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MAGNETOHYDRODYNAMIC FLOW OF CONDUCTING NON-NEWTONIAN [OLDROYD (1958) MODEL] FLUID WITH TRANSIENT PRESSURE GRADIENT THROUGH POROUS MEADIUM IN A RECTANGULAR CHANNEL

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ABSTRACT

The aim of the present paper is to study the unsteady flow of conducting non-Newtonian$[Oldroyed(1958)model]$ fluid of second order with transient pressure gradient through porous medium in a long rectangular channel under the influence of an uniform of magnetic field applied perpendicularly to the flow of fluid. The expression for velocity of fluid is obtained in elegant form. The various deductions have also been discussed in detail.

INTRODUCTION

The study of physics of flow of fluid through porous medium has become the basis of many scientific and engineering applications. This type of flow is of great importance in the petroleum engineering concerned with the movement of oil, gas and water through the reservoir of an oil or gas field to the hydrologist in the study of the migration of underground water and to the chemical engineer in the filtration process.

Many research workers have paid their attention towards the application of non-Newtonian fluid flow through porous medium in various types of channel with or without application of uniform transverse magnetic field, such as Kumar and Singh (1989), Gupta and Sharma (1990), Kapur, Bhatt and Sachetti (1992), Singh , Shankar and Singh (1995), Gupta and Gupta (1996), Singh and Kumar (1998), Hayat, Asghar and Siddiqui (2000), Sharma and Pareek (2001), Kundu and Sengupta (2001), Hassianien (2002), Sengupta and Basak (2002), Pundhir and Pundhir (2003), Rehman and Alam Sarkar (2004), Agarwal and Agarwal (2006), Sharma and Pareek (2006), Singh Kumar and Sharma (2008), Kumar Sharma and Singh (2008), Kumar and Singh (2009), Kumar, Mishra, and Singh (2011), and Kumar, Mishra and Sharma (2013) etc.

In the present paper, the unsteady laminar flow of conducting non-Newtonian $[Oldroyed(1958)model]$ fluid of second order with transient pressure gradient through porous medium in a long rectangular channel is studied when an uniform magnetic field be applied perpendicularly to the flow of fluid. The various deductions have also been discussed in detail.

BASIC THEORY AND EQUATIONS OF MOTIONS

For slow motion, the Rheological equations for second order non-Newtonian $[Oldroyed(1958)model]$ fluid are:

$$\tau_{ij} = -p\delta_{ij} + \tau_{ij}^{'} \tag{1}$$

$$\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\tau_{ij}^{'} = 2\mu\left(1+\mu_1\frac{\partial}{\partial t}+\mu_2\frac{\partial^2}{\partial t^2}\right)e_{ij} \tag{2}$$

$$e_{ij} = \frac{1}{2}\left(v_{i,j}+v_{j,i}\right) \tag{3}$$

where τ_{ij} is the stress tensor, $\tau_{ij}^{'}$ the deviatoric stress tensor, e_{ij} the rate of strain tensor, p the pressure, λ_1 the stress relaxation time parameter, μ_1 the strain rate retardation time parameter, λ_2 the additional material constant, μ_2 the additional material constant, δ_{ij} the metric tensor, μ the coefficient of viscosity and v_i is the velocity components.

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FORMULATION OF THE PROBLEM

Let us consider the walls of rectangular channel to be the $x = \pm a$ and $y = \pm b$. Z-axis is taken towards the direction of the flow of fluid. Again let 0, 0, w(x, y, t) be the velocity component along x, y, z-directions respectively at any point in the channel. A transient pressure gradient -$Pe^{-\omega t}$ varying with time is applied to the non-Newtonian fluid in the direction of flow.

Following the stress-strain relations (1)-(3), the equation for unsteady motion through porous medium under the influence of transverse uniform magnetic field is given by

$$\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial w}{\partial t}=-\frac{1}{\rho}(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2})\frac{\partial p}{\partial z}+\upsilon(1+\mu_1\frac{\partial}{\partial t}+\mu_2\frac{\partial^2}{\partial t^2})\nabla^2 \mathrm{w} - \left(\frac{\sigma B_0^2}{\rho}+\frac{\upsilon}{K}\right)\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)w \tag{4}$$

where K is coefficient of permeability, σ the conductivity of fluid and B_0 is the electromagnetic induction.

