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THE EFFECT OF THE RATIO γ ON MAGNETO-HYDRODYNAMIC VISCOELASTIC FLUID LAYER HEATED FROM BELOW

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ABSTRACT

The effect of the ratio γ (the strain retardation time to the sress relaxation time) on magneto-hydrodynamic viscoelastic fluid (Walters' lquid B') layer heated From below confined between tow horizontal planes has been studied. Linear stability theory and normal mode analysis are used to derive an differential equation of sixth order, and an exact solution for natural instability is obtained.Critical Rayleigh numbers and wave numbers for the onset of instability are discussed graphically as functions of γ, the Chandrasekhar number for various values of the Fourier number, and relaxation time at a constant value for Prandtl number .

Keywords: *Magnetohydrodynamic, Viscoelastic fluid, Temperature gradient, stability.*

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1. INTRODUCTION:

The phenomenal growth of energy requirements in recent years has been attracting considerable attention all over the word. This has resulted in a continuous exploration of new ideas and aveneus in harnessing various conventional energy sources, such as tidal waves, wind power, geo-thermal energy, etc. It is obvious that in order to utilize geo-thermal energy to a maximum, one should have a complete and precise knowledge of amount of perturbations needed to generate convection currents in geo-thermal fluid. Also, knowledge of the quantity of perturbations that are essential to initiate convection currents in mineral fluids found in the earth's crust helps one to utilize the minimal energy to extract the minerals. For example, in the recovery of hydrocarbons from underground petroleum deposits, the use of thermal process increasingly gaining importance as it enhances recovery. Heat is injected into the reservoir in the form of hot water or stream, or a burning part of the crude in the reservoir can generate heat. In all such thermal recovery processes, fluid flow takes place through a conducting medium and convection currents are deterimental.

To the author's knowledge, the first work, which deals directly with this problem, appears in a brief report by Green [17]. His analysis, which is restricted to the case when both bounding surfaces are free, was carried out in terms of a two-time-constant model due to Oldroyd [5,6]. The same problem was also attacked in some detail byVest and Arpaci [2] who employed a non-time-constant model due to Maxwell fluid [1,4]. This latter work has recently been extended by Takashima [9,10] to the case when the fluid layer is rotating about a vertical axis at a constant rate. All these investigations show that the presence of elasticity in a visco-elastic fluid distablized the fluid layer heated from below.

In technological fields there exists an important class of fluids, called non-Newtonian fluids,which are also being studied extensively because of their practical applications,such as fluid film lubrication, analysis of polymers in chemical engineering etc. One such fluid is called viscoelastic fluid and Walters [8] and Beard Walters [3] ded d the governing equation for the boundary flow for a prototype viscoelastic fluid which they have designated as liquid B', when this liquid had a very short memory.

The method of the matrix exponential, proposed by Bohar [9], and applied by Ezzat [9], which constitutes the basis of the state space approach oh modern control theory is applied to the non-dimensional equations of a visco-elastic fluid flow of hydromagnetic free convection flows. Ezzat and Abd-Elaal [11] studied the effects of the free convection currents with one relaxation time on the flow of a visco-elastic conduction fluid through a porous medium, which is bounded by a vertical plane surface. In these workes, more general model of magnetohydrodynamic free convection flow which also includes the relaxation time of heat convection and the electric permeability of the electomagnetic field are used. The inclusion of the relaxation time and electric permeability modify the governing thermal and electro-magnetic equations, changing them from parabolic to hyperbolic type, and there by eliminating the unrelastic result that thermal disturbance is realized instantaneously everywhere withen the fluid.Zaki [15]

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studied the stability of viscoelastic conducting liquid (Walters' liquid B') heated from below and presence of an electric field.Othman and Zaki [12] studied the effect of thermal relaxation time on a electrohydrodynamic viscoelastic fluid (Oldroyd liquid) layer heated from below.

The purpose of the present paper is to study the effect of the ratio γ on the onset of instability in a horizontal layer of viscoelastic conducting fluid (Walter's liquid B') heated from below with relaxation times in the presence of a magnetic field and elastic parameters.

