemical reaction and radiation absorption on the transient convective heat and mass transfer flow of a viscous, electrically conducting fluid in a vertical wavy channel with oscillatory flux. The equations governing the flow, heat and mass transfer are solved by employing a regular perturbation technique with the slope δ as a perturbation parameter. The velocity, temperature, concentration, the rate of heat and mass transfer are analysed for different parametric values.

Keywords: Heat and Mass transfer, Chemical reaction parameter, Radiation absorption, Heat sources.

1. INTRODUCTION

Coupled heat and mass transfer phenomenon in porous media is gaining attention due to its interesting applications. The flow phenomenon is relatively complex rather than that of the pure thermal convection process. Underground spreading chemical wastes and other pollutants, grain storage ,evaporation cooling and solidification are the few other application areas where the combined thermo-solutal natural convection in porous media are observed .Combined heat and mass transfer by free convection under boundary layer approximations has been studied by Bejan and Khair(2),Lai and Kulacki(9) and Murthy and Singh(11).Coupled heat and mass transfer by mixed convection in Darcian fluid-saturated porous media has been analysed by Lai(8).The free convection heat and mass transfer in a porous enclosure has been studied recently by Angirasa *et al*(1). The combined effects of thermal and mass diffusion in channel flows has been studied in recent times by a few authors, notably Nelson and Wood(14,15), Lee *at al*(10) and others(23,25).

In recent years, energy and material saving considerations have prompted an expansion of the efforts at producing efficient heat exchanger equipment through augmentation of heat transfer. It has been established (5a) that channels with diverging - converging geometries augment the transportation of heat transfer and momentum. As the fluid flows through a tortuous path viz., the dilated - constricted geometry, there will be more intimate contact between them. The flow takes place both axially (primary) and transversely (secondary) with the secondary velocity being towards the axis in the fluid bulk rather than confining within a thin layer as in straight channels. Hence it is advantageous to go for converging – diverging geometries for improving the design of heat transfer equipment. Vajravelu and Nayfeh (23a) have investigated the influence of the wall waviness on friction and pressure drop of the generated coquette flow. Vajravelu and Sastry (24) have analysed the free convection heat transfer in a viscous, incompressible fluid confined between long vertical wavy walls in the presence of constant heat source. Later Vajravelu and Debnath (25) have extended this study to convective flow in a vertical wavy channel in four different geometrical configurations. This problem has been extended to the case of wavy walls by McMichael and Deutsch (109), Deshikachar et al (49), Rao et al (17a) and Sree Ramachandra Murthy (20a). Hyan Goo Kwon et al (7a) have analyzed that the Flow and heat/mass transfer in a wavy duct with various corrugation angles in two dimensional flow regimes. Mahdy et al (79) have studied the mixed convection heat and mass transfer on a vertical wavy plate embedded in a saturated porous media (PST/PSE). Comini et al (3a) have analyzed the Convective heat and mass transfer in wavy finned-tube exchangers. Jer-Huan Jang et al (58) have analyzed that the Mixed convection heat and mass transfer along a vertical wavy surface.

In many chemical engineering processes, there does occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications viz., polymer production,

manufacturing of ceramics or glassware and food processing .Das *et al* (5) have studied the effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Muthukumaraswamy (13) has studied the effects of reaction on a long surface with suction. Recently Gnaneswar(6) has studied radiation and mass transfer on an unsteady two-dimensional laminar convective boundary layer flow of a viscous incompressible chemically reacting fluid along a semi-infinite vertical plate with suction by taking into account the effects of viscous dissipation.

The present trend in the field of chemical reaction analysis is to give a mathematical model for the system to predict the reactor performance. A large amount of research work has been reported in this field. In particular the study of heat and mass transfer with chemical reaction is of considerable importance in chemical and hydrometallurgical industries. Chemical reaction can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. Frequently the transformations proceed in a moving fluid, a situation encountered in a number of technological fields. A common area of interest in the field of aerodynamics is the analysis of thermal boundary layer problems for two dimensional steady and incompressible laminar flow passing a wedge. Simulataneous heat and mass transfer from different geometrics embedded in a porous media has many engineering and geophysical application such as geothermal reservoirs, drying of porous solids thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, and under ground energy transport. A very significant area of research in radiative heat transfer, at the present time is the numerical simulation of combined radiation and convection/conduction transport processes. The effort has arisen largely due to the need to optimize industrial system such as furnaces, ovens and boilers and the interest in our environment and in no conventional energy sources, such as the use of salt-gradient solar ponds for energy collection and storage. In particular, natural convection induced by the simultaneous action of buoyancy forces resulting from thermal and mass diffusion is of considerable interest in nature and in many industrial application such as geophysics, oceanography, drying process, solidification of binary alloy and chemical engineering. Kandaswamy et al (9a) have discussed the Effects of chemical reaction, heat and mass transfer on boundary layer flow over a porous wedge with heat radiation in the presence of suction or injection.

Recently Madhusudan Reddy (10b) has analysed the effect of chemical reaction on double diffusive heat transfer flow of a viscous fluid in a wavy channel.

In this paper we discuss the effect of chemical reaction and radiation absorption on unsteady free connective heat and mass transfer flow in a vertical wavy channel. The unsteadiness in the flow is due to the oscillatory flux in the flow region. The coupled equations governing the flow, heat and mass transfer have been solved by using a perturbation technique with the slope δ as the perturbation parameter. The expression for the velocity, the temperature, the concentration, and the rate of heat and mass transfer are derived and are analysed for different variations of the governing parameters G, M, β , k Q.

