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MIXED INTEGER LINEAR PROGRAMMING PROBLEMS WITH FUZZY VARIABLES

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ABSTRACT

In this paper, a new decomposition method is used for solving mixed integer linear programming problems with fuzzy variables by using classical mixed integer linear programming has been proposed. In the decomposition method, ranking functions are not used. This method provides the best solution to a variety of mixed integer linear programming problems with fuzzy variables in a simple and effective manner. The decomposition method is illustrated with the help of a numerical example.

Key Words: *Linear Programming, Integer Programming, Fuzzy Variables.*

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1. INTRODUCTION:

Mixed integer linear programming is linear programming which require some or all of the variables to be integers. In real world situation the available information in the system under consideration are not exact, therefore fuzzy linear programming (FLP) was introduced and studied by many researchers, see, e.g., [3, 4, 6, 8, 11]. Fuzzy set theory has been applied to many disciplines such as control theory and management sciences, mathematical modeling and industrial applications. The concept of FLP on a general level was first proposed by Tanaka et al. [10].

FLP problems have an essential role in fuzzy modeling, which can formulate uncertainty in actual environment. Afterwards, many authors have considered various types of the FLP problems and proposed several approaches for solving these problems. In particular, the most convenient methods are based on the concept of comparison of fuzzy numbers with help of ranking functions, see, e.g., [3, 8]. Usually in such methods authors define a crisp model which is equivalent to the FLP problem and then use optimal solution of the model as the optimal solution of the FLP problem.

Mahdavi-Amiri and Nasseri [7] introduced a dual simplex algorithm for solving linear programming problem with fuzzy variables and its dual by using a general linear ranking function and linear programming directly. Allahviranloo et al. [1] have proposed a new method based on fuzzy number ranking method for a FILP problem via crisp integer linear programming (ILP) problems. Nasseri [9] has proposed a new method for solving the FLP problems in which he has used the fuzzy ranking method for converting the fuzzy objective function into crisp objective function.
In this paper, we have proposed a new method namely, decomposition method for solving a MILP problem with fuzzy variables by using the classical MILP. The significance of this paper is providing a new method for solving MILP problems with fuzzy variables without using any ranking functions. This method can serve managers by providing the best solution to a variety of mixed integer linear programming problems with fuzzy variables in a simple and effective manner. The decomposition method is illustrated with the help of numerical example.

2. PRELIMINARIES:

We need the following definitions of the basic arithmetic operators on fuzzy triangular numbers based on the function principle which can be found in [9].
Definition: 2.1. A fuzzy number $\tilde{a}$ is a triangular fuzzy number denoted by (a_1, a_2, a_3).where a_1, a_2 and a_3 are real numbers and its membership function is given below.

$$\mu_{\tilde{a}}(x) = \begin{cases} (x-a_1)/(a_2-a_1) & for a_1 \le x \le a_2 \\ (a_3-x)/(a_3-a_2) & for a_2 \le x \le a_3 \\ 0 & otherwise \end{cases}$$

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Definition: 2.2. Let (a_1, a_2, a_3) be two triangular fuzzy numbers. Then

(a) $(a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
(b) $(a_1, a_2, a_3) - (b_1, b_2, b_3) = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$
(c) $k(a_1, a_2, a_3) = (ka_1, ka_2, ka_3)$ for $k \geq 0$
(d) $k(a_1, a_2, a_3) = (ka_3, ka_2, ka_1)$ for $k < 0$

Definition: 2.3. Let $\tilde{A} = (a_1, a_2, a_3)$ be in F(R).then

(*i*) $\tilde{A}$ is said to be positive if $a_i \geq 0$, for all i = 1 to 3;

(*ii*) $\tilde{A}$ is said to be integer if $a_i \geq 0$,for all i = 1 to 3 are integers and

(*iii*) $\tilde{A}$ is said to be symmetric if $a_2 - a_1 = a_3 - a_2$.

Definition: 2.4. A fuzzy vector $\tilde{b} = (\tilde{b}_i)_{m\times 1}$ is called nonnegative and denoted by $\tilde{b} \geq 0$, if each element of $\tilde{b}$ is a nonnegative real fuzzy number, that is, $\tilde{b}_i \geq 0$, $i = 1,2,\ldots\ldots,m$.

Consider the following $m \times n$ fuzzy linear system with nonnegative real fuzzy numbers:

$$A\tilde{x} \leq \tilde{b}$$

Where $A = (a_{ij})_{m\times n}$ is a nonnegative crisp matrix and $\tilde{x} = (\tilde{x}_j), \tilde{b} = (\tilde{b}i)$ are nonnegative fuzzy vectors and $\tilde{x}_j, \tilde{b}_i \in F(R)$,for all $1 \leq j \leq n$ and $1 \leq i \leq$ m.

3. FUZZY MIXED INTEGER LINEAR PROGRAMMING:

Consider the following integer linear programming problem with fuzzy variables

$\max \tilde{z} = c\tilde{x}$

subject to $A\tilde{x} \leq \tilde{b}$,

$\tilde{x} \geq 0$ and are integers

Where the coefficient matrix $A = (a_{ij})_{m\times n}$ is a nonnegative real crisp matrix.