2. MATHEMATICAL FORMULATION:

Let an infinite electrically conducting, viscoelastic fluid layer (Walters' liquid B') occupying the space between two horizontal rigid boundaries, which are at distance L apart. We choose the origin on the lower boundary, the Cartesian coordinate system x,y,z such that z is perpendicular to the boundaries and the fluid is permeated by a uniform external magnetic field $\mathbf{h} = (0,0,H_o)$ of intensity H_o aligned in the vertical direction. The lower surface at $z = 0$ and the upper surface at $z = L$ are maintained at constant temperatures T_o and T_1, respectively, and the fluid in the quiescent state is heated from below such that $\beta = \dfrac{T_o - T_1}{L}$ is the adverse temperature gradient.

The continuity equation

$$\frac{\partial v_i}{\partial x_i} = 0 \tag{1}$$

the momentum equation is the form [8]

$$\begin{aligned}(1+\lambda_1\frac{\partial}{\partial t})\left[\rho(\frac{\partial v_i}{\partial t} + v_k\frac{\partial v_i}{\partial x_k})\right] - \rho g_i - \frac{\partial P}{\partial x_i} + \frac{\mu_o h_j}{4\pi}\frac{\partial h_i}{\partial x_j} + K_o\Bigg[\frac{\partial}{\partial t}(\frac{\partial^2 v_i}{\partial x_k \partial x_k}) \\ + v_m(\frac{\partial^3 v_i}{\partial x_m \partial x_k \partial x_k}) - (\frac{\partial v_i}{\partial x_m})(\frac{\partial^2 v_m}{\partial x_k \partial x_k}) - 2(\frac{\partial v_m}{\partial x_k})(\frac{\partial^2 v_i}{\partial x_m \partial x_k})\Bigg] \\ = \eta_o(1+\lambda_2\frac{\partial}{\partial t})\frac{\partial^2 v_i}{\partial x_k \partial x_k}\end{aligned} \tag{2}$$

the energy equation [7]

$$\rho C_v\left[\frac{\partial T}{\partial t} + v_k\frac{\partial T}{\partial x_k}\right] = k_c\frac{\partial^2 T}{\partial x_k \partial x_k} - \tau\rho C_v\frac{\partial}{\partial t}\left[\frac{\partial T}{\partial t} + v_k\frac{\partial T}{\partial x_k}\right] \tag{3}$$

and

$$\frac{\partial h_i}{\partial x_i} = 0, \tag{4}$$

the equation of state is given by

$$\rho = \rho_o[1 - \alpha(T - T_o)]. \tag{5}$$

where ρ is the mass density, ρ_o is the reference density at the lower boundary, α is the coefficient of volume expansion, $v_i = (u, v, w)$ is the velocity of the fluid, P is the pressure, $g_i = (0, 0, -g)$ is the gravitational acceleration, η_o is the dynamic viscosity, K_o is the elastic constant of Walters' liquid B', λ_1 is the (stress) relaxation time, λ_2 is the (strain) retardation time, k_c is the thermal diffusivity, C_v is the specific heat at constant volume, T is the temperature of the fluid and τ is the relaxation time.

We first obtain the following steady solutions (denoted by an over bar)

$$\bar{u} = \bar{v} = \bar{w} = 0\,, \tag{6}$$

$$\bar{T} = T_o - \beta z\,, \tag{7}$$

$$\bar{\rho} = \rho_o (1 + \alpha\beta z) \tag{8}$$

$$\bar{h}_x = 0, \quad \bar{h}_y = 0, \quad \bar{h}_z = H_o\,. \tag{9}$$

Under Boussinesq approximation, the equations governing the disturbances can be written as (Chandrasekhar [16]):

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \tag{10}$$

$$(1+\lambda_1 \frac{\partial}{\partial t})\left[\frac{\partial}{\partial t}\nabla^2 w' - \alpha g \nabla_1^2 T' + \frac{K_o}{\rho_o}\frac{\partial}{\partial t}\nabla^4 w' - \frac{\mu_o H_o}{4\pi\rho_o}\frac{\partial}{\partial z}\nabla^2 h'\right] = \nu(1+\lambda_2)\nabla^4 w' \tag{11}$$

$$(1+\tau\frac{\partial}{\partial t})[\frac{\partial T'}{\partial t} - \beta w'] = k_c \nabla^2 T, \tag{12}$$

$$(\frac{\partial}{\partial t} - \eta\nabla^2)h' = H_o \frac{\partial w'}{\partial z}. \tag{13}$$

where ν is the kinematic viscosity. The dependent variables w', T' and h' represent respectively the z-component of the perturbation in the velocity, the temperature and the z-component of the perturbation in the magnetic flied. There $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $\nabla_1^{\,2} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ and t represents time.

