

DOMINATING SETS OF SQUARE OF CENTIPEDES

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ABSTRACTS

Let $G = (V, E)$ be a simple graph. A set $S \subseteq V$ is a dominating set of G , if every vertex in $V-S$ is adjacent to atleast one vertex in S . Let P_n^{*2} be the square of centipede corresponding to the path P_n and let $D(P_n^{*2}, i)$ denote the family of all dominating sets of P_n^{*2} with cardinality i . In this paper, some properties of the dominating sets of centipedes are exhibited. Also, we characterized the number of dominating sets of P_n^{*2} and $P_n^{*2} - \{2n\}$ of cardinality i .

Keywords: domination set, domination number.

1. INTRODUCTION

Let $G = (V, E)$ be a simple graph of order $|V| = n$. For any vertex $v \in V$, the open neighbourhood of v is the set $N(v) = \{u \in V / uv \in E\}$ and the closed neighbourhood of v is the set $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighbourhood of S is $N(S) = \bigcup_{v \in S} N(v)$ and the closed neighbourhood of S is $N[S] = N(S) \cup S$. A set $S \subseteq V$ is a dominating set of G , if $N[S] = V$, or equivalently, every vertex in $V-S$ is adjacent to atleast one vertex in S . The domination number of a graph G is defined as the minimum size of a dominating set of vertices in G and it is denoted as $\gamma(G)$. A path is a connected graph in which end vertices have degree one and the remaining vertices have degree two, and is denoted by P_n . The centipede P_n^* consists of a path P_n in which each vertex imbedded with a pendant edge and a pendant vertex.

Definition: 1.1: The 2nd power of a graph with the same set of vertices as G and an edge between two vertices if and only if there is a path of length at most 2 between them.

As usual we use $\lfloor x \rfloor$ for the largest integer less than or equal to x and $\lceil x \rceil$ for the smallest integer greater than or equal to x . Also, we denote the set $\{1, 2, \dots, n\}$ by $[n]$, throughout this paper.

2. DOMINATING SETS OF SQUARE OF CENTIPEDES

For the construction of the dominating sets of the square of centipede, P_n^{*2} , we need to investigate the dominating sets of $P_n^{*2} - \{2n\}$. In this section we investigate dominating sets of P_n^{*2} . Let $D(P_n^{*2}, i)$ be the family of dominating sets of P_n^{*2} with cardinality i . We shall find recursive formula for $|D(P_n^{*2}, i)|$. We need the following lemmas to obtain the result of this section:

Lemma 2.1: $\gamma(P_n^2) = \left\lceil \frac{n}{5} \right\rceil$

By lemma 2.1 and the definition of domination number, one has the following lemma:

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Lemma 2.2: For every $n \in N$,

- i) $\gamma(P_n^{*2}) = \left\lceil \frac{n}{3} \right\rceil$
- ii) $\gamma(P_n^{*2} - \{2n\}) = \left\lfloor \frac{n}{3} \right\rfloor$
- iii) $D(P_n^{*2}, i) = \Phi$ if and only if $i \prec \left\lceil \frac{n}{3} \right\rceil$ or $i \succ 2n$
- iv) $D(P_n^{*2} - \{2n\}, i) = \Phi$ if and only if $i \prec \left\lfloor \frac{n}{3} \right\rfloor$ or $i \succ 2n-1$

Proof:

- i) Clearly $\{3, 7, 11, \dots, 2n-1\}$ is a minimum dominating set for P_n^{*2} . If n is even or odd it contains $\left\lceil \frac{n}{3} \right\rceil$ elements.
Hence $\gamma(P_n^{*2}) = \left\lceil \frac{n}{3} \right\rceil$.
- ii) Clearly $\{3, 7, 11, \dots, 2n-3\}$ is a minimum dominating set for $P_n^{*2} - \{2n\}$. If n is even or odd it contains $\left\lfloor \frac{n}{3} \right\rfloor$ elements.
Hence $\gamma(P_n^{*2} - \{2n\}) = \left\lfloor \frac{n}{3} \right\rfloor$
- iii) It follows from (i) and the definition of dominating set.
- iv) It follows from (ii) and the definition of dominating set.

For the construction of $D(P_n^{*2}, i)$ we consider

$$D(P_n^{*2} - \{2n\}, i-1), D(P_{n-1}^{*2}, i-1), D(P_{n-1}^{*2} - \{2n-2\}, i-1), \text{ and } d(P_{n-3}^{*2}, i-1).$$

The families of these dominating sets can be empty or otherwise. Thus, we have eight combinations, whether these five families are empty or not. Two of these combinations are not possible (Lemma 2.3 (i), (ii) & (iii)).

Also, the combinations $D(P_n^{*2} - \{2n\}, i-1) = D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-3}^{*2}, i-1) = \Phi$ do not need to be considered because it implies that $D(P_n^{*2}, i) = \Phi$ (See lemma 2.2 (iii)). Thus we only need to consider four combinations or cases. We consider those cases in Lemma (2.3).

Lemma 2.3:

- i) If $D(P_n^{*2} - \{2n\}, i-1) = D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = \Phi$ then $D(P_{n-3}^{*2}, i-1) = \Phi$.
- ii) If $D(P_n^{*2} - \{2n\}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-3}^{*2}, i-1) = \Phi$ then $D(P_{n-1}^{*2}, i-1) = \Phi$.
- iii) If $D(P_n^{*2} - \{2n\}, i-1) = D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-3}^{*2}, i-1) = \Phi$,
then $D(P_n^{*2}, i) = \Phi$.
- iv) If $D(P_n^{*2} - \{2n\}, i-1) \neq \Phi$ $D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-3}^{*2}, i-1) = \Phi$,
then $D(P_n^{*2}, i) \neq \Phi$
- v) If $D(P_n^{*2} - \{2n\}, i-1) \neq \Phi$, $D(P_{n-1}^{*2} - \{2n-2\}, i-1) = \Phi$, then $D(P_{n-1}^{*2}, i-1) \neq \Phi$.

