International Journal of Mathematical Archive-2(4), Apr. - 2011, Page: 579-583
Available online through www.ijma.info ISSN 2229 – 5046

MAGNETOHYDRODYNAMIC (MHD) FLOW OF VISCO-ELASTIC [OLDROYD (1958) MODEL] LIQUID OF FIRST ORDER BETWEEN TWO PARALLEL FLAT PLATES

[1]Ravindra Kumar*, [1]K. K. Singh and [2]A. K. Sharma

[1]Department of Mathematics, Agra College Agra (U.P.), India
[2]Department of Mathematics, K.K.P.G.College Etawah (U.P.), India

Email ID: rkpal.maths@gmail.com ; kksingh.maths@gmail.com
aksharma.maths@gmail.com

(Received on: 28-03-11; Accepted on: 09-04-11)

ABSTRACT

Present paper is concerned with the oscillatory motion of conducting visco-elastic [Oldroyd (1958) model] liquid of first order between two infinite flat plates both of which execute simple harmonic motion in the plane of plates with different amplitudes and frequencies under the influence of an uniform magnetic field applied perpendicular to the flat plates. Some particular cases have also been discussed in detail.

INTRODUCTION:

Visco-elastic fluids are particular cases of non-Newtonian fluids which exhibit appreciable elastic behaviour and stress-strain velocity relations and are time dependent. The subject of Rheology is of great technological importance in many branches of industry, the problem arises of designing apparatus to transport or to process substances which can not be governed the classical stress-strain velocity relations. Examples of such substances and the process are many, in the extrussion of plastics, in the manufacture of rayon, nylon or other textiles, fibres, visco-elastic effects are encountered when the spinning solutions are transported or forced through spinnerts and in the manufacture of lubricating greases and rubbers.

In hydrodynamic flow we study the flow of electrically conducting fluid in the presence of Maxwell electromagnetic field. The flow of conducting fluid is effectively changed by the presence of magnetic field and the magnetic field also perturbed due to the motion of the conducting fluid. This phenomena is therefore interlocking in character and the discipline of this branch of science is called Magnetohydrodynamics (MHD). It is equally rich and admits wider application in Engineering technology, Cosmology, Astrophysics and other applied sciences. Choubey (1985) studied the hydromagnetic flow of an electrically conducting Rivlin-Ericksen liquid near an infinite horizontal flat plate. Yadav and Singh (1990) discussed the impulsive motion of a porous flat plate in an elastico-viscous Rivlin-Ericksen liquid. Recently Suvarna and Venkataramana (2002); Kundu and Sengupta (2003); Sengupta and Paul (2004); Krishna and Rao (2005) and Ghosh and Ghosh (2005); Kumar, Singh and Sharma (2009); Tripathi, Sharma and Singh(2009) etc. have studied the flow of various visco-elastic fluid through channels of different cross-sections.

The aim of the present paper is to study the oscillatory motion of visco-elastic Oldroyd liquid of first order under the influence of an uniform magnetic field between two infinite parallel flat plates. Both the plates are assumed to be oscillating harmonically with different amplitudes and frequencies. The uniform magnetic field is applied perpendicularly to the flat plates. Some particular cases have also been discussed in detail.

GOVERNING EQUATIONS OF MOTION:

The rheological equations for visco-elastic [Oldroyd (1958) model] liquid are:

$$P_{ik} = -p\delta_{ik} + P_{ik}^{'}$$

***Corresponding author: Ravindra Kumar*, *E-mail: rkpal.maths@gmail.com**
Department of Mathematics, Agra College Agra (U.P.), India

$$p'_{ik} + \lambda_1 \frac{D}{Dt} p'_{ik} + \mu_0 p'_{jj} e_{ik} - \mu_1 \left(p'_{ij} e_{jk} + p'_{jk} e_{ij} \right) - \upsilon_1 p'_{jl} e_{jl} \delta_{ik}$$
$$= 2\eta_0 \left(e_{ik} + \lambda_2 \frac{D}{Dt} e_{ik} - 2\mu_2 e_{ij} e_{jk} + \upsilon_2 e_{jl} \delta_{ik} \right)$$

with the equation of incompressibility

$$e_{ii} = 0$$

where

$$\frac{D}{Dt} b_{ik} = \frac{\partial}{\partial t} b_{ik} + \upsilon_{ij} b_{ik \cdot j} + w_{ij} b_{jk} + w_{kj} b_{ij}$$
$$e_{ij} = \frac{1}{2}\left(v_{k \cdot i} + v_{i \cdot k} \right)$$
$$w_{ik} = \frac{1}{2}\left(v_{k \cdot i} - v_{i \cdot k} \right)$$

e_{ik} = rate of strain tensor

p_{ik} = stress tensor

λ_1 = relaxation time

λ_2 = retardation time

η_0 = coefficient of viscosity

and $\mu_0, \mu_1, \mu_2, \upsilon_1$ *and* υ_2 are the material constants, each being of the dimension of time.