Now,introducing the nondimensional varaibles by given $L, \frac{L^2}{k_c}, \frac{k_c}{L}, \beta L$ and $\frac{k_c H_o}{\eta}$ as the units of length, time, velocity, temperature and magnetic flied, respectively, we obtain the equation governing the disturbances as:

$$(1+F\frac{\partial}{\partial t})\left[P_r^{-1}\frac{\partial}{\partial t}\nabla^2 w' - R\nabla^2 T' + P_r^{-1}K_o\frac{\partial}{\partial t}\nabla^4 w' - Q\frac{\partial}{\partial z}\nabla^2 h'\right] = (1+\gamma F\frac{\partial}{\partial t})\nabla^4 w', \tag{14}$$

$$\left[\frac{\partial}{\partial t}(1+\tau\frac{\partial}{\partial t}) - \nabla^2\right]T' = (1+\tau\frac{\partial}{\partial t})w', \tag{15}$$

$$\left(P_m P_r^{-1}\frac{\partial}{\partial t}-\nabla^2\right)h'=\frac{\partial w'}{\partial z}. \tag{16}$$

There, $P_r=\dfrac{\nu}{k_c}$ is the Prandtl number, $P_m=\dfrac{\nu}{\eta}$ is the magnetic Prandtl number, $F=\dfrac{\lambda_1 k_c}{L^2}$ is the Fourier number, $\gamma=\dfrac{\lambda_2}{\lambda_1}(\lambda_2<\lambda_1)$ is the ratio of the (strain) retardation time to the (stress) relaxation time, $R=\dfrac{\alpha g\beta L^4}{\nu k_c}$ is the Rayleigh number , $Q=\dfrac{\mu_o^2 H_o^2 L^4}{4\pi\sigma\nu\eta}$ is the Chandrasskhar number, $\sigma=\mu_o\rho_o$, $k_c=\dfrac{k}{\rho_o C_v}$ and $K_o^*=\dfrac{K_o}{\rho_o L^2}$ is the elastic parameter of (Walters' lquid B') .

Following the normal mode analysis we assume that the solutions of Eqs. (14)-(16) are given by

$$\left[w',T',h'\right](x,y,z,t)=\left[W(z),\Theta(z),H(z)\right]\exp\left[ct+i(ax+by)\right] \tag{17}$$

where a, b are the (real) wave numbers in the x and y directions and c is the time constant (which is complex in general) . Thus we arrive

$$(1+cF)\left[P_r^{-1}c\left(D^2-\lambda^2\right)W(z)+R\lambda^2\Theta(z)+P_r^{-1}K_o^*c\left(D^2-\lambda^2\right)^2W(z)-Q\left(D^2-\lambda^2\right)DH(z)\right]$$
$$=(1+\gamma cF)\left(D^2-\lambda^2\right)^2W(z), \tag{18}$$

$$\left[c(1+c\tau)-\left(D^2-\lambda^2\right)\right]\Theta(z)=(1+c\tau)W(z)\ , \tag{19}$$

$$\left[P_mP_r^{-1}-\left(D^2-\lambda^2\right)\right]H(z)=DW(z). \tag{20}$$

there, $D=\dfrac{d}{dz}$, $\lambda=\sqrt{a^2+b^2}$ is the horizontal wave number and c is the stability parameter which in general, a complex constant.

In seeking solutions of these equations we must impose certain boundary conditions at the lower surface z = 0 and the upper surface z = 1. In this paper we shall restrict ourselves to the case when both boundary surfaces are stress-free, non- deformable and isothermal.

The boundary conditions for W, Θ and H are given by

$$W=D^2W=\Theta=H=0 \quad \text{at} \quad z=0,1 \tag{21}$$

Since $P_m P_r^{-1}$ is exceedingly small under most terrestrial, the frist term on the left-hand of Eq. (21) may be ignored. Consequently, we can eliminate H from Eqs. (19) and (21) without any differentiation; thus,

$$(1+cF)\left[P_r^{-1}c\left(D^2-\lambda^2\right)W(z)+R\lambda^2\Theta(z)+P_r^{-1}K_o^*c\left(D^2-\lambda^2\right)^2W(z)+QD^2W(z)\right]$$
$$=(1+\gamma cF)\left(D^2-\lambda^2\right)^2W(z). \tag{22}$$

This means that under the above application the solution the underlying problem can be carried out independently of the bondary conditions on the magnetic flied.