Proof:

- i) If $D(P_n^{*2} - \{2n\}, i-1) = D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = \Phi$, by lemma 2.2(iii),(iv)
 $i-1 \prec \left\lfloor \frac{n}{3} \right\rfloor$ or $i-1 \succ 2n-1$, $i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$ or $i-1 \succ 2n-2$, $i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor$ or $i-1 \succ 2n-3$.

Therefore, $i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$ or $i-1 \succ 2n-1$

Suppose, $D(P_{n-3}^{*2}, i-1) \neq \Phi$ then $\left\lceil \frac{n-3}{3} \right\rceil \leq i-1 \leq 2(n-3)$.

We have $\left\lfloor \frac{n-1}{3} \right\rfloor \leq \left\lceil \frac{n-3}{3} \right\rceil \leq i-1$. But $i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$, which is a contradiction.

Therefore, $D(P_{n-3}^{*2}, i-1) = \Phi$.

ii) If $D(P_n^{*2} - \{2n\}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-3}^{*2}, i-1) = \Phi$, by lemma 2.2(iii),(iv)

$i-1 \prec \left\lfloor \frac{n}{3} \right\rfloor$ or $i-1 \succ 2n-1$, $i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$ or $i-1 \succ 2n-3$, $i-1 \prec \left\lceil \frac{n-3}{3} \right\rceil$ or $i-1 \succ 2n-6$.

Therefore

$i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$ or $i-1 \succ 2n-1$. Therefore $i-1 \prec \left\lceil \frac{n-3}{3} \right\rceil$ or $i-1 \succ 2n-2$ holds.

Hence, $D(P_{n-1}^{*2}, i-1) = \Phi$.

iii) If $D(P_n^{*2} - \{2n\}, i-1) = D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-3}^{*2}, i-1) = \Phi$. by lemma 2.2(iii),(iv),

$i-1 \prec \left\lfloor \frac{n}{3} \right\rfloor$ or $i-1 \succ 2n-1$, $i-1 \prec \left\lceil \frac{n-1}{3} \right\rceil$ or $i-1 \succ 2n-2$, $i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$ or

$i-1 \succ 2n-3$, $i-1 \prec \left\lceil \frac{n-3}{3} \right\rceil$ or $i-1 \succ 2n-6$.

Therefore, $i-1 \prec \left\lceil \frac{n-3}{3} \right\rceil$ or $i-1 \succ 2n-1$.

That is, $i \prec \left\lceil \frac{n-3}{3} \right\rceil + 1 \leq \left\lfloor \frac{n}{3} \right\rfloor$ or $i \succ 2n$.

Therefore, $i \prec \left\lfloor \frac{n}{3} \right\rfloor$ or $i \succ 2n$

Hence, $D(P_n^{*2}, i) = \Phi$.

iv) If $D(P_n^{*2} - \{2n\}, i-1) \neq \Phi$, $D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-3}^{*2}, i-1) = \Phi$,

then $\left\lfloor \frac{n}{3} \right\rfloor \leq i-1 \leq 2n-1$,

$i-1 \prec \left\lceil \frac{n-1}{3} \right\rceil$ or $i-1 \succ 2n-2$, $i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$ or $i-1 \succ 2n-3$ and

$i-1 \prec \left\lceil \frac{n-3}{3} \right\rceil$ or $i-1 \succ 2n-6$.

Therefore, we have $2n-2 \prec i-1 \leq 2n-1$.

Therefore, $2n-1 \leq i-1 \leq 2n-1$.

Therefore, $i-1=2n-1$

Therefore, $i=2n$

Therefore $D(P_n^{*2}, i) \neq \Phi$.

v) If $D(P_n^{*2} - \{2n\}, i-1) \neq \Phi$, $D(P_{n-1}^{*2} - \{2n-2\}, i-1) = \Phi$, then $D(P_{n-1}^{*2}, i-1) \neq \Phi$

We have $\left\lfloor \frac{n}{3} \right\rfloor \leq i-1 \leq 2n-1$ and $i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$ or $i-1 \succ 2n-3$.

Therefore, $2n-2 \leq i-1 \leq 2n-1$.

Therefore, $2n-1 \leq i \leq 2n$.

Therefore, $i=2n-1$ or $i=2n$.

Therefore, in either case, we have $D(P_{n-1}^{*2}, i-1) \neq \Phi$.

Theorem 2.4: For every $n \geq 7$

i) $D(P_n^{*2} - \{2n\}, i-1) \neq \Phi$ and $D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-3}^{*2}, i-1) = \Phi$, if and only if $i=2n$.

ii) $D(P_n^{*2} - \{2n\}, i-1) \neq \Phi$, $D(P_{n-1}^{*2}, i-1) \neq \Phi$, $D(P_{n-1}^{*2} - \{2n-2\}, i-1) = \Phi$, $D(P_{n-3}^{*2}, i-1) = \Phi$, if and only if $i=2n-1$

iii) $D(P_n^{*2} - \{2n\}, i-1) \neq \Phi$, $D(P_{n-1}^{*2}, i-1) \neq \Phi$, $D(P_{n-1}^{*2} - \{2n-2\}, i-1) \neq \Phi$, and $D(P_{n-3}^{*2}, i-1) = \Phi$ if and only if $i=2n-2$ or $2n-3$ or $2n-4$.

Proof:

i) (\Rightarrow) Since $D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-3}^{*2}, i-1) = \Phi$, we have

$i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$ or $i-1 \succ 2n-2$, $i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$ or $i-1 \succ 2n-3$, $i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor$ or $i-1 \succ 2(n-3)$.

Also, $\left\lfloor \frac{n}{3} \right\rfloor \leq i-1 \leq 2n-1$.

Therefore, $i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$ does not hold.

Therefore, $i-1 \succ 2n-2$.

Therefore $i-1 \geq 2n-1$.

Therefore, $i \geq 2n$

Together, we have $i=2n$.