The cost vector $c = (c_1,\ldots,c_n)$ is nonnegative crisp vector and $\tilde{x} = (\tilde{x}_j)_{n\times 1}$ and $\tilde{b} = (\tilde{b}_i)_{m\times 1}$ are nonnegative real fuzzy vectors such that $\tilde{x}_j, \tilde{b}_i \in F(R)$ for all $1 \leq j \leq n$ and $1 \leq i \leq m$. If some or all variables in $\tilde{x} = (\tilde{x}_j)_{n\times 1}$ are restricted to be integer then the problem is called fuzzy mixed integer linear programming problem.

4. THE ALGORITHM:

***Step 1*:** Consider the mixed integer linear programming problem with fuzzy variables

$\max\limits_{\tilde{x}_1, \tilde{x}_2} \tilde{z}$

subject to $\tilde{x}_1 \geq 0$ and integer, $\tilde{x}_2 \geq 0$

.

***Step 2*:** Let $\tilde{z} = (z_1, z_2, z_3), \tilde{x}_1 = (y_1, x_1, t_1)$ and $\tilde{x}_2 = (y_2, x_2, t_2)$.

***Step 3*:** Then the above problem is converted into three crisp mixed integer linear programming problem.

(P_2) $\max\limits_{x_1, x_2} z_2$

subject to $x_1 \geq 0$ and integer $x_2 \geq 0$.

***Step 4*: Introduce slack or surplus variables to convert the inequalities into equations and obtain the optimum solution of given problem by the dual simplex method

***Step 5*: Let (u_1, u_2) be its optimal solution.

***Step 6*: Test the integrality condition of the optimum.

i. If $u_1 \geq 0$ and are integer, $u_2 \geq 0$,the current solution is an optimal one then goto Step 16.

ii. If $u_1 \geq 0$ but not integer then go to the next step.

***Step 7*: Express the number in two parts as the sum of the integer and a non-negative fraction.

***Step 8*: Apply the tree of Branch and Bound method to get the integer solution.

***Step 9*: Let (U_1, U_2) be its optimal solution.

***Step 10*: Write the second mixed integer linear programming problem.

(P$_1$) $\max\limits_{y_1, y_2} z_1$

subject to $y_1 \leq U_1$

$y_2 \leq U_2$

$y_1 \geq 0$ and integer $y_2 \geq 0$.

***Step 11*: Go to step 4 to step8 and repeat the procedure until we get the optimum solution which satisfy the integrality condition.

***Step 12*: Let (V_1, V_2) be its optimal solution.

***Step 13*: Write the third mixed integer linear programming problem

(P$_1$) $\max\limits_{t_1, t_2} z_3$

subject to $t_1 \leq V_1$

$t_2 \leq V_2$

$t_1 \geq 0$ and integer $t_2 \geq 0$.

***Step 14*: Go to step 4 to step 8 and repeat the procedure until we get the optimum solution which satisfy the integrality condition.

***Step 15*: Let (W_1, W_2) be its optimal solution.

Step 16: We get the final required optimal solution is $\tilde{x}_1 = (y_1, x_1, t_1) = (U_1, V_1, W_1)$
$\tilde{x}_2 = (y_2, x_2, t_2) = (U_2, V_2, W_2)$, $\tilde{z} = (z_1, z_2, z_3)$

5. NUMERICAL EXAMPLE:

The proposed method is illustrated by the following example.

Consider the following integer linear programming problem with

$$\max.\tilde{z} = 5\tilde{x}_1 \oplus 6\tilde{x}_2$$

$$\text{subject to } 10\tilde{x}_1 \oplus 3\tilde{x}_2 = (48,52,48)$$

$$2\tilde{x}_1 \oplus 3\tilde{x}_2 = (12,18,12)$$

$$\tilde{x}_1 \geq 0 \text{ and integer}$$

$$\tilde{x}_2 \geq 0$$

Solution:

Let $\tilde{z} = (\tilde{z}_1, \tilde{z}_2, \tilde{z}_3)$, $\tilde{x}_1 = (y_1, x_1, t_1)$ and $\tilde{x}_2 = (y_2, x_2, t_2)$

Now, the problem (P_2) is given below

(P_2) $\max z_2 = 5x_1 + 6x_2$

subject to $10x_1 + 3x_2 \leq 52$

$$2x_1 + 3x_2 \leq 18$$

$x_1 \geq 0$ and integer

$$x_2 \geq 0$$

Now, using dual simplex method

Table-1

Basic variables		C_j	5	6	0	0	Min X_B / X_k
	C_B	X_B	x_1	x_2	x_3	x_4	
x_3	0	52	10	3	1	0	52/3
x_4	0	18	2	**3**	0	1	18/3
		Z_j - C_j	-5	-6	0	0	

Table-2

Basic variables		C_j	5	6	0	0	Min X_B / X_k
	C_B	X_B	x_1	x_2	x_3	x_4	
x_3	0	34	**8**	0	1	-1	34/8
x_2	6	6	2/3	1	0	1/3	18/2
		Z_j - C_j	-1	0	0	2	