For $\eta_0 > 0,\ \lambda_1 = \mu_1 = \mu_2 = \lambda_2 = 0,\ \ \mu_0 = \upsilon_1 = \upsilon_2 = 0$ the liquid will behave as ordinary viscous liquid.

FORMULATION OF THE PROBLEM:

Let *d* be the distance between parallel flat plates, x-axis along the lower plate in the direction of flow of liquid and y-axis perpendicular to the plates. Supposing that the lower and upper plates execute oscillations with different amplitudes v_1, v_2 and frequencies ω_1, ω_2 respectively.

Following Ghosh (1968) the equation of motion for visco-elastic Oldroyd liquid of first order under the influence of an uniform magnetic field between oscillating plates is given by

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = -\frac{1}{\rho}\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial p}{\partial x} + \upsilon\left(1 + \lambda_2 \frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)u \tag{1}$$

where $\upsilon = \dfrac{\mu}{\rho}$ = kinetic viscosity, μ the coefficient of viscosity, *u* the velocity of liquid in the direction of oscillation, *p* the fluid pressure, *t* the time and B_0 is the magnetic inductivity.

Assuming the pressure gradient to be zero, the equation (1) becomes

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = \upsilon\left(1 + \lambda_2 \frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)u \tag{2}$$

The boundary conditions are:

$$\left.\begin{aligned} u &= v_1 e^{-i\omega_1 t} \qquad when \quad y = 0 \\ u &= v_2 e^{-i\omega_2 t} \qquad when \quad y = d \end{aligned}\right\} \tag{3}$$

Introducing the following non-dimensional quantities:

$$y^* = \frac{1}{d}y, \quad t^* = \frac{\upsilon}{d^2}t, \quad u^* = \frac{d}{\upsilon}u, \quad \lambda_1^* = \frac{\upsilon}{d^2}\lambda_1, \quad \lambda_2^* = \frac{\upsilon}{d^2}\lambda_2,$$

$$v_1^* = \frac{d}{\upsilon}v_1, \quad v_2^* = \frac{d}{\upsilon}v_2, \quad \omega_1^* = \frac{d^2}{\upsilon}\omega_1, \quad \omega_2^* = \frac{d^2}{\upsilon}\omega_2,$$

in (2) and (3) and then dropping stars, we get

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - M^2\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)u \tag{4}$$

and

$$\left.\begin{aligned} u &= v_1 e^{-i\omega_1 t} \qquad when \quad y = 0 \\ u &= v_2 e^{-i\omega_2 t} \qquad when \quad y = 1 \end{aligned}\right\} \tag{5}$$

where $M = dB_0\sqrt{\frac{\sigma}{\mu}}$ (Hartmann Number).

SOLUTION OF THE PROBLEM:

We look for a solution of equation (4) in the form

$$u = v_1 f(y) e^{-i\omega_1 t} + v_2 g(y) e^{-i\omega_2 t} \tag{6}$$

which is evidently periodic in *t*.

Substituting (6) in (4), we get

$$\begin{aligned} &v_1 e^{-i\omega_1 t}\left[(1 - i\omega_1\lambda_2)\frac{d^2 f}{dy^2} + \left\{\lambda_1\omega_1^2 + i\omega_1 - M^2(1 - i\omega_1\lambda_1)\right\}f\right] \\ &\quad + v_2 e^{-i\omega_2 t}\left[(1 - i\omega_2\lambda_2)\frac{d^2 g}{dy^2} + \left\{\lambda_1\omega_2^2 + i\omega_2 - M^2(1 - i\omega_2\lambda_1)\right\}g\right] = 0 \end{aligned} \tag{7}$$

By assumption v_1 *and* v_2 are not zero, we have

$$\frac{d^2 f}{dy^2} + \frac{i\omega_1(1 - i\omega_1\lambda_1) - M^2(1 - i\omega_1\lambda_1)}{(1 - i\omega_1\lambda_2)} f = 0$$

or $$\frac{d^2 f}{dy^2} + \frac{(1 - i\omega_1\lambda_1)(i\omega_1 - M^2)}{(1 - i\omega_1\lambda_2)} f = 0$$

or $$\frac{d^2 f}{dy^2} + m^2 f = 0 \tag{8}$$

and

$$\frac{d^2 g}{dy^2} + \frac{i\omega_2(1 - i\omega_2\lambda_1) - M^2(1 - i\omega_2\lambda_1)}{(1 - i\omega_2\lambda_2)} g = 0$$

or $$\frac{d^2 g}{dy^2} + \frac{(1 - i\omega_2\lambda_1)(i\omega_2 - M^2)}{(1 - i\omega_2\lambda_2)} g = 0$$

or $$\frac{d^2 g}{dy^2} + n^2 g = 0 \tag{9}$$

where

$$m^2 = \frac{(1 - i\omega_1\lambda_1)(i\omega_1 - M^2)}{(1 - i\omega_1\lambda_2)} \text{ and } n^2 = \frac{(1 - i\omega_2\lambda_1)(i\omega_2 - M^2)}{(1 - i\omega_2\lambda_2)}$$