2. FORMULATION OF THE PROBLEM

We consider the effect of chemical reaction on the unsteady motion of viscous, incompressible fluid through a porous

medium in a vertical channel bounded by wavy walls. The thermal buoyancy in the flow field is created by an oscillatory flux in the fluid region. The walls are maintained at constant temperature and concentration. The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. The viscous and Darcy dissipations are neglected in comparison with heat by conduction and convection in the energy equation. Also the Kinematic viscosity ν , the thermal conducting k are treated as constants. We choose a rectangular Cartesian system 0(x, y) with x-axis in the vertical direction and y-axis normal to the walls. The

walls of the channel are at $y = \pm Lf(\frac{\delta x}{L})$

 $T = T_1$ $C = C_1$ H_0 $y = - Lf(\delta x/L)$ $y = + Lf(\delta x/L)$

Configuration of the Problem

The equations governing the unsteady flow, heat and mass transfer are

Equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

Equation of linear momentum

$$\rho_{e}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^{2} u}{\partial x^{2}} + \frac{\partial^{2} u}{\partial y^{2}}\right) - \rho g - (\sigma \mu_{e}^{2} H_{o}^{2})u$$
(2.2)

$$\rho_{e}\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^{2} v}{\partial x^{2}} + \frac{\partial^{2} v}{\partial y^{2}}\right)$$
(2.3)

Equation of Energy:

$$\rho_{e}C_{p}\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \lambda \left(\frac{\partial^{2} T}{\partial x^{2}} + \frac{\partial^{2} T}{\partial y^{2}}\right) - Q(T - T_{e}) + Q_{1}(C - C_{e})$$
(2.4)

Equation of diffusion

$$\left(\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y}\right) = D_1 \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) - k_1 (C - C_e)$$
(2.5)

Equation of state

$$\rho - \rho_{e} = -\beta \rho_{e} (T - T_{e}) - \beta^{*} \rho_{e} (C - C_{e}) \tag{2.6}$$

where P_e is the density of the fluid in the equilibrium state, Te, Ce are the temperature and concentration in the equilibrium state, (u,v) are the velocity components along O(x,y) directions, p is the pressure, T, C are the temperature and Concentration in the flow region, p is the density of the fluid, μ is the constant coefficient of viscosity, Cp is the specific heat at constant pressure, λ is the coefficient of thermal conductivity, μ is the permeability of the porous medium, μ is the coefficient of thermal expansion, Q is the strength of the constant internal heat source, μ is the electrical conductivity, μ is the magnetic permeability, μ is the volumetric expansion with mass fraction coefficient μ is the molecular diffusivity and μ is the chemical reaction coefficient.

The flow is maintained by an oscillatory volume flux for which a characteristic velocity is defined as

$$q(1+k e^{i\omega t}) = \frac{1}{L} \int_{-Lf}^{Lf} u \, dy.$$
 (2.7)

The boundary conditions for the velocity and temperature fields are

$$u = 0, v = 0, T = T_1, C = C_1$$
 on $y = -Lf\left(\frac{\delta x}{L}\right)$
$$u = 0, v = 0, T = T_2, C = C_2$$
 on $y = +Lf\left(\frac{\delta x}{L}\right)$ (2.8)

In view of the continuity equation we define the stream function ψ as

$$\mathbf{u} = -\mathbf{\psi}_{\mathbf{v}}, \mathbf{v} = \mathbf{\psi}_{\mathbf{x}} \tag{2.9}$$

Eliminating pressure p from equations (2.2) & (2.3) and using the equations governing the flow in terms of ψ are

$$[(\nabla^{2}\psi)_{t} + \psi_{x}(\nabla^{2}\psi)_{y} - \psi_{y}(\nabla^{2}\psi)_{x}] = v\nabla^{4}\psi - \beta g(T - T_{0})_{y} = \beta^{*} g(C - C_{0})_{y} - (\sigma\mu_{e}^{2}H_{o}^{2})\frac{\partial^{2}\psi}{\partial v^{2}}$$
(2.10)

$$\rho_{e}C_{p}\left(\frac{\partial T}{\partial t} - \frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} + \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y}\right) = \lambda \nabla^{2}T - Q(T - T_{o})$$
(2.11)

$$\left(\frac{\partial C}{\partial t} - \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y}\right) = D\nabla^2 C - k_1 (C - C_o) \tag{2.12}$$

Introducing the non-dimensional variables in (2.8) & (2.9) as

$$x' = x/L, \ y' = y/L, t' = t\omega, \Psi' = \Psi/\nu, \theta = \frac{T - T_2}{T_1 - T_2}, C' = \frac{C - C_2}{C_1 - C_2}$$
(2.13)

The governing equations in the non-dimensional form (after dropping the dashes) are

$$R\left(\gamma^{2}(\nabla^{2}\psi)_{t} - \frac{\partial(\psi, \nabla^{2}\psi)}{\partial(x, y)}\right) = \nabla^{4}\psi + \left(\frac{G}{R}\right)(\theta_{y} + NC_{y}) - M^{2}\frac{\partial^{2}\psi}{\partial y^{2}}$$
(2.14)

$$P\left(\gamma^{2} \frac{\partial \theta}{\partial t} - \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y}\right) = \nabla^{2} \theta - \alpha \theta + Q_{1} C \tag{2.15}$$

$$Sc\left(\gamma^2 \frac{\partial C}{\partial t} - \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y}\right) = \nabla^2 C - KC \tag{2.16}$$

where

$$R = \frac{UL}{V}$$
 (Reynolds number) $G = \frac{\beta g \Delta T_e L^3}{V^2}$ (Grashof number)

$$P = \frac{\mu c_p}{k_1} \text{ (Prandtl number)}, \ M^2 = \frac{\sigma \mu_e^2 H_o^2 L^2}{v^2} \text{ (Hartman Number)}$$

$$Sc = \frac{V}{D_1}$$
 (Schmidt Number) $\alpha = \frac{QL^2}{\lambda}$ (Heat source parameter)

$$K = \frac{K_1 L^2}{D_1}$$
 (Chemical reaction parameter) $\gamma^2 = \frac{\omega L^2}{V}$ (Wormsely Number)

$$Q_{1} = \frac{Q_{1}^{'}(C - C_{e})L^{2}}{D_{1}C_{p}}$$
 (Radiation absorption parameter) $\nabla^{2} = \frac{\partial^{2}}{\partial x^{2}} + \frac{\partial^{2}}{\partial y^{2}}$

The corresponding boundary conditions are

$$\psi(+f)-\psi(-f)=1$$

$$\frac{\partial \psi}{\partial x} = 0, \quad \frac{\partial \psi}{\partial y} = 0 \quad aty \eta = \pm f$$
 (2.17)

$$\theta(x, y) = 1, C = 1$$
 on $y = -f$

$$\theta(x, y) = 0, C = 0$$
 on $y = f$

$$\frac{\partial \theta}{\partial y} = 0, \frac{\partial C}{\partial y} = 0 \quad at \quad y = 0 \tag{2.18}$$

The value of ψ on the boundary assumes the constant volumetric flow in consistant with the hyphothesis (2.7). Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function t .