Therefore, the boundary conditions for W and Θ are given by

$$W = D^2 W = \Theta = 0 \quad \text{at} \quad z = 0, 1 \tag{23}$$

Equations (19) and (22) subject to the boundary conditions (23) constitute an eigenvalue system of sixth order.

1. Solution:

The eigenvalue system defined by Eqs.(19), (22) and (23) can readily combined

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$$(1+cF)\left[c(1+c\tau)-\left(D^2-\lambda^2\right)\right]\left[P_r^{-1}c\left(D^2-\lambda^2\right)W(z)+P_r^{-1}K_o^* c\left(D^2-\lambda^2\right)^2 W(z)+Q\,D^2 W\right]$$
$$+\lambda^2 R(1+c\tau)(1+cF)W(z)$$
$$=(1+\gamma cF)\left[c(1+c\tau)-\left(D^2-\lambda^2\right)\right]\left(D^2-\lambda^2\right)^2 W(z). \tag{24}$$

together with

$$W = D^{2m} W = 0 \quad \text{at} \quad z = 0, 1 \qquad (m = 1, 2, 3, \ldots) \tag{25}$$

Examination of (25) and (26) indicates that relevant solution for W (chracterizing the lowest mode) is given by

$$W = W_o \sin(\pi z) \tag{26}$$

$$(1+cF)\left[c(1+c\tau)+B\right]\left[-P_r^{-1}cB+P_r^{-1}K_o^* cB^2-Q\pi^2\right]+\lambda^2 R(1+c\tau)(1+cF)$$
$$=(1+\gamma Fc)\left[c(1+c\tau)+B\right]B^2. \tag{27}$$

then, we can rewrite Eq. (27) in the form

$$R = \left[c(1+c\tau)+B\right]\left[\pi^2 Q+BcP_r^{-1}(1-K_o^* B)\right]\left[\lambda^2(1+c\tau)\right]^{-1}$$
$$+B^2(1+\gamma Fc)\left[c(1+c\tau)+B\right]\left[\lambda^2(1+c\tau)(1+cF)\right]^{-1} \tag{28}$$

where W_o is a constant , it must be remember that c can be complex and

$$B = \pi^2 + \lambda^2 \tag{29}$$

1. OVERSTABILITY MOTIONS AND CONCLUSIONS:

Let us now separate the right-hand side of Eq. (28) into the real and imaginary parts after setting $c = i\omega$ with ω being real. Then, we have

$$R = R_x + i\omega R_y \tag{30}$$

There, R_x and R_y are real-valued functions of $P_r, Q, K_o^*, \tau, \lambda$ and ω, and explicit expansions for these functions are follows:

$$R_x = B\left[\pi^2 P_r Q-\omega^2(1-K_o^* B)(1-\tau B)+\omega^4(1-K_o^* B)\tau^2\right]\left[\lambda^2 P_r(1+\omega^2\tau^2)\right]^{-1} (31)$$
$$+B^2\left[B+\omega^2(1-\gamma)(1+\omega^2\tau^2)F+\omega^2 F(\gamma\tau-\tau+\gamma F)B\right]\left[\lambda^2(1+\omega^2\tau^2)(1+\omega^2F^2)\right]^{-1} \tag{31}$$

$$R_y = \left[\pi^2 P_r Q(1-\tau B+\omega^2\tau^2)+B^2(1-K_o^* B)\right]\left[\lambda^2 P_r(1+\omega^2\tau^2)\right]^{-1}$$
$$+B^2\left[1-\tau B+\omega^2\tau^2(1+\omega^2\gamma F)+BF(\gamma-1)\right]\left[\lambda^2(1+\omega^2\tau^2)(1+\omega^2F^2)\right]^{-1} \tag{32}$$

It is apparent from Eq. (29) that for arbitrary assigned values of P_r, τ, λ, K_o^*, Q, F and ω, R will be complex, but the physical meaning of R requires it to be real.

Consequently, from the condition that R must be real, so we have either

$$R = R_x \qquad \text{and} \qquad \omega = 0 \tag{33}$$

or

$$R = R_x \qquad \text{and} \qquad R_y = 0 \tag{34}$$

From Eq. (34) we obtain the eigenvalue equation for a neutral stationary instability,

$$R = \frac{B^2}{\lambda^2}\left[\pi^2 Q + B\right]. \tag{35}$$

For Newtonian viscous fluid, when the magnetic flied is absent i.e. $Q = 0$, Eq. (36) reduces to

$$R = \frac{B^3}{\lambda^2} \;. \tag{36}$$

which agrees with the classical result (Chandrasekhar [16]) Equation (28) will give the critical Rayleigh number R_c for the onset of stationary instability.