The solution of (8) is

$$f(y) = A \cos m\, y + B \sin m\, y \tag{10}$$

and corresponding boundary conditions for $f(y)$ are:

$$\left.\begin{array}{lll} f(y) = 1 & when & y = 0 \\ f(y) = 0 & when & y = 1 \end{array}\right\} \tag{11}$$

$\therefore$ From (10) and (11), we get

$$f(y) = \cos\, m\, y - \frac{\cos m}{\sin m} \sin m\, y \tag{12}$$

Similarly from (9) with boundary conditions $g(0) = 0$ *and* $g(1) = 1$, we get

$$g(y) = \frac{\sin n\, y}{\sin n} \tag{13}$$

Now putting the values of $f(y)$ *and* $g(y)$ in (6), we get the velocity of visco-elastic Oldroyd (1958) type liquid between two parallel flat plates under the influence of magnetic field.

$$u = v_1 \frac{\sin m\,(1 - y)}{\sin m} e^{-i\omega_1 t} + v_2 \frac{\sin n\, y}{\sin n} e^{-i\omega_2 t} \tag{14}$$

PARTICULAR CASES:

Case I: If both the plates oscillate with same amplitudes and different frequencies then $v_1 = v_2 = v$ (say) and from (14)

$$u = v\left\{\frac{\sin m\,(1 - y)}{\sin m} e^{-i\omega_1 t} + \frac{\sin n\, y}{\sin n} e^{-i\omega_2 t}\right\} \tag{15}$$

Case II: If both the plates oscillate with same frequencies and different amplitudes then $\omega_1 = \omega_2 = \omega$ (say) and from (14)

$$u = \left\{ v_1 \frac{\sin m\,(1-y)}{\sin m} + v_2 \frac{\sin n\,y}{\sin n} \right\} e^{-i\omega t} \tag{16}$$

Where $m^2 = \dfrac{(1-i\omega\lambda_1)(i\omega - M^2)}{(1-i\omega\lambda_2)} = n^2$

Case III: If both the plates oscillate with same amplitudes and frequencies then $v_1 = v_2 = v$ (say) and $\omega_1 = \omega_2 = \omega$ (say) and from (14)

$$u = v\{(1-\cot m)\sin m\,y + \cos m\,y\} e^{-i\omega t} \tag{17}$$

where $m^2 = \dfrac{(1-i\omega\lambda_1)(i\omega - M^2)}{(1-i\omega\lambda_2)}$

Case IV: If magnetic field is withdrawn then $M = 0$ and from (14)

$$u = v_1 \frac{\sin m\,(1-y)}{\sin m} e^{-i\omega_1 t} + v_2 \frac{\sin n\,y}{\sin n} e^{-i\omega_2 t} \tag{18}$$

where $m^2 = \dfrac{i\omega_1(1-i\omega_1\lambda_1)}{(1-i\omega_1\lambda_2)}$ and $n^2 = \dfrac{i\omega_2(1-i\omega_2\lambda_1)}{(1-i\omega_2\lambda_2)}$

REFERENCES:

[1] Choubey, K.R. (1985): Ind. Jour. Pure Appl. Math., Vol.19, No.8, p-931.

[2] Ghosh, A.K. (1968): Bull. Cal. Math. Soc., Vol.64, No.4, p-163.

[3] Ghosh, S. and Ghosh, A.K. (2006): Ind. Jour. Pure Appl. Math. Vol.36, No.10, p-529.

[4] Krishna, D.V. and Rao, P.N. (2005): Acta Ciencia Indica, Vol.XXXI M, No.3, p-917.

[5] Kumar, R.; Singh. K.K. and Sharma, A.K. (2009): Jour. of Purvanchal Acad. Sci., Vol.15, p-80

[6] Kundu, S.K. and Sengupta, P.R. (2003):Bull. of Allahabad Mathematical Soc. Vol.18, p-73.

[7] Oldroyd, G.J. (1958): Proc. Roy. Soc. (London), Vol.254, p-278.

[8] Rehman, M.M. and Alam Sarkar, M.S. (2004): Bull. Cal. Math. Soc., Vol.96, No.6, p-463.

[9] Sengupta, P.R. and Paul, S. (2004): Ind. Jour. Theo. Phy. Vol.52, No.4, p-327.

[10] Suvarna, K. and Venkataramana, S. (2002): Acta Ciencia Indica, Vol.XXVIII M, No.4, p-485.

[11] Tripathi, A.; Sharma, A.K. and Singh, K.K. (2009): Ultra Scientist, Vol.21 M, No.3, p-671.

[12] Yadav, B.S. and Singh, K.K. (1990); Proc. Nat. Acad. Sci. India, Vol.60 (A), No.I, p-103.

****