3. METHOD OF SOLUTION

The main aim of the analysis is to discuss the perturbations created over a combined free and forced convection flow due to traveling thermal wave imposed on the boundaries. The perturbation analysis is carried out by assuming that the aspect ratio δ to be small.

Introduce the transformation such that

$$\overline{x} = \delta x, \frac{\partial}{\partial x} = \delta \frac{\partial}{\partial \overline{x}}$$

Then
$$\frac{\partial}{\partial x} \approx O(\delta) \rightarrow \frac{\partial}{\partial \overline{x}} \approx O(1)$$

For small values of $\delta <<1$, the flow develops slowly with axial gradient of order δ

And hence we take $\frac{\partial}{\partial \overline{x}} \approx O(1)$

Using the above transformation the equations (2.15-2.17) reduces to

$$\delta R \left(\gamma^2 (\nabla_1^2 \psi)_t + \frac{\partial (\psi, \nabla_1^2 \psi)}{\partial (x, y)} \right) = \nabla_1^4 \psi - \left(\frac{G}{R} \right) (\theta_y + NC_y) - M^2 \frac{\partial^2 \psi}{\partial y^2}$$
(3.1)

$$\delta P \left(\gamma^2 \frac{\partial \theta}{\partial t} - \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) = \nabla_1^2 \theta - \alpha \theta + Q_1$$
(3.2)

$$\delta Sc \left(\gamma^2 \frac{\partial C}{\partial t} - \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} \right) = \nabla_1^2 C - KC$$
(3.3)

where

$$\nabla_1^2 = \delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

Introducing the transformation

$$\eta = \frac{y}{f(\bar{x})}$$

the equations(3.1-3.3) reduces to

$$\delta Rf \left(\gamma^2 (F^2 \psi)_t + \frac{\partial (\psi, F^2 \psi)}{\partial (\overline{x}, \eta)} \right) = F^4 \psi - \left(\frac{Gf^3}{R} \right) (\theta_\eta + NC_\eta) - (M^2 f^2) \frac{\partial^2 \psi}{\partial \eta^2}$$
(3.4)

$$\delta P \left(\gamma^2 \frac{\partial \theta}{\partial t} - f \left(\frac{\partial \psi}{\partial \eta} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial \eta} \right) \right) = F^2 \theta - \alpha f^2 \theta + Q_1 f^2 C$$
(3.5)

$$\delta Sc \left(\gamma^2 \frac{\partial C}{\partial t} - f \left(\frac{\partial \psi}{\partial \eta} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial \eta} \right) \right) = F^2 C - KC$$
(3.6)

where

$$F^{2} = \delta^{2} \frac{\partial^{2}}{\partial \overline{x}^{2}} + \frac{\partial^{2}}{\partial \eta^{2}}$$

We adopt the perturbation scheme and write

$$\psi(x,\eta,t) = \psi_0(x,\eta,t) + ke^{it}\overline{\psi}_0(x,\eta,t) + \delta(\psi_1(x,\eta,t) + ke^{it}.\overline{\psi}_1(x,\eta,t)) + \dots$$

$$\theta(x,\eta,t) = \theta_0(x,\eta,t) + ke^{it}\overline{\theta_0}(x,\eta,t) + \delta(\theta_1(x,\eta,t) + ke^{it}.\overline{\theta}_1(x,\eta,t)) + \dots$$

$$C(x,\eta,t) = C_0(x,\eta,t) + ke^{it}C\overline{\phi}_0(x,\eta,t) + \delta(C_1(x,\eta,t) + ke^{it}.\overline{C}_1(x,\eta,t)) + \dots$$
(3.7)

•

On substituting (3.1) in (3.4) - (3.6) and separating the like powers of δ the equations and respective conditions to the zeroth order are

$$\psi_{0,\eta\eta\eta\eta} - (M_1^2 f^2) \psi_{0,\eta\eta\eta} = + \left(\frac{Gf^3}{R}\right) (\theta_{0,\eta} + NC_{0,\eta})$$
(3.8)

$$\theta_{om} - (\alpha f^2)\theta_0 = -Q_1 C_0 \tag{3.9}$$

$$C_{o,nn} - (KScf^{2})C_{0} = 0 (3.10)$$

with

 $\Psi_0(+1)-\Psi_0(-1)=1$,

$$\psi_{0, \eta} = 0, \psi_{0, x} = 0 \text{ at } \eta = \pm 1$$
 (3.11)

$$\theta_o = 1$$
, $C_0 = 1$ on $\eta = -1$

(3.12)

$$\theta_0 = 0$$
, $C_0 = 0$ on $\eta = 1$

$$\overline{\theta}_{0,\mu\eta} - (iP\gamma^2 f^2)\overline{\theta}_0 = -Q_1 \overline{C_0}$$
(3.13)

$$\overline{C}_{0,nn} - (KSc\gamma^2 f^2)\overline{C}_o = 0 \tag{3.14}$$

$$\overline{\psi}_{0,\eta\eta\eta\eta} - ((M_1^2 + i\gamma^2)f^2)\overline{\psi}_{0,\eta\eta} = -\left(\frac{Gf^3}{R}\right)(\overline{\theta}_{0,\eta} + N\overline{C}_{0,\eta})$$
(3.15)

$$\overline{\theta}_{a}(\pm 1) = 0$$
, $\overline{C}_{a}(\pm 1) = 0$

$$\overline{\psi}_{o}(+1) - \overline{\psi}_{o}(-1) = 1 \quad \overline{\psi}_{o,\eta}(\pm 1) = 0, \overline{\psi}_{o,x}(\pm 1) = 0 \tag{3.16}$$