(\Leftarrow) It follows from Lemma 2.2(iii), (iv).

ii) (\Rightarrow) Since $D(P_n^{*2} - \{2n\}, i-1) \neq \Phi$, $D(P_{n-1}^{*2}, i-1) \neq \Phi$, $D(P_{n-1}^{*2} - \{2n-2\}, i-1) = \Phi$, by Lemma

2.2(iii),(iv), we have $\left\lfloor \frac{n}{3} \right\rfloor \leq i-1 \leq 2n-1$, $\left\lfloor \frac{n-1}{3} \right\rfloor \leq i-1 \leq 2n-2$, and $i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$ or $i-1 \succ 2n-3$. Also

$i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$

Then $2n-3 \prec i-1 \leq 2n-2$.

Therefore, $i-1=2n-2$

Thus, $i=2n-1$

(\Leftarrow) It follows from Lemma 2.2(iii), (iv).

iii) (\Rightarrow) Since

$$D(P_n^{*2} - \{2n\}, i-1) \neq \Phi, D(P_{n-1}^{*2}, i-1) \neq \Phi, D(P_{n-1}^{*2} - \{2n-2\}, i-1) \neq \Phi, \text{and } D(P_{n-3}^{*2}, i-1) = \Phi, \text{ by}$$

Lemma 2.2(iii),(iv), we have $\left\lfloor \frac{n}{3} \right\rfloor \leq i-1 \leq 2n-1, \left\lceil \frac{n-1}{3} \right\rceil \leq i-1 \leq 2n-2, \left\lfloor \frac{n-1}{3} \right\rfloor \leq i-1 \leq 2n-3$.Also .

$$i-1 < \left\lceil \frac{n-3}{3} \right\rceil \text{ or } i-1 > 2n-6. \text{Also } i-1 \geq 2n-5.$$

Therefore, $2n-5 \leq i-1 \leq 2n-3$.Therefore, $2n-4 \leq i \leq 2n-2$.

Hence $i=2n-2, 2n-3, 2n-4$

(\Leftarrow) It follows from Lemma 2.2(iii), (iv).

Now, we construct following Theorem.

Theorem 2.5:

i) If $D(P_n^{*2} - \{2n\}, i-1) \neq \Phi$,and $D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-3}^{*2}, i-1) = \Phi$ then

$$D(P_n^{*2}, i) = \{X \cup \{2n\} / X \in D(P_n^{*2} - \{2n\}, i-1)\}$$

ii) If $D(P_n^{*2} - \{2n\}, i-1) \neq \Phi$, $D(P_{n-1}^{*2}, i-1) \neq \Phi$, $D(P_{n-1}^{*2} - \{2n-2\}, i-1) \neq \Phi$

and $D(P_{n-3}^{*2}, i-1) = \Phi$,then

$$D(P_n^{*2}, i) = \{X_1 \cup \{2n\} / X_1 \in D(P_n^{*2} - \{2n\}, i-1), X_2 \cup \{\{2n-1\} / D(P_{n-1}^{*2}, i-1)\}$$

$$\{X_3 \cup \{2n-2, 2n-3\} / X_3 \in D(P_{n-1}^{*2} - \{2n-2\}, i-1)\}$$

iii) If $D(P_n^{*2} - \{2n\}, i-1) \neq \Phi$ $D(P_{n-1}^{*2}, i-1) \neq \Phi$ $D(P_{n-1}^{*2} - \{2n-2\}, i-1) \neq \Phi$ and $D(P_{n-3}^{*2}, i-1) \neq \Phi$,then

$$\{X_1 \cup \{2n\} / X_1 \in D(P_n^{*2} - \{2n\}, i-1),$$

$$X_2 \cup \{2n-1\} / X_2 \in D(P_{n-1}^{*2}, i-1),$$

$$X_3 \cup \{2n-3\} / X_3 \in D(P_{n-1}^{*2} - \{2n-2\}, i-1)\},$$

$$X_4 \cup \begin{cases} \{2n-1\} / X_4 \in D(P_{n-3}^{*2}, i-1), 1 \notin D(P_{n-3}^{*2}, i-1) \\ \{2n-3\} / X_4 \in D(P_{n-3}^{*2}, i-1), 1 \in D(P_{n-3}^{*2}, i-1) \end{cases}$$

Proof:

i) By theorem 2.1(i), $i=2n$.Since in this case $D(P_n^{*2}, i) = D(P_n^{*2}, 2n) = \{[2n]\}$ and

$$D(P_n^{*2} - \{2n\}, i-1) = D(P_n^{*2} - \{2n\}, 2n-1) = \{[2n-1]\},$$

then we have the result.

ii) Let

$$Y_1 = \{X_1 \cup \{2n\} / X_1 \in D(P_n^{*2} - \{2n\}, i-1)\}$$

$$Y_2 = \{X_2 \cup \{2n-1\} / X_2 \in D(P_{n-1}^{*2}, i-1)\} \text{ and}$$

$$Y_3 = \{X_3 \cup \{2n-2, 2n-3\} / X_3 \in D(P_{n-1}^{*2} - \{2n-2\}, i-1)\}.$$

Obviously, $Y_1 \cup Y_2 \cup Y_3 \subseteq D(P_n^{*2}, i)$ (2.1)

Now let $Y \in D(P_n^{*2}, i)$.If $2n \in Y$, then at least one of the vertives labeled $2n-1$ or $2n-2$ is in Y.In either cases,

$$Y = X_1 \cup \{2n\} \text{ forsome } X_1 \in D(P_n^{*2} - \{2n\}, i-1) \text{ ,that is } Y \in Y_1.$$

If $2n \notin Y$ and $2n-1 \in Y$, then $2n-2 \in Y$, So $Y = X_2 \cup \{2n-1\} \text{ forsome } D(P_{n-1}^{*2}, i-1)$,that is $Y \in Y_2$.Now

suppose that $2n-2 \in Y$, $2n \notin Y$ and $2n-1 \notin Y$,then at least one of the vertices labeled

$2n-3, 2n-4$ is in Y. If $2n-4 \in Y$, then

$$Y = \{X_3 \cup \{2n-2, 2n-3\} / X_3 \in D(P_{n-1}^{*2} - \{2n-2\}, i-1)\}; \text{that is } Y \in Y_3.$$

Thus, we have proved that $D(P_n^{*2}, i) \subseteq Y_1 \cup Y_2 \cup Y_3$. (2.2)

