Almost slightly \( \beta \)-continuity, Slightly \( \beta \)-open and Slightly \( \beta \)-closed mappings

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**ABSTRACT**

In this paper we discuss new type of continuous functions called Almost slightly pre\(--\)-continuous, slightly \( \beta \)-open and slightly \( \beta \)-closed functions; its properties and interrelation with other such functions are studied.

**Keywords:** slightly continuous functions; slightly semi-continuous functions; slightly pre-continuous; slightly \( \beta \)--continuous functions and slightly \( \nu \)--continuous functions.

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1. INTRODUCTION

In 1995 T. M. Nour introduced slightly semi-continuous functions. After him T. Noiri and G. I. Chae further studied slightly semi-continuous functions in 2000. T. Noiri individually studied about slightly \( \beta \)--continuous functions in 2001. C. W. Baker introduced slightly precontinuous functions in 2002. Arse Nagli Uresin and others studied slightly \( \delta \)--continuous functions in 2007. Recently S. Balasubramanian and P.A.S.Vyjayanthi studied slightly \( \nu \)--continuous functions in 2011. Mappings play an important role in the study of modern mathematics, especially in Topology and Functional analysis. Closed mappings are one such mappings which are studied for different types of closed sets by various mathematicians for the past many years. N.Biswas, discussed about semiopen mappings in the year 1970, A.S.Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb studied preopen mappings in the year 1982 and S.N.El-Deeb, and I.A.Hasanen defin and studied about preclosed mappings in the year 1983. Further Asit kumar sen and P. Bhattacharya discussed about pre-closed mappings in the year 1993. A.S.Mashilour, I.A.Hasanene and S.N.El-Deeb introduced \( \alpha \)-open and \( \alpha \)-closed mappings in the year in 1983, F.Cammaroto and T.Noiri discussed about semipre-open and semipre-closed mappings in the year 1989 and G.B.Navalagi further verified few results about semipreclosed mappings. M.E.Abd El-Monsef, S.N.El-Deeb and R.A.Mahmoud introduced \( \beta \)-open mappings in the year 1983 and Saeid Jafari and T.Noiri, studied about \( \beta \)-closed mappings in the year 2000. In the year 2010, S. Balasubramanian and P.A.S.Vyjayanthi introduced \( \nu \)-open mappings and in the year 2011, further defined almost \( \nu \)-open mappings and also they introduced \( \nu \)-closed and Almost \( \nu \)-closed mappings. C.W.Baker studied slightly-open and slightly-closed mappings in the year 2011. Inspired with these developments we introduce in this paper Almost slightly \( \beta \)-continuous, slightly \( \beta \)-open and slightly \( \beta \)-closed functions and study its basic properties and interrelation with other type of such functions. Throughout the paper \((X, \tau)\) and \((Y, \sigma)\) (or simply \(X\) and \(Y\)) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned.

2. PRELIMINARIES

**Definition 2.1:** \( A \subseteq X \) is called g-closed [rg-closed] if \( \text{cl}_A \subseteq U \) whenever \( A \subseteq U \) and \( U \) is open in \( X \).

**Definition 2.2:** A function \( f: X \rightarrow Y \) is said to be

(i) continuous [resp: nearly-continuous; \( r\alpha \)-continuous; \( \alpha \)-continuous; semi-continuous; \( \beta \)--continuous; pre-continuous] if inverse image of each open set is open[resp: regular-open; \( r\alpha \)-open; \( \alpha \)-open; semi-open; \( \beta \)-open; preopen].

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Example 3.1: \( X = Y = \{a, b, c\}; \tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\} \) and \( \sigma = \{\emptyset, \{a\}, \{b, c\}, Y\}. \) Let \( f \) be defined as \( f(a) = b; f(b) = c; \) and \( f(c) = a \), then \( f \) is sl.\( \beta \)-c function and al.sl.\( \beta \)-c function shorty.

Example 3.2: \( X = Y = \{a, b, c\}; \tau = \{\emptyset, \{a\}, \{b, c\}, X\} \) and \( \sigma = \{\emptyset, \{a\}, \{b, c\}, Y\}. \) Let \( f \) be defined as \( f(a) = b; f(b) = c; \) and \( f(c) = a \), then \( f \) is not sl.\( \beta \)-c, and al.sl.\( \beta \)-c.