The first order equations are

$$\psi_{1,\eta\eta\eta\eta} - (M_1^2 f^2) \psi_{1,\eta\eta} = -\left(\frac{Gf^3}{R}\right) (\theta_{1,\eta} + NC_{1,\eta}) + (Rf)(\psi_{0,\eta} \psi_{0,\chi\eta\eta} - \psi_{0,\chi} \psi_{0,\eta\eta\eta})$$
(3.17)

$$\theta_{1,m} - (\alpha f^2) \theta_1 = (PRf) (\psi_{0,x} \theta_{o,\eta} - \psi_{0,\eta} \theta_{ox}) - Q_1 C_1$$
(3.18)

$$C_{1,m} - (KScf^{2})C_{1} = (Scf)(\psi_{0,x}C_{o,\eta} - \psi_{0,\eta}C_{ox})$$
(3.19)

$$\overline{\psi}_{1,\eta\eta\eta\eta} - ((M_1^2 + i\gamma^2)f^2)\overline{\psi}_{1,\eta\eta} = -\left(\frac{Gf^3}{R}\right)(\overline{\theta}_{1,\eta} + N\overline{C}_{1,\eta}) + (Rf)(\overline{\psi}_{0,\eta}\psi_{0,\eta\eta\eta} + \psi_{0,\eta}\overline{\psi}_{0,\eta\eta\eta} - \psi_{0,\eta}\overline{\psi}_{0,\eta\eta\eta} - \overline{\psi}_{0,\eta\eta\eta})$$
(3.20)

$$\overline{\theta}_{1,\eta\eta} - ((iP\gamma^2 + \alpha)f^2)\overline{\theta}_1 = (PRf)(\psi_{0,\eta}\overline{\theta}_{o,x} + \overline{\psi}_{0,\eta}\theta_o - \overline{\psi}_{0,x}\theta_{o,\eta} - \psi_{0,x}\overline{\theta}_{o\eta}) - Q_1\overline{C_1}$$
(3.21)

$$\overline{C}_{1,n} - ((K + i\gamma^2)Scf^2)\overline{C}_{1} = (Scf)(\psi_{0,n}\overline{C}_{0,x} + \overline{\psi}_{0,n}C_{0,n} - \overline{\psi}_{0,x}C_{0,n} - \psi_{0,x}\overline{C}_{0,n})$$
(3.22)

with

$$\psi_{1(+1)} - \psi_{1(-1)} = 0$$

$$\psi_{1,\eta} = 0, \psi_{1,x} = 0 \text{ at } \eta = \pm 1$$
 (3.23)

$$\theta_1(\pm 1) = 0$$
 $C_1(\pm 1) = 0$

$$\overline{\theta}_1(\pm 1) = 0$$
 $\overline{C}_1(\pm 1) = 0$

$$\overline{\psi}_1(+1) - \overline{\psi}_1(-1) = 1 \quad \overline{\psi}_{1,n}(\pm 1) = 0, \overline{\psi}_{1,n}(\pm 1) = 0 \tag{3.24}$$

4. SOLUTION OF THE PROBLEM

Solving the equations (3.2)- (3.13) subject to the relevant boundary conditions we obtain

$$C_0 = 0.5 \left(\frac{Ch(\beta_1 \eta)}{Ch(\beta_1)} - \frac{Sh(\beta_1 \eta)}{Sh(\beta_1)} \right)$$

$$\psi_{0} = a_{13}Ch(M_{1}\eta) + a_{14}Sh(M_{1}\eta) + a_{15}\eta + a_{16} + f_{1}(\eta)$$

$$f_1(\eta) = a_0 Sh(\beta_1 \eta) + a_{10} Ch(\beta_1 \eta) + a_{11} Sh(\beta_2 \eta) + a_{12} Ch(\beta_2 \eta)$$

$$\overline{C}_0 = 0$$

$$\overline{\theta}_0 = 0$$

$$\overline{\psi}_0 = a_{16}Sh(M_1\eta) + a_{17}\eta + a_{18}$$

$$C_1 = a_{92}Ch(\beta_2\eta) + a_{93}Sh(\beta_2\eta) + f_2(\eta)$$

$$\begin{split} f_{2}(\eta) &= (a_{67} + \eta a_{79})Ch(\beta_{6}\eta) + (a_{68} + \eta a_{80})Ch(\beta_{7}\eta) + a_{69}Sh(\beta_{6}\eta) + a_{70}Ch(\beta_{7}\eta) + \\ &\quad + (a_{71} + \eta a_{83})Sh(\beta_{8}\eta) + a_{72}Sh(\beta_{9}\eta) + (a_{73} + \eta a_{81})Ch(\beta_{8}\eta) + (a_{74} + \eta a_{82})Ch(\beta_{9}\eta) + \\ &\quad + (a_{75} + \eta (a_{78} + a_{88}))Sh(2\beta_{1}\eta) + (a_{76} + \eta (a_{77} + a_{87}))Ch(2\beta_{1}\eta) + a_{84}\eta Sh(\beta_{5}\eta) + a_{85}\eta Sh(\beta_{1}\eta) + \\ &\quad + a_{96}\eta Ch(\beta_{1}\eta) + a_{99}\eta^{2}Ch(\beta_{1}\eta) + a_{99}\eta^{2}Sh(\beta_{1}\eta) - a_{91}\eta^{2} \end{split}$$