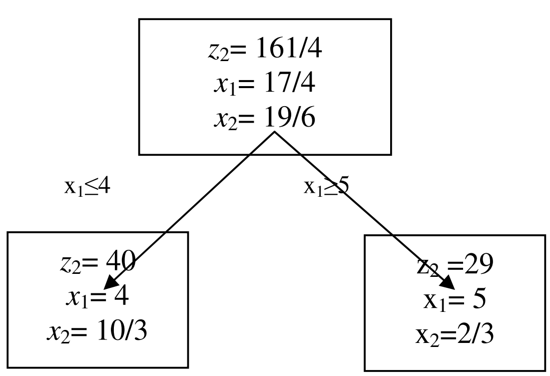
Table-3

Basic variables		C_j	5	6	0	0	Min X_B / X_k
	C_B	X_B	x_1	x_2	x_3	x_4	
x_1	5	17/4	1	0	1/8	-1/8	___
x_2	6	19/6	0	1	-1/12	5/12	___
		Z_j - C_j	0	0	1/4	15/8	

Since all Z_j - $C_j \geq 0$, thus Table-3 is an optimal one.

The solution of the problem (P_2) is x_1 = 17/4, x_2 = 19/6, max z_2 = 161/4

But x_1 is an integer value, so for make it integer we use branch and bound method.



Now, the problem (P_1) is given below:

(P_1)
$$\max z_1 = 5y_1 + 6y_2$$
$$\text{subject to } 10y_1 + 3y_2 \le 48$$
$$2y_1 + 3y_2 \le 12$$
$$y_1 \le 4 \text{ and integer}$$
$$y_2 \le 10/3$$

Similarly using dual simplex method, we obtain optimal solution of (P_1)

$y_1 = 9/2$, $y_2 = 1$ and max $z_1 = 57/2$.

For making y_1 to be an integer, we use branch and bound method and obtain the solution

$y_1 = 4$, $y_2 = 4/3$ and max $z_1 = 28$

Now, the problem (P_3) is given below:

(P_3)
$$\max z_3 = 5t_1 + 6t_2$$
$$\text{subject to } 10t_1 + 3t_2 \le 48$$
$$2t_1 + 3t_2 \le 12$$
$$t_1 \le 4 \text{ and integer } t_2 \le 4/3$$

Similarly using dual simplex method and branch and bound method, we can obtain the optimal solution of (P_3) is

$t_1 = 4$, $t_2 = 4/3$ and max $z_3 = 28$

Therefore, the solution for the given fuzzy mixed integer linear programming problem is

$$\tilde{x}_1 = (y_1, x_1, t_1) = (4,4,4)$$
$$\tilde{x}_2 = (y_2, x_2, t_2) = (4/3, 10/3, 4/3)$$
$$\tilde{z} = (z_1, z_2, z_3) = (28, 40, 28)$$

6. CONCLUSION:

The decomposition method provides an optimal solution to FMILP problems without using ranking functions and applying classical mixed integer linear programming. This method can serve managers by providing the best solution to a variety of mixed integer linear programming problems with fuzzy variables in a simple and effective manner.

REFERENCES:

[1] Allahviranloo, T., Shamsolkotabi, KH., Kiani, N, A. and Alizadeh, L., Fuzzy integer linear programming problems, Int. J. Contemp. Math. Sciences, 2 (2007), 167 – 181.

[2] Bernard Kolman, Robert E. Beck. Computer science and scientific computing, Elementary linear programming with application, (1980).

[3] Campos, L. and Verdegay, J. L., Linear programming problems and ranking of fuzzy numbers, Fuzzy Sets and Systems, 32, (1989), 1-11.

[4] Delgado, M., Verdegay, J.L. and Vila, M.A., A general model for fuzzy linear programming, Fuzzy Sets and Systems, 29, (1989), 21-29.

[5] Herrera, F. and Verdegay, J. L., Three models of fuzzy integer linear programming, European Journal of Operational Research, 83, (1995), 581-593.

[6] Lai, Y. J. and Hwang, C. L., Fuzzy mathematical programming methods and applications, Springer, Berlin, (1992).

[7] Mahdavi-Amiri, N. and Nasseri, S. H., Duality results and a dual simplex method for linear programming problems with trapezoidal fuzzy variables, Fuzzy Sets and Systems, 158, (2007), 1961-1978.

[8] Maleki, H.R., Tata, M. and Mashinchi, M., Linear programming with fuzzy variables, Fuzzy Sets and Systems, 109, (2000), 21-33.

[9] Nasseri, S. H., A new method for solving fuzzy linear programming by solving linear programming, Applied Mathematical Sciences, 2 (2008), 37-46.

[10] Tanaka, H., Okuda, T. and Asai, K., On fuzzy mathematical programming, The Journal of Cybernetics, 3 (1974), 37-46.

[11] Verdegay, J. L., A dual approach to solve the fuzzy linear programming problem, Fuzzy Sets and Systems, 14 (1984), 131-141.

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