On the other hand Eq. (35) leads,

$$\begin{aligned} R = {} & B\left[\pi^2 P_r Q - \omega^2 (1-K_o^* B)(1-\tau B) + \omega^4 (1-K_o^* B)\tau^2\right]\left[\lambda^2 P_r (1+\omega^2\tau^2)\right]^{-1} \\ & + B^2\left[B + \omega^2 (1-\gamma)(1+\omega^2\tau^2)F + \omega^2 F(\gamma(-\tau+\gamma F)B\right]\left[\lambda^2 (1+\omega^2\tau^2)(1+\omega^2 F^2)\right]^{-1} \end{aligned} \tag{37}$$

and

$$C_o\delta^2 + C_1\delta + C_2 = 0. \tag{38}$$

where

$$\delta = \omega^2, \tag{39}$$

$$C_o = \gamma\tau^2 F P_r B^2 + \pi^2\tau^2 F^2 P_r Q, \tag{40}$$

$$C_1 = -K_o^* F^2 B^3 + (\tau^2 P_r + F^2)B^2 + \pi^2\tau F^2 P_r Q B + \pi^2 P_r (\tau^2 + F^2) Q, \tag{41}$$

$$C_2 = \left[F P_r (\gamma - 1) - \tau P_r - K_o^*\right]B^3 + (1 + P_r)B^2 - \pi^2\tau P_r Q B + \pi^2 P_r Q, \tag{42}$$

For assigned values of P_r, τ, λ, F, K_o^* and Q Eqs. (38) and (39) define R as a function of λ, the minimum of this function determines the critical Rayleigh number R_c for the onset of oscillatory convection (i.e. overstability) should be compared with that the onset of stationary convection (i.e. ordinary instability). The type of instability, which takes place in practice, will be that corresponding to the lower value of the critical Rayleigh number.

In order to determine the conditions under which instability sets in overstability P_r, , τ, λ, K_o^*, F and Q were assigned fixed values and the value of δ was evaluated numerically from Eq. (38).Using this value of δ, the value of

R was evaluated numerically from Eq. (37). If the value δ was negative, the neutral state was considered to be stationary. The procedure was then repeated for various values of λ in order to locate the minimum of R. .

We have plotted the variation of the Rayleigh number R with the wave numbere λ using Eq. (37) satisfying (38) for the onset of overstable and stationary case for values of the dimensionless parameters $P_r = 100, K_o^* = 0.2, \tau = 0.02, 0.1, \gamma = 0.4$ and $F = 0.1, 0.5$ of the Fourier number (elastic parameter).

Figures 1 and 2 correspond to two values $Q = 100, 1000$ respectively, of the Chandrasekhar number. Figures 1 and 2 show that the Rayleigh number R increases with an increase in the magnetic field and decreases as the relaxation time τ and elastic parameter F increases i.e the onset instability is delayed as Q increases while it is hastened as τ and F increases.

The critical Rayleigh number R_c and the critical wave number λ_c obtained in that manner for both stationary instability and oscillatory instability (over-stability) is shown in Figes 3 and 4 respectively, as functions of Q for values of dimensionless parameters $P_r = 100,\ K_o^* = 0.2, \tau = 0.02,\ 0.1,\ \gamma = 0.4$ and $F = 0.1, 0.5$. It is seen from Fig. 3 that the critical Rayleigh number R_c decreases as the relaxation time τ and Foureir number F increases. From Fig. 4 we notice that the critical wave number λ_c increases with an increase in the elastic parameter F , the relaxation time τ and magnetic flied , also we see that the value of λ_c for an oscillatory instability is greater than that of a stationary instability. Figures 5 and 6 correspond to two values $Q = 100, 1000,$ respectively of the magnetic flied, for values of the dimensionless parameters $P_r = 100,\ K_o^* = 0.2, \tau = 0.1, F = 0.1$ and $\gamma = 0.04,\ 0.4$. Figures 1 and 2 show that the Rayleigh number R increases with an increase in the ratio of of the (strain) retradation to the the (stress) relaxation times γ i.e the onset instability is delayed as γ increases.