$$\theta_1 = b_{92}Ch(\beta_1\eta) + b_{93}Sh(\beta_1\eta) + f_3(\eta)$$

$$\begin{split} f_3(\eta) &= d_1 Ch(\beta_{10}\eta) + d_2 Ch(\beta_{11}\eta) + d_3 Sh(\beta_{10}\eta) + d_4 Sh(\beta_{11}\eta) + d_5 Ch(\beta_9\eta) + d_6 Sh(\beta_9\eta) \\ &+ d_7 Ch(\beta_8\eta) + d_8 Sh(\beta_8\eta) + d_9 Ch(\beta_7\eta) + d_{10} Sh(\beta_7\eta) + d_{11} Ch(\beta_6\eta) + d_{12} Sh(\beta_6\eta) \\ &+ d_{13} Sh(2\beta_1\eta) + d_{14} Ch(2\beta_1\eta) + d_{15} Ch(\beta_2\eta) + d_{16} Sh(\beta_2\eta) + d_{17} Sh(2\beta_2\eta) + d_{18} Ch(2\beta_2\eta) \\ &+ d_{19} \eta Ch(\beta_1\eta) + d_{20} \eta S \quad (\beta_1\eta) + d_{21} \eta^2 Ch(\beta_1\eta) + d_{22} \eta^2 S \quad (\beta_1\eta) + d_{23} \eta^3 Ch(\beta_1\eta) \\ &+ d_{24} \eta^3 Sh(\beta_1\eta) + d_{25} \eta Sh(2\beta_1\eta) + d_{26} \eta Sh(2\beta_2\eta) + d_{27} \eta Ch(2\beta_1\eta) + d_{28} \eta Ch(\beta_2\eta) \\ &+ d_{29} \eta Sh(\beta_2\eta) + d_{30} \eta Ch(\beta_2\eta) + d_{31} \eta Ch(\beta_6\eta) + d_{32} \eta Ch(\beta_7\eta) + d_{34} \eta Ch(\beta_7\eta) + \\ &+ d_{35} \eta Sh(\beta_8\eta) + d_{36} \eta Sh(\beta_9\eta) + d_{37} \eta Sh(\beta_6\eta) + d_{38} \eta Sh(\beta_7\eta) + d_{39} \eta Sh(\beta_{10}\eta) \\ &+ d_{40} \eta Sh(\beta_{11}\eta) + d_{41} \eta Sh(\beta_{11}\eta) + d_{42} \eta Ch(\beta_{10}\eta) + d_{43} \eta Ch(\beta_{11}\eta) - d_{43} \eta^2 - d_{44} \end{split}$$

$$\psi_1 = p_1 + p_2 \eta + p_3 Ch(M_1 \eta) + p_4 Sh(M_1 \eta) + f_4(\eta)$$

$$f_{4}(\eta) = g_{1}Sh(\beta_{1}\eta) + g_{2}Ch(\beta_{1}\eta) + g_{3}Sh(\beta_{2}\eta) + g_{4}Ch(\beta_{2}\eta) + g_{5}Sh(\beta_{6}\eta)$$

$$+ g_{6}Sh(\beta_{7}\eta) + g_{7}Sh(\beta_{8}\eta) + g_{8}Sh(\beta_{9}\eta) + g_{9}Sh(\beta_{10}\eta) + g_{10}Sh(\beta_{11}\eta)$$

$$+ g_{11}Ch(\beta_{6}\eta) + g_{12}Ch(\beta_{7}\eta) + g_{13}Ch(\beta_{8}\eta) + g_{14}Ch(\beta_{9}\eta) + g_{15}Ch(\beta_{10}\eta)$$

$$+ g_{16}Ch(\beta_{11}\eta) + g_{17}Ch(2\beta_{1}\eta) + E_{97}\eta^{2}Sh(M_{1}\eta) + E_{98}\eta^{2}Ch(M_{1}\eta)$$

$$+ E_{99}\eta Sh(M_{1}\eta) + E_{104}\eta Sh(2M_{1}\eta) + E_{105}\eta^{2}Sh(2M_{1}\eta) + E_{106}Ch(2M_{1}\eta)$$

$$\begin{split} &+a_{21}Sh(\beta_{12}\eta)+a_{22}Sh(\beta_{13}\eta)+a_{23}Sh(\beta_{14}\eta)+a_{24}Sh(\beta_{15}\eta)+a_{25}Ch(\beta_{12}\eta)\\ &+a_{26}Ch(\beta_{13}\eta)+a_{27}Ch(\beta_{14}\eta)+a_{28}Ch(\beta_{15}\eta)+E_{57}\eta^3Sh(\beta_{1}\eta)+g_{29}\eta^2Sh(\beta_{1}\eta...+g_{30}\eta)\\ &+g_{30}\eta Sh(\beta_{1}\eta+E_{72}\eta^3Ch(\beta_{1}\eta)+g_{31}\eta^2Ch(\beta_{1}\eta)+g_{32}\eta)Ch(\beta_{1}\eta)\\ \end{split}$$

5. NUSSELT NUMBER and SHERWOOD NUMBER

$$(\tau)_{y=-1} = d_6 + Ecd_7 + \delta d_8 + O(\delta^2)$$

The local rate of heat transfer coefficient (Nusselt number Nu) on the walls has been calculated using the formula

$$Nu = \frac{1}{\theta_m - \theta_w} \left(\frac{\partial \theta}{\partial y} \right)_{\eta = \pm 1}$$

where $\theta_m = 0.5 \int_{-1}^{1} \theta \, d\eta$

and the corresponding expressions are

$$(Nu)_{\eta=+1} = \frac{(d_9 + \delta d_{11})}{(\theta_m - Sin(x + \gamma t))}, \ (Nu)_{\eta=-1} = \frac{(d_8 + \delta d_{10})}{(\theta_m - 1)}, \quad \text{where } \theta_m = d_{14} + \delta d_{15}$$

The local rate of mass transfer coefficient(Sherwood Number Sh) on the walls has been calculated using the formula

$$Sh = \frac{1}{C_m - C_w} \left(\frac{\partial C}{\partial y} \right)_{y=\pm 1} \qquad \text{where} \qquad C_m = 0.5 \int_{-1}^{1} C \, dy$$

and the corresponding expressions are

$$(Sh)_{\eta=+1} = \frac{(d_4 + \delta d_6)}{(C_m)}$$
 $(Sh)_{\eta=-1} = \frac{(d_5 + \delta d_7)}{(C_m - 1)}$,

where $C_m = d_{12} + \delta d_{13}$, where $d_1.d_2, \dots, d_{14}$ are constants.