For the present problem, the boundary conditions are:

$$\left.\begin{aligned} w&=0, \quad when\ x=\pm a, \quad -b\le y\le b\\ w&=0, \quad when\ y=\pm b, \quad -a\le x\le a\end{aligned}\right\} \tag{5}$$

Introducing the following non-dimensional quantities:

$$w^*=\frac{a}{\nu}w,\quad (x^*,y^*,z^*)=\frac{1}{a}(x,y,z),\ \ p^*=\frac{a^2}{\rho\nu^2}p,\ \ t^*=\frac{\nu}{a^2}t,$$

$$\lambda_1^*=\frac{\nu}{a^2}\lambda_1,\ \ \mu_1^*=\frac{\nu}{a^2}\mu_1,\ \ \lambda_2^*=\frac{\nu^2}{a^4}\lambda_2,\ \ \mu_2^*=\frac{\nu^2}{a^4}\mu_2,\ \ K^*=\frac{1}{a^2}K$$

In equation (4), we get (after dropping the stars)

$$(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2})\frac{\partial w}{\partial t}=-(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2})\frac{\partial p}{\partial z}+(1+\mu_1\frac{\partial}{\partial t}+\mu_2\frac{\partial^2}{\partial t^2})\nabla^2\mathrm{w} - \left(H+\frac{1}{K}\right)(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2})w \tag{6}$$

where

$$H=aB_0\sqrt{\frac{\sigma}{\mu}}\quad (Hartmann\ number)$$

The boundary conditions (5) reduces to

$$w=0,\qquad when\ x=\pm 1,\qquad -c\le y\le c \tag{7}$$

$$w=0,\qquad when\ y=\pm c,\qquad -1\le x\le 1 \tag{8}$$

$$where\qquad c=\frac{b}{a}$$

SOLUTION OF THE PROBLEM

Now we have to consider those types of situations of flow of fluid which is transient in nature with respect to time and periodic in nature with respect to y. Subject to the nature of the boundary conditions (7) and (8) we choose the solution of (6) as

$$w=W(x)\cos my.\,e^{-\omega t} \tag{9}$$

The boundary conditions (7) and (8) corresponding to (9) are

$$W=0,\qquad when\ x=\pm 1,\qquad -c\le y\le c \tag{10}$$

$$W=0,\qquad when\ y=\pm c,\qquad -1\le x\le 1 \tag{11}$$

The boundary condition (11) will be satisfied if

$$\cos mc=0$$

$$or \quad mc = (2n+1)\frac{\pi}{2}$$

$$or \quad m = (2n+1)\frac{\pi}{2c};\ n = 0,1,2,3 \ldots\ldots\ldots\ldots.. \tag{12}$$

We may construct the solution as the sum of all possible solutions for each value of n of the form

$$w = \sum_{n=0}^{\infty} W(x)\cos my . e^{-\omega t} \tag{13}$$

$$After\ putting\ the\ value\ of\ \frac{\partial p}{\partial z} = -Pe^{-\omega t}\ , \omega > 0\ in\ (6)\ we\ get$$

$$\sum_{n=0}^{\infty}\frac{d^2W(x)}{dx^2}\cos my - \sum_{n=0}^{\infty} m^2 W(x)\cos my + \frac{P(1-\lambda_1\omega+\lambda_2\omega^2)}{(1-\mu_1\omega+\mu_2\omega^2)} + \sum_{n=0}^{\infty}\frac{(1-\lambda_1\omega+\lambda_2\omega^2)\left\{\omega - H - \frac{1}{K}\right\}}{(1-\mu_1\omega+\mu_2\omega^2)} W(x)\cos my = 0$$