From (2.1) and (2.2), We have

$$D(P_n^{*2}, i) = \left\{ X_1 \cup \{2n\} / X_1 \in D(P_n^{*2} - \{2n\}, i-1), X_2 \cup \{2n-1\} / D(P_{n-1}^{*2}, i-1) \right\} \\ \left\{ X_3 \cup \{2n-2, 2n-3\} / X_3 \in D(P_{n-1}^{*2} - \{2n-2\}, i-1), \right\}$$

iii) Let

$$Y_1 = X_1 \cup \{2n\} / X_1 \in D(P_n^{*2} - \{2n\}, i-1), \\ Y_2 = X_2 \cup \{2n-1\} / X_2 \in D(P_{n-1}^{*2}, i-1), \\ Y_3 = X_3 \cup \{2n-3\} / X_3 \in D(P_{n-1}^{*2} - \{2n-2\}, i-1) \text{, and} \\ Y_4 = X_4 \cup \left\{ \begin{array}{l} \{2n-1\} / X_4 \in D(P_{n-3}^{*2}, i-1), 1 \notin D(P_{n-3}^{*2}, i-1) \\ \{2n-3\} / X_4 \in D(P_{n-3}^{*2}, i-1), 1 \in D(P_{n-3}^{*2}, i-1) \end{array} \right\} .$$

Obviously, $Y_1 \cup Y_2 \cup Y_3 \cup Y_4 \subseteq D(P_n^{*2}, i)$ (2.3)

Now, let $Y \in D(P_n^{*2}, i)$. If $2n \in Y$, then at least one of the vertices labeled $2n-1$ or $2n-2$ is in Y . In either cases, $Y = X_1 \cup \{2n\}$ for some $X_1 \in D(P_n^{*2} - \{2n\}, i-1)$; that is $Y \in Y_1$.

If $2n \notin Y$ and $2n-1 \in Y$, then $2n-2 \in Y$, so $Y = X_2 \cup \{2n-1\}$ for some $D(P_{n-1}^{*2}, i-1)$ that is $Y \in Y_2$. Now suppose that $2n-2 \in Y$, $2n \notin Y$ and $2n-1 \notin Y$, then at least one of the vertices labeled $2n-3, 2n-4$ is in Y . If $2n-4 \in Y$, then $Y = \{X_3 \cup \{2n-3\}$ for some $X_3 \in D(P_{n-1}^{*2} - \{2n-2\}, i-1)\}$, that is $Y \in Y_3$. Now suppose that $2n-3 \in Y$, $2n-2 \notin Y$, $2n-1 \notin Y$, then at least one of the vertices labeled $2n-4, 2n-5$ is in Y . If $2n-5 \in Y$, then $Y = X_4 \cup \left\{ \begin{array}{l} \{2n-1\} \text{ for some } X_4 \in D(P_{n-3}^{*2}, i-1), 1 \notin D(P_{n-3}^{*2}, i-1) \\ \{2n-3\} \text{ for some } X_4 \in D(P_{n-3}^{*2}, i-1), 1 \in D(P_{n-3}^{*2}, i-1) \end{array} \right\}$.

Thus we have proved that $D(P_n^{*2}, i) \subseteq Y_1 \cup Y_2 \cup Y_3 \cup Y_4$. (2.4)

From (2.3) and (2.4)

$$\left\{ X_1 \cup \{2n\} / X_1 \in D(P_n^{*2} - \{2n\}, i-1), \right. \\ \left. X_2 \cup \{2n-1\} / X_2 \in D(P_{n-1}^{*2}, i-1), \right. \\ \left. X_3 \cup \{2n-3\} / X_3 \in D(P_{n-1}^{*2} - \{2n-2\}, i-1) \right\}, \\ X_4 \cup \left\{ \begin{array}{l} \{2n-1\} / X_4 \in D(P_{n-3}^{*2}, i-1), 1 \notin D(P_{n-3}^{*2}, i-1) \\ \{2n-3\} / X_4 \in D(P_{n-3}^{*2}, i-1), 1 \in D(P_{n-3}^{*2}, i-1) \end{array} \right\}$$

Theorem 2.6: For every $n \geq 7$,

$$|D(P_n^{*2}, i)| = |D(P_n^{*2} - \{2n\}, i-1)| + |D(P_{n-1}^{*2}, i-1)| + |D(P_{n-1}^{*2} - \{2n-2\}, i-1)| + |D(P_{n-3}^{*2}, i-1)|$$

Proof: We consider the three cases in Theorem 2.2

- i) By Theorem 2.2(i), $D(P_n^{*2}, i) = \{X \cup \{2n\} / X \in D(P_n^{*2} - \{2n\}, i-1)\}$. Therefore we have the result in this case.
- ii) By Theorem 2.2(ii), we have

$$D(P_n^{*2}, i) = A_1 \cup A_2 \cup A_3 \text{ Where,}$$

$$|A_1| = D(P_n^{*2} - \{2n\}, i-1), |A_2| = D(P_{n-1}^{*2}, i-1), |A_3| = D(P_{n-1}^{*2} - \{2n-2\}, i-1). \text{ Since for every}$$

$X_1 \in A_1, X_2 \in X_1$, and for every $X_2 \in A_2, 2n \notin X_2$, so $A_1 \cap A_2 = \Phi$. Also since for every $X_2 \in A_2$ and $X_3 \in A_3, 2n-1 \in X_2$, and $2n-1 \notin X_3$, we have $A_2 \cap A_3 = \Phi$. For every $X_3 \in A_3, 2n-1 \notin X_3$, and $2n-3 \notin X_3$, but for every $X_1 \in A_1$, at least one of $2n-3$ or $2n-1$ is in X_3 ; because $X_3 = Y_3 \cup \{2n\}$ for some $Y_3 \in D(P_n^{*2} - \{2n\}, i-1)$, so at least one of $2n-3$ or $2n-1$ must be in Y_3 . Hence we have the result.