Natural convection instability of a visco-elastic fluid (Walter's lquid B') heated from below in the presence of magnetic field has been analyzed numerically. The study focused on the effect of a magnetic field , relaxation time, the ratio of of the (strain) retradation to the the (stress) relaxation times γ , and Foureir number F (elastic parameter) on the convection phenomenon. From the above analysis, we conclude that the elastic parameter, , the ratio of of the (strain) retradation to the the (stress) relaxation times γ , the relaxation time and the presence of magnetic field have a profound influence on the threshold of instability.

REFERENCES:

[1] A. G. Fredrickson, Principles and Application of Rheology, Prentice- Hell, Inc, 1969.

[2] C. M. Vest and V. S. Arpaci**,** Overstability of viscoelastic fluid layer heated from below, J. Fluid Mech. 36 (2006) 613-623.

[3] D. W. Beard and K. Walters, Elasto-viscous boundary-layer flows. I. Two- dimensional flow near *a* stagnation point. Proc.Camb.Phil. Soc. vol. 60 (1969), 667- 671.

[4] J.C Maxwell, Phil. Trans. Roy. Soc. vol 157 (1867) 49.

[5] J. G. Oldroyd**,** On formation of rheological equation of state**,** Proc.Roy. Soc. A200(1950) 523- 541 .

[6] J. G Oldroyd, Non Newtonian effects in steady motion of some idealized elastic- viscous liquids Proc. Roy. Soc. A245 (1958) 278-298.

[7] K. A. Helmyt, Can. J. Phy. 77,(1997), 279.

[8] K. Walters, Secod-order effects in elasticity, Plasticity and Fluid Dynamics, p.507 Pergamon, Oxford, 1969.

[9] L. Y. Bahar, J. Acount.Soc. Am. 75(1975),606.

[10] M. A. Ezzat, Can. J. Phys. **72**, 311 (1994).

[11] M. A. Ezzat and Abd-Elaal J. Franklin Inst. 334B, 685 (1997)

[12] M. I. Othman, and S. A. Zaki**,** The effect of thermal relaxation time on electrodynamic viscoelastic fluid with one relaxation time, Can. J. Phys.vol. 81, (2003)779-787.

[13] M. Takashima, Thermal instability in viscoelastic fluid layer, J. Phys Soc. No. 3(1972), 511-518**.**

[14] M. Takashima, J. Phys. Soc. Japan. vol 47 (1979), 1717.

[15] S. A. Zaki, Instability of viscoelastic fluid layer heated from below with relaxation time and presence an electric field, Appl. Math. Sci.Vol.*5*(2011),no.32,1567- 1587.

[16] S. Chandrasekhar**,** Hydrodynamic and Hydromagnetic Stability, (Oxford Univ. London, 1961).

[17] T. Green, Phys. of Fluids vol. 11(1968), 1410.

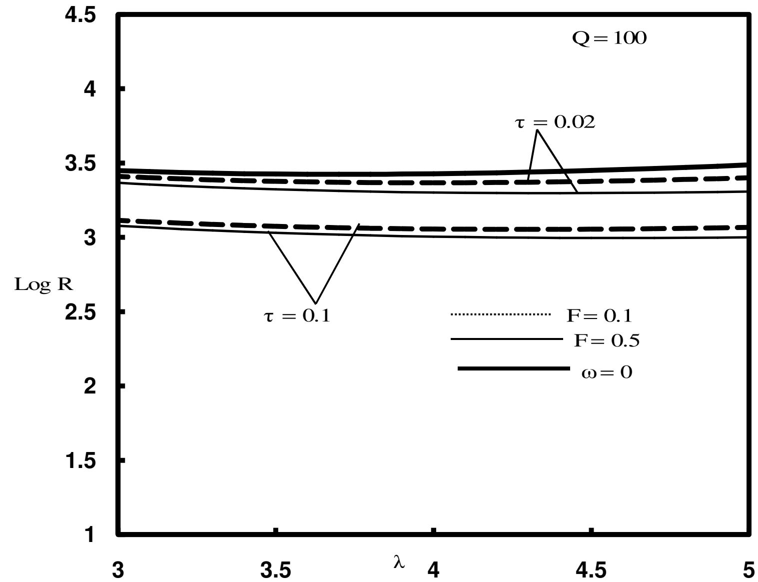

Fig. 1. Variation of R with λ for various values of F and τ at K_o^* =0.2, $P_r = 100$, $\gamma = 0.3$ and $Q = 100$. $\omega = 0$ represents the onset stationary convection.