Or

$$\sum_{n=0}^{\infty}\frac{d^2W(x)}{dx^2}\cos my + \frac{4P}{\pi}\frac{(1-\lambda_1\omega+\lambda_2\omega^2)}{(1-\mu_1\omega+\mu_2\omega^2)}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)}\cos my - \sum_{n=0}^{\infty}\left[m^2 - \frac{(1-\lambda_1\omega+\lambda_2\omega^2)\left\{\omega - H - \frac{1}{K}\right\}}{(1-\mu_1\omega+\mu_2\omega^2)}\right] W(x)\cos my = 0 \tag{14}$$

$$\because \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)}\cos my = \frac{\pi}{4}(by\ Fourier\ cosine\ series) in\ interval - c \le y \le c$$

Equating the coefficient of $\cos my$ *equal to zero, the value of W(x) can be determined from*

$$\frac{d^2W(x)}{dx^2} - \frac{Q^2}{a^2}W(x) + A_n = 0 \tag{15}$$

where

$$Q^2 = a^2\left\{m^2 - \frac{(1-\lambda_1\omega+\lambda_2\omega^2)\left\{\omega - H - \frac{1}{K}\right\}}{(1-\mu_1\omega+\mu_2\omega^2)}\right\},$$

$$and \quad A_n = \frac{4P(-1)^n(1-\lambda_1\omega+\lambda_2\omega^2)}{(2n+1)\pi(1-\mu_1\omega+\mu_2\omega^2)}$$

The general solution of (15) is

$$W(x) = C_1 cosh\frac{Q}{a}x + C_2 sinh\frac{Q}{a}x + \frac{a^2A_n}{Q^2} \tag{16}$$

Using boundary conditions (10), we get

$$W(x) = \frac{(-1)^n 4P(1-\lambda_1\omega+\lambda_2\omega^2)a^2}{(2n+1)\pi(1-\mu_1\omega+\mu_2\omega^2)Q^2}\left\{1 - \frac{\cosh\frac{Q}{a}x}{\cosh\frac{Q}{a}}\right\} \tag{17}$$

Hence the velocity of Oldroyd fluid is given by

$$W(x,y,t) = \sum_{n=0}^{\infty}\left[\frac{(-1)^n 4P(1-\lambda_1\omega+\lambda_2\omega^2)a^2}{(2n+1)\pi(1-\mu_1\omega+\mu_2\omega^2)Q^2}\left\{1 - \frac{\cosh\frac{Q}{a}x}{\cosh\frac{Q}{a}}\right\}\right] .\cos(2n+1)\frac{\pi}{2c}y.e^{-\omega t} \tag{18}$$

DEDUCTIONS

I. *If magnetic field is withdrawn i.e. H=0 in equation (18), we obtain all expressions of Kumar, Mishra and Sharma (2013) with slight change of notations.*

II. *If porous medium is withdrawn i.e. K→ ∞ in equation (18), we obtain all expressions of Kumar and Singh (2009) with slight change of notations.*

III. *If we put* $\lambda_2 = 0$, $\mu_2 = 0$ *in the equations (18), we obtain the velocity expression for first order Oldroyd visco-elastic fluid through porous medium under the influence of uniform magnetic field.*

IV. *If we put* $\lambda_2 = 0$, $\mu_2 = 0$ *and magnetic field is withdrawn i.e. H=0 in equation (18), we obtain all expressions for first order Oldroyd fluid of Singh, Kumar and Sharma (2008).*

V. *If we put* $\lambda_2 = 0$ *and* $\mu_1 = 0$, $\mu_2 = 0$ *in the equation (18), we get velocity expression for Maxwell visco-elastic fluid through porous medium under the influence of uniform magnetic field.*

VI. *If we take* $\lambda_1 = 0$, $\lambda_2 = 0$, $\mu_1 = 0$, $\mu_2 = 0$ *in the equations (18), we get velocity expression for purely viscous fluid through porous medium under the influence of uniform magnetic field.*

VII. *If porous medium and magnetic field are withdrawn i.e. K→ ∞ and H=0 in equation (18), we obtain all expressions of Kundu and Sengupta (2001).*

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