- iii) By Theorem 2.2(ii) $D(P_n^{*2}, i) = A_1 \cup A_2 \cup A_3 \cup A_4$, Where,

$$|A_1| = D(P_n^{*2} - \{2n\}, i-1), |A_2| = D(P_{n-1}^{*2}, i-1), |A_3| = D(P_{n-1}^{*2} - \{2n-2\}, i-1) \text{ and } |A_4| = D(P_{n-3}^{*2}, i-1)$$

Since for every $X_1 \in A_1, X_2 \in X_1$, and for every $X_2 \in A_2, 2n \notin X_2$, so $A_1 \cap A_2 = \Phi$. Also since for every $X_2 \in A_2$ and $X_3 \in A_3, 2n-1 \in X_2$, and $2n-1 \notin X_3$, we have $A_2 \cap A_3 = \Phi$. For every $X_3 \in A_3, 2n-1 \notin X_3$, and $2n-3 \notin X_3$, but for every $X_1 \in A_1$, at least one of $2n-3$ or $2n-1$ is in X_3 ; because $X_3 = Y_3 \cup \{2n\}$ for some $Y_3 \in D(P_n^{*2} - \{2n\}, i-1)$ so at least one of $2n-3$ or $2n-1$ must be in Y_3 . For every $X_4 \in A_4, 2n-1 \notin X_4$ and $2n-3 \notin X_4$ but for every $X_1 \in A_1$, at least one of $2n-3$ or $2n-1$ is in X_4 because $X_4 = Y_4 \cup \{2n\}$ for some $Y_4 \in D(P_n^{*2} - \{2n\}, i-1)$, at least one of $2n-3$ or $2n-1$ must be in Y_4 . Hence we have the result.

3. Domination sets of $(P_n^{*2} - \{2n\})$

For the construction of $(P_n^{*2} - \{2n\})$, we consider

$$D(P_{n-1}^{*2}, i-1), D(P_{n-1}^{*2} - \{2n-2\}, i-1), D(P_{n-2}^{*2}, i-1), D(P_{n-2}^{*2} - \{2n-4\}, i-1) \text{ and } D(P_{n-3}^{*2} - \{2n-6\}, i-1)$$

The families of these dominating sets can be empty or otherwise. Thus, we have eight combinations, whether these five families are empty or not. Two of these combinations are not possible (Lemma 2.3 (i), (ii) & (iii)).

Also, the combinations

$D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-2}^{*2}, i-1) = D(P_{n-2}^{*2} - \{2n-4\}, i-1) = D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$ does not need to be considered because it implies that $D(P_n^{*2} - \{2n\}, i-1) = \Phi$

(See lemma 2.5 (iii)). Thus we only need to consider four combinations or cases. We consider those cases in theorem (2.7).

Lemma 3.1:

i) If $D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-2}^{*2}, i-1) = D(P_{n-2}^{*2} - \{2n-4\}, i-1) = \Phi$, then

$$D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi .$$

ii) If $D(P_{n-1}^{*2}, i-1) = D(P_{n-2}^{*2}, i-1) = D(P_{n-2}^{*2} - \{2n-4\}, i-1) = D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$, then

$$D(P_{n-1}^{*2} - \{2n-2\}, i-1) = \Phi .$$

iii) If $D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-2}^{*2}, i-1) = \Phi$,

$$D(P_{n-2}^{*2} - \{2n-4\}, i-1) = D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi , \text{ then } D(P_n^{*2} - \{2n\}, i-1) = \Phi .$$

iv) $D(P_{n-1}^{*2}, i-1) \neq \Phi$ and

$$D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-2}^{*2}, i-1) = D(P_{n-2}^{*2} - \{2n-4\}, i-1) = D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi \text{ then}$$

$$D(P_n^{*2} - \{2n\}, i-1) \neq \Phi .$$

v) If $D(P_{n-1}^{*2}, i-1) \neq \Phi, D(P_{n-1}^{*2} - \{2n-2\}, i-1) \neq \Phi$ and

$$D(P_{n-2}^{*2} - \{2n-4\}, i-1) = \Phi \text{ th } \& D(P_{n-2}^{*2}, i-1) = \Phi$$

Proof:

i) Since $D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-2}^{*2}, i-1) = D(P_{n-2}^{*2} - \{2n-4\}, i-1) = \Phi$, by lemma 2.2(iii),(iv) we have,

$$i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor \text{ or } i-1 \succ 2n-2, \quad i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor \text{ or } i-1 \succ 2n-3, \quad i-1 \prec \left\lfloor \frac{n-2}{3} \right\rfloor \text{ or } i-1 \succ 2n-4 \\ i-1 \prec \left\lfloor \frac{n-2}{3} \right\rfloor \text{ or } i-1 \succ 2n-5.$$

From the above relations, it follows that $i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor$ or $i-1 \succ 2n-7$.

Therefore, $D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$.

- ii) Since $D(P_{n-1}^{*2}, i-1) = D(P_{n-2}^{*2}, i-1) = D(P_{n-2}^{*2} - \{2n-4\}, i-1) = D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$
so by Lemma 2.2(iii),(iv), we have

$$i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor \text{ or } i-1 \succ 2n-2, \quad i-1 \prec \left\lfloor \frac{n-2}{3} \right\rfloor \text{ or } i-1 \succ 2n-4, \quad i-1 \prec \left\lfloor \frac{n-2}{3} \right\rfloor \text{ or } i-1 \succ 2n-5, \\ i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor \text{ or } i-1 \succ 2n-7.$$

Therefore, $i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor$ or $i-1 \succ 2n-2$.

Hence, $i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor$ or $i-1 \succ 2n-3$ hold.

Hence, $D(P_{n-1}^{*2} - \{2n-2\}, i-1) = \Phi$.

- iv) If $D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-2}^{*2}, i-1) = \Phi$,

$D(P_{n-2}^{*2} - \{2n-4\}, i-1) = D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$, so by Lemma 2.2(iii),(iv), we have,

$$i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor \text{ or } i-1 \succ 2n-2, \quad i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor \text{ or } i-1 \succ 2n-3, \quad i-1 \prec \left\lfloor \frac{n-2}{3} \right\rfloor \text{ or } i-1 \succ 2n-4, \\ i-1 \prec \left\lfloor \frac{n-2}{3} \right\rfloor \text{ or } i-1 \succ 2n-5, \quad i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor \text{ or } i-1 \succ 2n-7.$$

Hence, $i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor$ or $i-1 \succ 2n-2$. $i \succ 2n-1$

Therefore, $i \prec \left\lfloor \frac{n}{3} \right\rfloor$ or $i \succ 2n-1$.