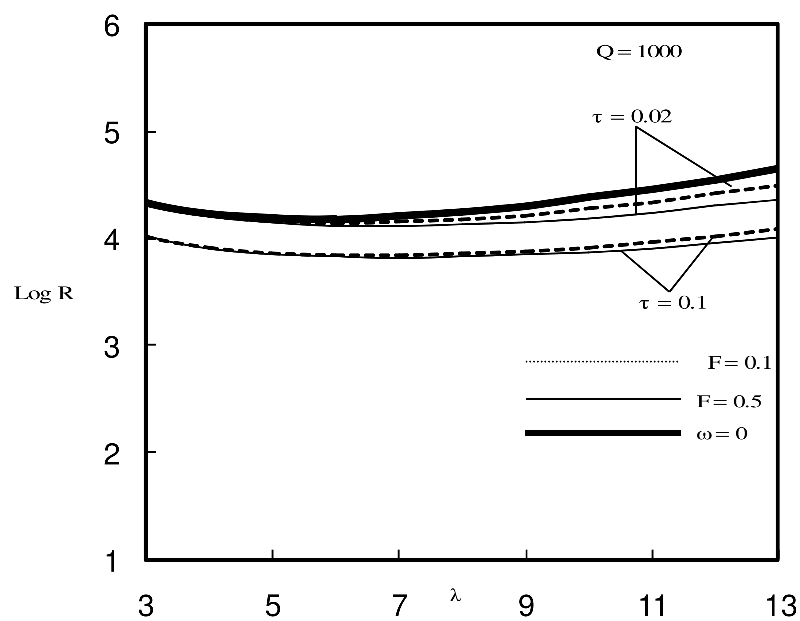

Fig. 2. Variation of R with λ for various values of F and τ at $K_o^* = 0.2$, $P_r = 100$, $\gamma = 0.3$ and $Q = 1000$. $\omega = 0$ represents the onset stationary convection.

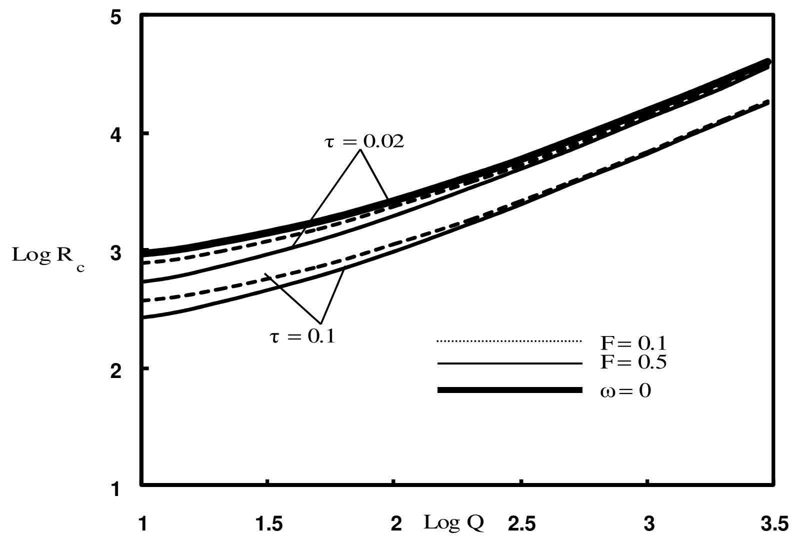

Fig. 3. critical Rayleigh number R_C as a function of Q for various values of F at $P_r = 100$, $K_o^* = 0.2$, $\gamma = 0.3$, and $\tau = 0.02, 0.0.1$. $\omega = 0$ represents the onset stationary convection.