6. DISCUSSION OF THE RESULTS

In this analysis, we investigate the effect of chemical reaction and radiation absorption on unsteady convective heat and mass transfer flow of a viscous electrically conducting fluid in a vertical wavy channel in the presence of heat generating sources. The axial velocity is shown in figure 1-11 for different values of G, M, Q_1,β , k. The actual axial flow is in the vertically downward direction i.e. u < 0 is actual flow and u > 0 is a reversal flow.

Fig 1 represents the variation of u with Grashof number G. It is found that u exhibits a reversal flow in entire fluid region and the region of reversal flow enlarges with increase in |G| with maximum attained at $\eta = -0.4$. The magnitude of u enhances with increase in |G|.

Higher the Lorentz force lesser |u| in the flow region (fig.2).

An increase in the strength of the heat source enhances |u| in fluid region. Also for $\alpha \ge 4$, u exhibits a reversal flow in the entire flow region (fig.3).

An increase in the radiation absorption parameter Q_1 results in an enhancement in |u| with reversal flow in the vicinity of $\eta = 1$ (fig.4).

The influence of the surface geometry on u is shown in fig.5. It is found that higher the dilation of the channel walls, larger |u| for $\beta \le 0.5$ and larger for $|\beta| \ge 0.7$.

The variation of u with chemical reaction parameter k shows that the axial velocity enhances with increase in k (fig.6).

The secondary velocity (v) which is due to the wall waviness of the channel is shown in figs 7-12.

From fig.7 that for G>0, v is towards the midregion and is towards the boundary for G<0. |v| enhances with increase in |G| with maximum attained at $\eta = 0.4$.

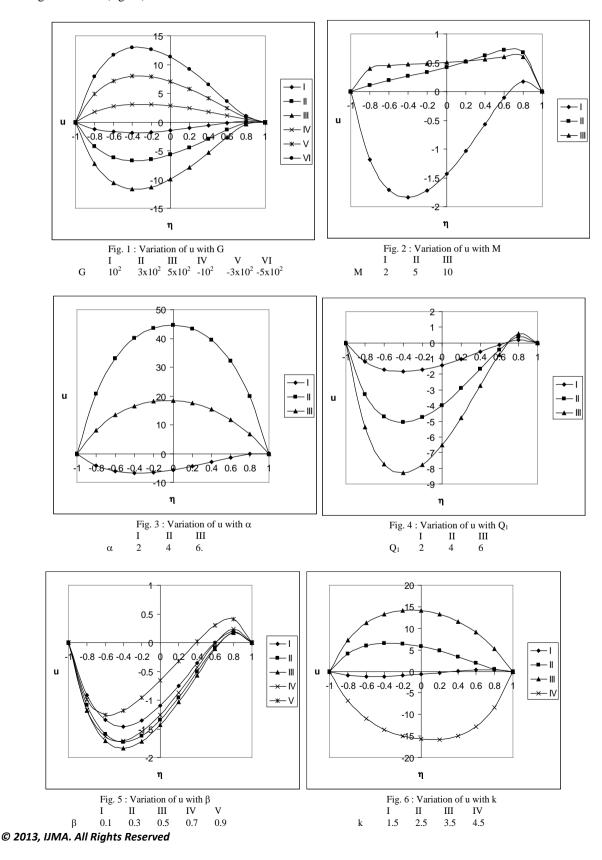
The depreciation in |v| is remarkable at M = 5 and is marginal at higher M=10. (fig.8).

The variation of v with heat source parameter α shows that the magnitude of v enhances remarkably with increase in $\alpha \le 4$ and depreciates with higher $\alpha \ge 6$. (fig.9).

An increase in Q_1 leads to an enhancement in |v| (fig.10).

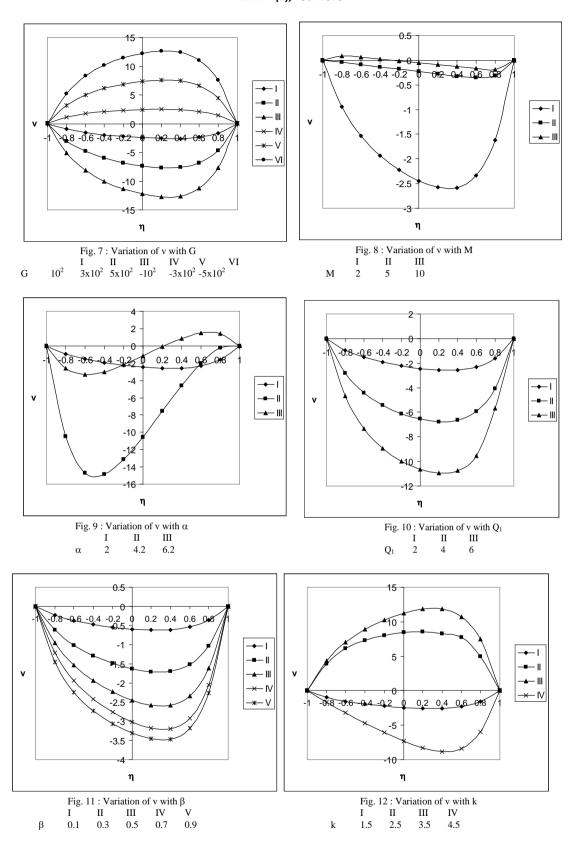
From fig.11 we find that higher the dilation of the channel walls larger |v| in the flow region.