Therefore, $D(P_n^{*2} - \{2n\}, i-1) = \Phi$.

- v) Since $D(P_{n-1}^{*2}, i-1) \neq \Phi$ and

$D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-2}^{*2}, i-1) = D(P_{n-2}^{*2} - \{2n-4\}, i-1) = D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$ So,

by Lemma 2.2(iii),(iv), we have,

$$\left\lfloor \frac{n-1}{3} \right\rfloor \leq i-1 \leq 2n-2, \quad i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor \text{ or } i-1 \succ 2n-3, \quad i-1 \prec \left\lfloor \frac{n-2}{3} \right\rfloor \text{ or } i-1 \succ 2n-4,$$

$$i-1 \prec \left\lfloor \frac{n-2}{3} \right\rfloor \text{ or } i-1 \succ 2n-5, \quad i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor \text{ or } i-1 \succ 2n-7.$$

Since $i-1 \geq \left\lfloor \frac{n-1}{3} \right\rfloor$, the possibilities of $i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor, i-1 \prec \left\lfloor \frac{n-2}{3} \right\rfloor$.

$i-1 \prec \left\lfloor \frac{n-2}{3} \right\rfloor$ or $i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor$ do not occur. Therefore, from $\left\lfloor \frac{n-1}{3} \right\rfloor \prec i-1 \leq 2n-2$,

We have $i-1 \leq 2n-2$.

The other possibilities, we obtain that $i-1 \succ 2n-3$.

Therefore, $i-1 \geq 2n-2$. But $i-1 \leq 2n-2$.

Therefore, $i-1 = 2n-2$.

Therefore, $i = 2n-1$.

Hence, $D(P_n^{*2} - \{2n\}, i-1) \neq \Phi$.

- vi) Since $D(P_{n-1}^{*2}, i-1) \neq \Phi$, $D(P_{n-1}^{*2} - \{2n-2\}, i-1) \neq \Phi$ and $D(P_{n-2}^{*2} - \{2n-4\}, i-1) = \Phi$, by Lemma 2.2(iii),(iv), we have,

$$\left\lceil \frac{n-1}{3} \right\rceil \leq i-1 \leq 2n-2, \quad \left\lfloor \frac{n-1}{3} \right\rfloor \leq i-1 \leq 2n-3 \text{ and } i-1 \prec \left\lfloor \frac{n-2}{3} \right\rfloor \text{ or } i-1 \succ 2n-5.$$

$$\text{Then } \left\lceil \frac{n-1}{3} \right\rceil \leq i-1 \leq 2n-3 \text{ and } i-1 \succ 2n-5.$$

Therefore, $i-1 \leq 2n-3$ and $i-1 \geq 2n-4$.

Therefore, $i-1 = 2n-4$ or $i-1 = 2n-3$.

Therefore, $i = 2n-3$ or $i-1 = 2n-2$.

Therefore, $D(P_n^{*2} - \{2n\}, i-1) \neq \Phi$.

Now we state when these cases for the families $D(P_{n-1}^{*2}, i-1)$, $D(P_{n-1}^{*2} - \{2n-2\}, i-1)$, $D(P_{n-2}^{*2}, i-1)$,

$D(P_{n-2}^{*2} - \{2n-4\}, i-1)$, $D(P_{n-3}^{*2} - \{2n-6\}, i-1)$

can occur.

Theorem 3.2:

For every $n \geq 7$

- i) $D(P_{n-1}^{*2}, i-1) \neq \Phi$ and
 $D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-2}^{*2}, i-1) = D(P_{n-2}^{*2} - \{2n-4\}, i-1) = D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$ if and only if $i=2n-1$.
- ii) $D(P_{n-1}^{*2}, i-1) \neq \Phi$, $D(P_{n-1}^{*2} - \{2n-2\}, i-1) \neq \Phi$, $D(P_{n-2}^{*2}, i-1) = \Phi$ and $D(P_{n-2}^{*2} - \{2n-4\}, i-1) = \Phi$ if and only if $i = 2n-2$.
- iii) $D(P_{n-1}^{*2}, i-1) \neq \Phi$, $D(P_{n-1}^{*2} - \{2n-2\}, i-1) \neq \Phi$, $D(P_{n-2}^{*2}, i-1) \neq \Phi$, $D(P_{n-1}^{*2} - \{2n-4\}, i-1) \neq \Phi$ and $D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$ if and only if $i=2n-4$, $i=2n-5$.

Proof:

- i) (\Rightarrow) Since $D(P_{n-1}^{*2}, i-1) \neq \Phi$ and,

$D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-2}^{*2}, i-1) = D(P_{n-2}^{*2} - \{2n-4\}, i-1) = D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$ we have,

$$i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor \text{ or } i-1 \succ 2n-3, \quad i-1 \prec \left\lfloor \frac{n-2}{3} \right\rfloor \text{ or } i-1 \succ 2n-4, \quad i-1 \prec \left\lfloor \frac{n-2}{3} \right\rfloor \text{ or } i-1 \succ 2n-5,$$

$$i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor \text{ or } i-1 \succ 2n-7.$$

Also, $\left\lceil \frac{n-1}{3} \right\rceil \leq i-1 \leq 2n-2$.

Therefore $i-1 < \left\lfloor \frac{n-1}{3} \right\rfloor$ does not hold.

Therefore $i-1 > 2n-3$. Hence $i-1 \geq 2n-2$. Therefore $i \geq 2n-1$.

But $i-1 \leq 2n-2$, $i \leq 2n-1$.

Therefore $i=2n-1$.