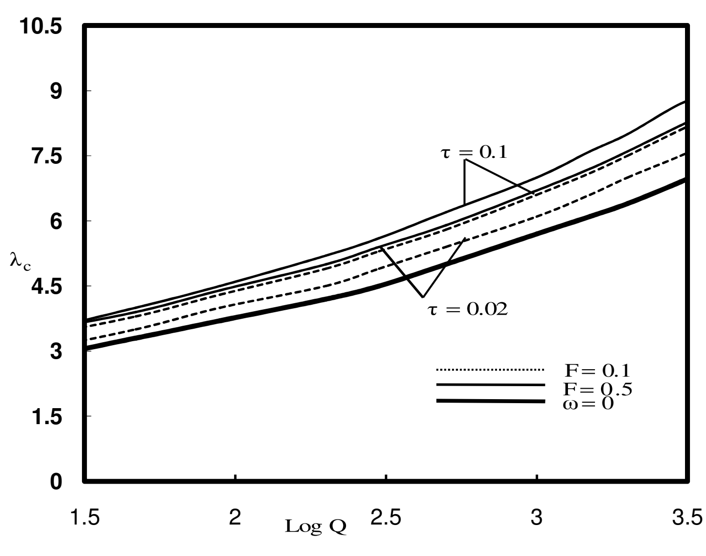

Fig. 4. critical wave number λ_C as a function of Q for various values of F at $P_r = 100$, $K_o^* = 0.2$, $\gamma = 0.3$ and $\tau = 0.02, 0.0.1$. $\omega = 0$ represents the onset stationary convection.

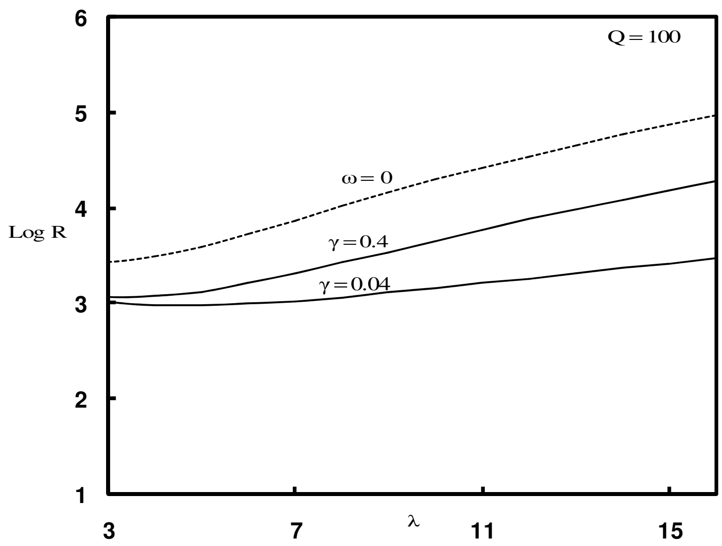

Fig. 5. Variation of R with λ for various values of γ at $\tau = 0.1$, $F = 0.1$, $K_o^* = 0.2$, $P_r = 100$ and $Q = 100$. $\omega = 0$ represents the onset stationary convection.

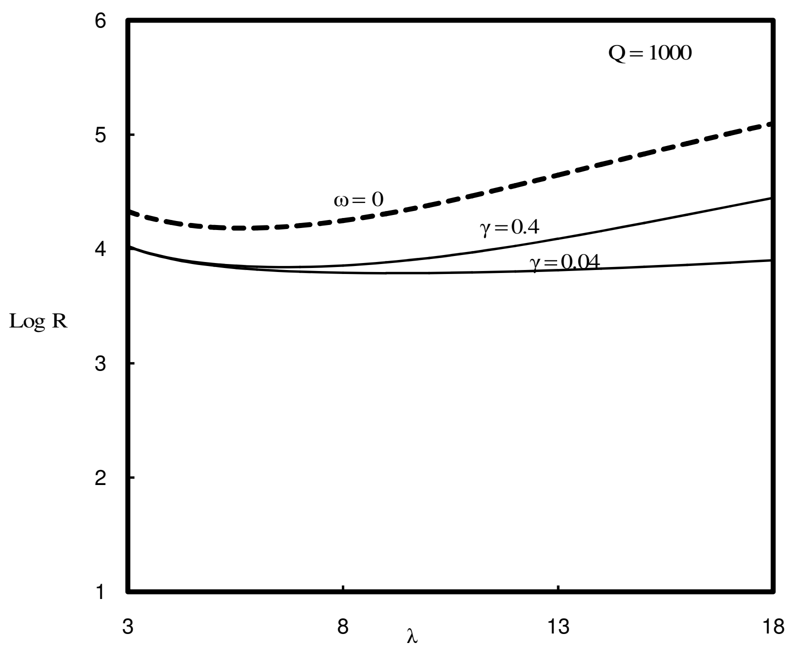

Fig. 6. Variation of R with λ for various values of γ at $\tau = 0.1$, $F = 0.1$, $K_o^* = 0.2$, $P_r = 100$ and $Q = 1000$. $\omega = 0$ represents the onset stationary convection.