The variation of v with chemical reaction parameter k shows that |v| enhances with increase in $k \le 3.5$ and depreciates with higher $k \ge 4.5$ (fig.12).



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The non-dimensional temperature (θ) is shown in figures 13-18 for different parametric values. We follow the convention that the non-dimensional temperature is positive or negative according as the actual temperature is greater (or) lesser than T_2 . Fig. 13 represents θ with Grashof number G. It is found the actual temperature enhances with G>0 and depreciates with G<0.

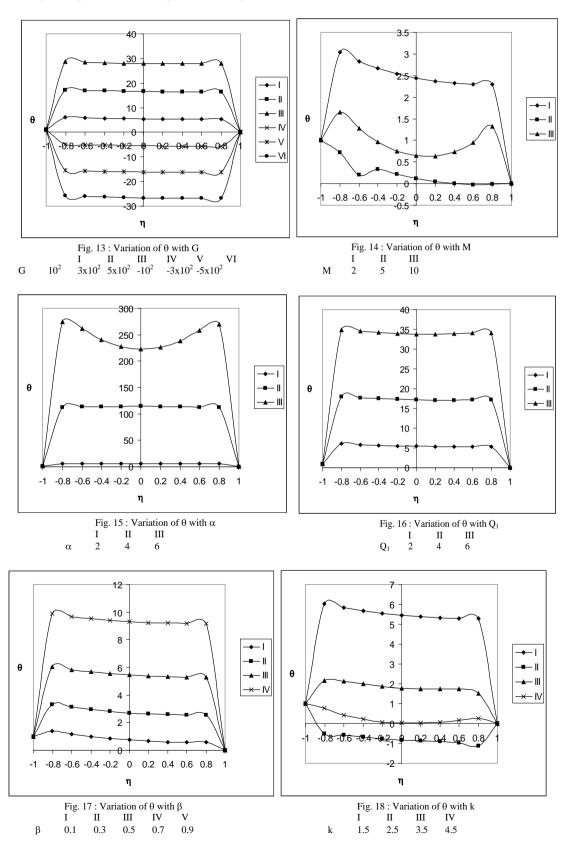
The actual temperature depreciates with M (fig. 14).

The variation of θ with heat source parameter α exhibits an increasing tendency with α (fig.15).

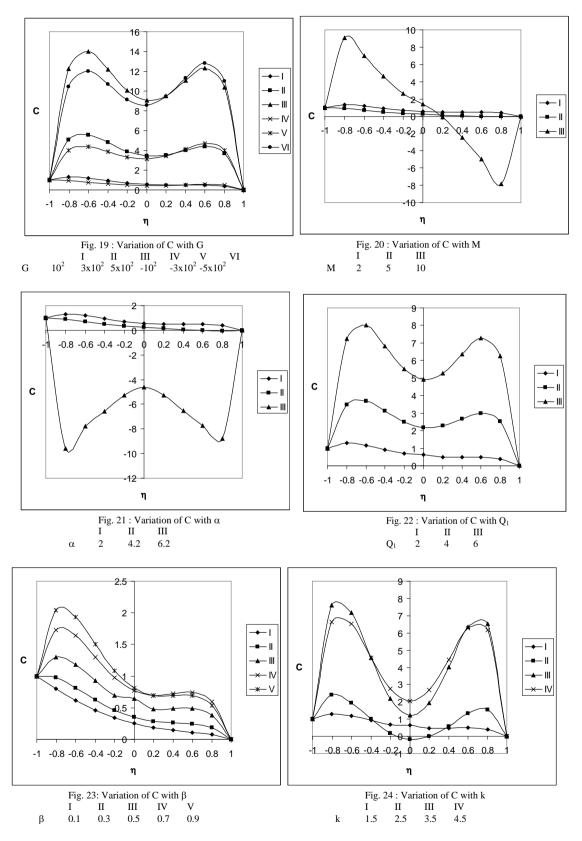
From fig 16 we notice that the actual temperature enhances with increase in the radiation absorption parameter Q₁.

The influence of the wall waviness is shown in (fig.17). It is found that higher the dilation of the channel walls larger the actual temperature.

An increase in the chemical reaction parameter $k \le 2.5$ leads to a depreciation in the actual temperature, enhances at k = 3.5 and again depreciates with higher $k \ge 4.5$ (fig.18).



The non-dimensional concentration (C) is shown in figs 19-24 for different parametric values. Fig.19 represents the concentration C with Grashof number G. It is found that the actual concentration enhances with increase in |G| with maximum at $\eta=0.6$.



The variation of C with Hartmann number M shows that higher the Lorentz force smaller the actual concentration and for further higher Lorentz force larger in the left half and smaller in the right half (fig.20).

An increase in the heat source parameter α reduces the actual concentration. It is found that for higher $\alpha = 6$ we notice a remarkable depreciation in the actual concentration (fig.21).

An increase in Q_1 enhances in the entire flow region (fig.22).

Higher the dilation of the channel walls larger the actual concentration in entire flow region (fig.23).

For $k \le 2.5$, we find that the actual concentration enhances in the region, except in the region (-0.2, 0.2_ where it depreciates and for higher $k \ge 3.5$, it depreciates in the regions adjacent to $\eta = \pm 1$ and enhances in the central region (fig.24).

The rate of heat transfer (Nu) at $\eta=\pm 1$ is shown tables 1-2 for different values of G, Q_1 , k, β . It is found that the rate of heat transfer enhances at $\eta=1$ and reduces at $\eta=-1$ with G>0 and for G<0, we notice an enhancement at both the walls. An increase in R enhances |Nu| at $\eta=\pm 1$. The variation of Nu with chemical reaction parameter k shows that |Nu| enhances with k at $\eta=\pm 1$. An increase in Q_1 reduces it at $\eta=+1$ and enhances at $\eta=-1$. Moving along the axial direction the Nusselt number reduces at $\eta=+1$ and enhances at $\eta=-1$ for all G. Higher the dilation of the walls larger the magnitude of rate of heat transfer at both walls (tables 1&2).