(\Leftarrow) It follows from Lemma 2.2(iii), (iv).

ii) Since $D(P_{n-1}^{*2}, i-1) \neq \Phi$, $D(P_{n-1}^{*2} - \{2n-2\}, i-1) \neq \Phi$,

$D(P_{n-2}^{*2}, i-1) = \Phi$ and $D(P_{n-2}^{*2} - \{2n-4\}, i-1) = \Phi$, so by Lemma 2.2(iii),(iv), we have,

$$\left\lceil \frac{n-1}{3} \right\rceil \leq i-1 \leq 2n-2, \left\lceil \frac{n-1}{3} \right\rceil \leq i-1 \leq 2n-3.$$

Also, $i-1 < \left\lceil \frac{n-2}{3} \right\rceil$ or $i-1 > 2n-4$, $i-1 < \left\lceil \frac{n-2}{3} \right\rceil$ or $i-1 > 2n-5$.

Therefore, $\left\lceil \frac{n-1}{3} \right\rceil \leq i-1 \leq 2n-3$.

Therefore, $i-1 < \left\lceil \frac{n-2}{3} \right\rceil$ and $i-1 < \left\lceil \frac{n-1}{3} \right\rceil$ do not hold.

Therefore, $i-1 > 2n-4$. Therefore, $i-1 \geq 2n-3$.

But $i-1 \leq 2n-3$.

Therefore, $i=2n-2$.

(\Leftarrow) It follows from Lemma 2.2(iii), (iv).

iii) $D(P_{n-1}^{*2}, i-1) \neq \Phi$, $D(P_{n-1}^{*2} - \{2n-2\}, i-1) \neq \Phi$, $D(P_{n-2}^{*2}, i-1) \neq \Phi$ $D(P_{n-1}^{*2} - \{2n-4\}, i-1) \neq \Phi$ and $D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$, So by Lemma 2.2(iii),(iv), we have,

$$\left\lceil \frac{n-1}{3} \right\rceil \leq i-1 \leq 2n-2, \left\lceil \frac{n-1}{3} \right\rceil \leq i-1 \leq 2n-3, \left\lceil \frac{n-2}{3} \right\rceil \leq i-1 \leq 2n-4, \left\lceil \frac{n-2}{3} \right\rceil \leq i-1 \leq 2n-5$$

.Therefore $\left\lceil \frac{n-1}{3} \right\rceil \leq i-1 \leq 2n-5$.

Also, $i-1 < \left\lceil \frac{n-3}{3} \right\rceil$ or $i-1 > 2n-7$. Therefore, $i-1 < \left\lceil \frac{n-3}{3} \right\rceil$ does not hold.

Therefore, $i-1 > 2n-7$. Therefore, $i-1 \geq 2n-6$. But $i-1 \leq 2n-5$.

Therefore, $i=2n-6$ or $2n-5$.

Therefore, $i=2n-5$ or $2n-4$.

Theorem 3.3:

i) $D(P_{n-1}^{*2}, i-1) = \Phi$, $D(P_{n-1}^{*2} - \{2n-2\}, i-1) \neq \Phi$, $D(P_{n-2}^{*2}, i-1) \neq \Phi$ and $D(P_{n-3}^{*2} - \{2n-6\}, i-1) \neq \Phi$ then

- $$D(P_n^{*2} - \{2n\}, i) = \{X_1 \cup \{2n-1\} / X_1 \in D(P_{n-1}^{*2} - \{2n-2\}, i-1), \\ X_2 \cup \{2n-2\} / X_2 \in D(P_{n-1}^{*2}, i-1), \\ X_3 \cup \{2n-3\} / X_3 \in D(P_{n-2}^{*2} - \{2n-4\}, i-1), \\ X_4 \cup \{2n-5\} / X_4 \in D(P_{n-3}^{*2} - \{2n-6\}, i-1), \}$$
- ii) $D(P_{n-1}^{*2}, i-1) \neq \Phi$ a n $D(P_{n-1}^{*2} - \{2n-2\}, i-1) = \Phi, D(P_{n-2}^{*2}, i-1) = \Phi, D(P_{n-3}^{*2} - \{2n-6\}, i-1) \neq \Phi$
then $D(P_n^{*2} - \{2n\}, i) = \{X \cup \{2n-1\} / X \in D(P_{n-1}^{*2}, i-1)\}$

Proof:

i) Let

$$Y_1 = \{X_1 \cup \{2n-1\} / X_1 \in D(P_{n-1}^{*2} - \{2n-2\}, i-1)\}$$

$$Y_2 = \{X_2 \cup \{2n-2\} / X_2 \in D(P_{n-1}^{*2}, i-1)\}$$

$$Y_3 = \{X_3 \cup \{2n-3\} / X_3 \in D(P_{n-2}^{*2} - \{2n-4\}, i-1)\}$$

$$Y_4 = \{X_4 \cup \{2n-5\} / X_4 \in D(P_{n-3}^{*2} - \{2n-6\}, i-1)\}$$

Obviously, $Y_1 \cup Y_2 \cup Y_3 \cup Y_4 \subseteq D(P_n^{*2} - \{2n\}, i)$ (2.1)

Now let $Y \in D(P_n^{*2} - \{2n\}, i)$. If $2n \in Y$, then at least one of vertices labeled 2n-1 or 2n-2 is in Y. In either cases, $Y = X_1 \cup \{2n\}$ for some $X_1 \in D(P_{n-1}^{*2} - \{2n-2\}, i-1)$; that is $Y \in Y_1$.

If $2n \notin Y$ and $2n-1 \in Y$, then $2n-2 \in Y$, so $Y = X_2 \cup \{2n-1\}$ for some $D(P_{n-1}^{*2}, i-1)$ that is $Y \in Y_2$. Now suppose that $2n-2 \in Y$, $2n \notin Y$ and $2n-1 \notin Y$, then at least one of vertices labeled 2n-3, 2n-4 is in Y. If $2n-4 \in Y$, then $Y = X_3 \cup \{2n-3\}$ for some $X_3 \in D(P_{n-2}^{*2} - \{2n-4\}, i-1)$, that is $Y \in Y_3$. Now suppose that $2n-5 \in Y$, $2n-3 \notin Y$ and $2n-4 \notin Y$, then at least one of vertices labeled 2n-3, 2n-5 is in Y. If $2n-4 \in Y$, then $Y = X_4 \cup \{2n-5\}$ for some $X_4 \in D(P_{n-3}^{*2} - \{2n-6\}, i-1)$;
that is $Y \in Y_4$.

