THREE SUMMATION FORMULAE USING CONTIGUOUS RELATION AND RELATED TO HYPERGEOMETRIC FUNCTION

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ABSTRACT

The main aim of this paper is to evaluate two summation formulae involving Contiguous Relation associated with Recurrence relation and Hypergeometric function.

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A. INTRODUCTION:

Generalized Gaussian Hypergeometric function of one variable

\[ {}_A F_b(a_1,a_2,...,a_A;b_1,b_2,...,b_B;z) = \sum_{k=0}^{\infty} \frac{(a_1)_k(a_2)_k...(-a_A)_k}{(b_1)_k(b_2)_k...(-b_B)_k} \frac{z^k}{k!} \]  

(1)

where the parameters \( b_1 , b_2 , ....,b_B \) are neither zero nor negative integers and \( A , B \) are non negative integers.

Contiguous Relations

[Andrews p.363(9.16) , E.D. p.51(10), H.T.F.I. p.103(32)]

\( (a-b) \; _2 F_1(a, b ; c; z) = a \; _2 F_1(a+1,b; c; z) - b \; _2 F_1(a,b+1; c; z) \) 

(2)

[Abramowitz p.558 (15.2.19)]

\( (a-b) \; (1-z) \; _2 F_1(a, b ; c; z) = (c-b) \; _2 F_1(a,b-1; c; z) + (a-c) \; _2 F_1(a-1,b; c; z) \) 

(3)

A New Summation Formula

[Ref. [2] p.337 (10)]

\[ _2 F_1(a, b ; \frac{a+b-1}{2}, \frac{1}{2}) = 2^{b-1} \frac{\Gamma(\frac{a+b}{2})}{\Gamma(b)} \left( \frac{\Gamma(\frac{b}{2})}{\Gamma(b)} \right)^{1/2} + 2 \frac{\Gamma(\frac{b+1}{2})}{\Gamma(b)} \]  

(4)

B. MAIN SUMMATION FORMULAE:

For all the formulae \( a \neq b \)

For \( a<1 \) and \( a>8 \)

\[ _2 F_1(a, b ; \frac{a-b-1}{2}, \frac{1}{2}) = 2^{b-1} \frac{\Gamma(\frac{a+b}{2})}{\Gamma(b)} \left( \frac{\Gamma(\frac{b}{2})}{\Gamma(b)} \right)^{1/2} \frac{384a^6-60a^4+25a^2-1+10a^2b+25ab+34a^2b}{(a-7)(a-5)(a-3)(a-1)} + \]

\[ -\frac{192a^6+b+78a^4+b+48a^2+b^2+72a^3b+42a^2b^2+140ab^3+153ab^2+90a^2b^2+201ab^3+b^3}{(a-7)(a-5)(a-3)(a-1)} \]  

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C. DERIVATIONS OF THE SUMMATION FORMULAE:

**Derivation of (5):** Substituting \( c = \frac{a+b-8}{2} \) and \( z = \frac{1}{2} \) in equation (3), we get

\[
(a-b) \sum_{j=1}^{a+b-8} \binom{a+b-8}{j} \binom{b-1}{j} = (a-b-8) \sum_{j=1}^{a-9} \binom{a-9}{j} \binom{b-1}{j} + (a-b+8) \sum_{j=1}^{a-1} \binom{a-1}{j} \binom{b-1}{j}.
\]

Now with the help of the derived result from equation (4), we get

\[
L.H.S = (a-b-8) 2^{b-1} \sum_{j=1}^{a+b-8} \binom{a+b-8}{j} \binom{b}{j} - 384 = \frac{2^{b-1}}{b} \left\{ -96a + 16 \sum_{j=1}^{a-9} \binom{a-9}{j} \binom{b}{j} \right\} + \frac{1}{b} \left\{ -96a + 16 \sum_{j=1}^{a-1} \binom{a-1}{j} \binom{b}{j} \right\}.
\]
Three summation formulae using contiguous relation and related to hypergeometric function /IJMA- 2(4), Apr.-2011, Page: 518-520

On simplification , we get the formula (5)

Similarly, we can prove the other formulae.

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