The rate of mass transfer (Sh) at $\eta \pm 1$ is shown in tables 3-4 for different parametric values. An increase in G>0 enhances |Sh| at $\eta = \pm 1$ while it enhances at $\eta = +1$ and reduces at $\eta = -1$ for G<0. An increase in the radiation absorption parameter Q_1 enhances the rate of mass transfer at $\eta = +1$ and at $\eta = -1$, it reduces with $Q_1 \le 4$ for G>0 and enhances for G<0 and for higher $Q_1 \ge 6$, it reduces for all G. An increase in k ≤ 1.5 enhance |Sh| and depreciates with higher k ≥ 2.5 in the heating case and cooling case a reversed effect is observed in the behaviour of |Sh| at $\eta = +1$. At $\eta = -1$ the rate of mass transfer depreciate increase in k for all G. Higher the dilation of the channel walls larger |Sh| at $\eta = +1$ and smaller |Sh| at $\eta = -1$. (tales 3 & 4).

Table-1
Nusselt number (Nu) at $\eta = +1$

				110				
G	I	II	III	IV	V	VI	VII	VIII
10^{3}	-0.01355	-0.15009	-0.2512	-0.04482	-0.0835	-0.08638	-0.05328	-0.05940
$3x10^{3}$	-0.05273	-0.17771	-0.2715	-0.02960	-0.07830	-0.07833	0.01036	0.09965
$5x10^{3}$	-0.08850	-0.17728	-0.2911	-0.01811	-0.07154	-0.07159	0.05678	0.21915
-10^{3}	0.04165	-0.06977	-0.0714	-0.06613	-0.09612	-0.09614	-1.17251	-0.36295
$-3x10^3$	-0.31573	0.11382	-0.1192	-0.09837	-0.10823	-0.10823	-0.89185	6.60215
$-5x10^3$	0.02425	0.51517	0.5314	-0.15335	-0.12361	-0.12359	0.36967	0.14976
k	0.5	1.5	2.5	0.5	0.5	0.5	0.5	0.5
β	0.5	0.5	0.5	0.7	0.9	1.5	0.5	0.5
Q_1	2	2	2	2	2	2	4	6

Table-2 Nusselt number (Nu) at $\eta = -1$

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G	I	II	III	IV	V	VI	VII	VIII
10^{3}	0.14333	0.61643	0.71643	0.57149	0.64032	0.64034	0.41680	0.50759
$3x10^{3}$	0.03876	0.39136	0.41138	0.47128	0.56305	0.56310	0.32303	0.44518
$5x10^{3}$	-0.02551	0.26869	0.36769	0.40397	0.50255	0.50261	0.29137	0.47656
-10^{3}	0.50148	1.05419	1.15324	0.73607	0.74243	0.74241	0.78730	1.01501
$-3x10^3$	-0.66386	2.02311	2.25113	1.05548	0.88367	0.88356	-5.25261	-1.78571
$-5x10^3$	-0.15408	4.92682	5.45236	1.93890	1.09187	1.09158	-0.60105	-0.66296
k	0.5	1.5	2.5	0.5	0.5	0.5	0.5	0.5
β	0.5	0.5	0.5	0.7	0.9	1.5	0.5	0.5
Q_1	2	2	2	2	2	2	4	6

Table-3
Sherwood number (Sh) at n = +1

Shel wood number (Sh) at $ =+1$									
G	I	II	III	IV	V	VI	VII	VIII	
10^{3}	-0.30188	0.85633	-0.52111	0.40905	0.51252	0.51257	1.06125	2.36756	
$3x10^{3}$	-0.59315	1.51080	0.55367	0.85223	0.96660	0.96666	2.65128	6.41349	
$5x10^{3}$	-1.16595	1.95975	0.56229	1.30990	1.42070	1.42075	4.77216	13.31212	
-10^{3}	-0.27264	0.16688	-0.65218	-0.02066	0.05845	0.05847	-0.13264	-0.08798	
$-3x10^3$	-0.49281	-0.30296	0.10005	-0.43783	-0.39561	-0.39562	-1.01993	-1.53827	
$-5x10^3$	-0.95572	-0.46478	0.25704	-0.84331	-0.84966	-0.84971	-1.66209	-2.28564	
k	0.5	1.5	2.5	0.5	0.5	0.5	0.5	0.5	
β	0.5	0.5	0.5	0.7	0.9	1.5	0.5	0.5	
Q_1	2	2	2	2	2	2	4	6	

Table-4
Sherwood number (Sh) at n = -1

Shel wood number (Sh) at 1 = -1								
G	I	II	III	IV	VI	VII	VIII	IX
10^{3}	-10.51477	-9.27785	-1.23367	-7.56091	-7.50540	-7.50545	-10.25605	-11.40274
$3x10^{3}$	-14.33007	-15.05510	-2.36074	-8.36586	-8.54437	-8.54464	-9.56771	-8.12803
$5x10^{3}$	-19.48473	1.49560	-2.49667	-9.10470	-9.58328	-9.58384	-7.99190	-3.36845
-10^{3}	-8.44286	-8.39664	-3.99941	-6.68520	-6.46636	-6.46625	-9.37967	-10.48924
$-3x10^3$	-8.48669	-8.75085	-3.08048	-5.73381	-5.42726	-5.42705	-5.65396	2.82965
$-5x10^3$	-10.92910	1.18119	-2.92544	-4.70151	-4.38809	-4.38786	3.60951	64.41862
k	0.5	1.5	2.5	0.5	0.5	0.5	0.5	0.5
β	0.5	0.5	0.5	0.7	0.9	1.5	0.5	0.5
Q_1	2	2	2	2	2	2	4	6

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