Thus we have proved

$$D(P_n^{*2} - \{2n\}, i) \subseteq Y_1 \cup Y_2 \cup Y_3 \cup Y_4 \quad D(P_n^{*2}, i) \subseteq Y_1 \cup Y_2 \cup Y_3 \quad (2.2)$$

From (2.1) and (2.2)

We have

$$D(P_n^{*2} - \{2n\}, i) = \{X_1 \cup \{2n-1\} / X_1 \in D(P_{n-1}^{*2} - \{2n-2\}, i-1), \\ X_2 \cup \{2n-2\} / X_2 \in D(P_{n-1}^{*2}, i-1), \\ X_3 \cup \{2n-3\} / X_3 \in D(P_{n-2}^{*2} - \{2n-4\}, i-1), \\ X_4 \cup \{2n-5\} / X_4 \in D(P_{n-3}^{*2} - \{2n-6\}, i-1), \}$$

- ii) By lemma 2.1, i=2n-1. Therefore $D(P_n^{*2} - \{2n\}, i) = D(P_n^{*2} - \{2n\}, 2n-1) = \{[2n-1]\}$
and $D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2}, 2n-2) = \{[2n-2]\}$. So we have the result.

Theorem3.4: For every $n \geq 7$

$$\begin{aligned} |D(P_n^{*2} - \{2n\}, i)| &= |D(P_{n-1}^{*2}, i-1)| + |D(P_{n-1}^{*2} - \{2n-2\}, i-1)| + |D(P_{n-2}^{*2}, i-1)| \\ &\quad + |D(P_{n-2}^{*2} - \{2n-4\}, i-1)| + |D(P_{n-3}^{*2} - \{2n-6\}, i-1)| \end{aligned}$$

Proof: We consider the three cases in Theorem 2.2

i) By Theorem 2.2 $D(P_n^{*2} - \{2n\}, i) = A_1 \cup A_2 \cup A_3 \cup A_4$, where. $|A_1| = D(P_{n-1}^{*2} - \{2n-2\}, i-1)$,

$$|A_2| = D(P_{n-2}^{*2}, i-1), |A_3| = D(P_{n-2}^{*2} - \{2n-4\}, i-1), |A_4| = D(P_{n-3}^{*2} - \{2n-6\}, i-1).$$

Since for every $X_1 \in A_1, X_2 \in X_1$, and for every $X_2 \in A_2, 2n \notin X_2$, so $A_1 \cap A_2 = \emptyset$. Also since for every $X_2 \in A_2$ and $X_3 \in A_3, 2n-1 \in X_2$, and $2n-1 \notin X_3$, we have $A_2 \cap A_3 = \emptyset$. For every $X_3 \in A_3$,

$2n-1 \notin X_3$, and $2n-3 \notin X_3$, but for every $X_1 \in A_1$, atleast one of $2n-3$ or $2n-1$ is in X_3 , because $A_3 \cap A_4 = \emptyset$. For every $X_4 \in A_4$ for some so atleast one of $2n-3$ or $2n-1$ must be in X_4 . $2n-1 \notin X_3$, and $2n-3 \notin X_3$, but for every $X_1 \in A_1$, atleast one of the $2n-3$ or $2n-1$ is in X_4 , because $X_4 = Y_4 \cup \{2n\}$ for some

$Y_4 \in D(P_n^{*2} - \{2n\}, i)$ so atleast one of the $2n-3$ or $2n-1$ must be Y_4 . Hence we have the result.

ii) By Theorem 2.2(ii), $|D(P_n^{*2} - \{2n\}, i-1)| = |D(P_{n-1}^{*2}, i-1)|$. Since in this case

$$|D(P_{n-1}^{*2} - \{2n-2\}, i-1)| = |D(P_{n-2}^{*2}, i-1)| = 0 \text{ and } |D(P_{n-2}^{*2} - \{2n-4\}, i-1)| = |D(P_{n-3}^{*2} - \{2n-6\}, i-1)|$$

= 0, we have the result in this case.

The following table illustrates the number of dominating sets are of $D(P_n^{*2}, i)$ and $D(P_n^{*2} - \{2n\}, i)$, for $n = 1, 2, \dots, 10$ and $i = 1, 2, \dots, 20$.

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n																				
P_1^{*2}	2	1																		
$P_2^{*2} - \{4\}$	3	3	1																	
P_3^{*2}	2	6	4	1																
$P_4^{*2} - \{6\}$	2	9	10	5	1															
P_5^{*2}	1	8	18	15	6	1														
$P_6^{*2} - \{8\}$	1	10	29	33	21	7	1													
P_7^{*2}	0	6	30	58	53	28	8	1												
$P_8^{*2} - \{10\}$	0	7	41	97	115	83	36	9	1											
P_9^{*2}	0	3	32	110	193	190	118	45	10	1										
$P_{10}^{*2} - \{12\}$	0	3	39	158	331	402	316	164	55	11	1									
P_{11}^{*2}	0	1	22	141	398	660	682	471	218	66	12	1								
$P_{12}^{*2} - \{14\}$	0	1	25	183	595	1149	1415	1189	698	285	78	13	1							
P_{13}^{*2}	0	0	11	127	540	1377	2239	2421	1825	971	362	91	14	1						
$P_{14}^{*2} - \{16\}$	0	0	12	138	695	1979	3671	4688	4254	2797	1333	453	105	15	1					
P_{15}^{*2}	0	0	4	97	606	2161	4907	7641	8462	6832	4064	1774	557	120	16	1				
$P_{16}^{*2} - \{18\}$	0	0	4	92	595	2767	7068	12548	16103	15294	10896	5838	2331	677	136	17	1			
P_{17}^{*2}	0	0	1	43	510	2251	8056	17061	26066	29517	25208	16371	8078	2994	812	153	18	1		
$P_{18}^{*2} - \{20\}$	0	0	1	47	553	3001	10307	25117	43127	55583	54725	41579	39749	11080	3806	965	171	19	1	
P_{19}^{*2}	0	0	0	17	320	2353	9998	29102	59414	89550	103191	92162	64241	50263	14766	31755	1135	190	20	1

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