Example 3.3: \( X = Y = \{a, b, c\}; \tau = \{\emptyset, \{a\}, \{b, c\}, X\} \) and \( \sigma = \{\emptyset, \{a\}, \{b, c\}, Y\}. \) Let \( f \) be defined as \( f(a) = b; f(b) = c; \) and \( f(c) = a \), then \( f \) is sl.\( \beta \)-c, sl.p.c., al.sl.\( \beta \)-c, al.sl.p.c., but not sl.c., sl.s.c., al.sl.c., and al.sl.s.c.,

Example 3.4: In Example 3.1, \( f \) is sl.\( \beta \)-c, sl.p.c., sl.s.c., sl.c., al.sl.\( \beta \)-c, al.sl.p.c., al.sl.s.c., and al.sl.c.

Example 3.5: In Example 3.2, \( f \) is sl.p.c., sl.c., al.sl.p.c., and al.sl.c., but not sl.\( \beta \)-c, and sl.s.c., but not al.sl.\( \beta \)-c, and al.sl.s.c.,

Example 3.6: In Example 3.3, \( f \) is sl.p.c., sl.\( \beta \)-c, al.sl.p.c., and al.sl.\( \beta \)-c, but not sl.c., sl.s.c., al.sl.c., and al.sl.s.c.,

Theorem 3.1: The following are equivalent:

(i) \( f \) is al.sl.\( \beta \)-c.
(ii) \( f^{-1}(V) \) is \( \beta \)-open for every r-clopen set \( V \) in \( Y \).
(iii) \( f^{-1}(V) \) is \( \beta \)-closed for every r-clopen set \( V \) in \( Y \).
(iv) \( f(\beta cl(A)) \subseteq \beta cl(f(A)) \).

Corollary 3.1: The following are equivalent.

(i) \( f \) is al.sl.\( \beta \)-c.
(ii) For each \( x \) in \( X \) and each \( V \in RCO(Y, f(x)) \) \( \exists U \in \beta O(X, x) \) such that \( f(U) \subseteq V \).
Theorem 3.2: Let $\sum = \{ U_i ; i \in I \}$ be any cover of $X$ by regular open sets in $X$. A function $f$ is \alslbc iff $f_{\lambda}$ is \alslbc for each $\lambda \in I$.

Proof: Let $i \in I$ be an arbitrarily fixed index and $U_i \in \text{RO}(X)$. Let $x \in U_i$ and $V \in \text{RCO}(Y, f_{U_i}(x))$

Since $f$ is \alslbc, $\exists U \subseteq \beta O(X, x)$ such that $f(U) \subset V$. Since $U_i \in \text{RO}(X)$, by Lemma 2.1 $x \in U \cap U_i \subseteq \beta O(U_i)$ and $(f_{U_i})^{-1}(U \cap U_i) = f(U \cap U_i) \subset f(U) \subset V$. Hence $f_{U_i}$ is \alslbc.

Conversely Let $x$ in $X$ and $F$ in $\text{RO}(X, f(x))$, $\exists i \in I$ such that $x \in U_i$. Since $f_{U_i}$ is \alslbc, $\exists U \subseteq \beta O(U_i, x)$ such that $f_{U_i}(U_i) \subset V$. By Lemma 2.1, $U \subseteq \beta O(X)$ and $f(U) \subset V$. Hence $f$ is \alslbc.

Theorem 3.3: If $f$ is almost continuous and $g$ is continuous, then $g \circ f$ is \alslbc.

Theorem 3.4: If $f$ is almost continuous, open and $g$ be any function, then $g \circ f$ is \alslbc iff $g$ is \alslbc.

Proof: If part: Theorem 3.3

Only if part: Let $A \in \text{RCO}(Z)$. Then $(g \circ f)^{-1}(A) \subseteq \tau(X)$. Since $f$ is open, $f(g \circ f)^{-1}(A) = g^{-1}(A)$ is open in $Y$. Thus $g$ is \alslbc.

Corollary 3.2: If $f$ is \alr, open and bijective, $g$ is a function. Then $g$ is \alslbc iff $g \circ f$ is \alslbc.

Theorem 3.5: If $g: X \to X \times Y$, defined by $g(x) = (x, f(x))$ for all $x$ in $X$ be the graph function of $f: X \to Y$. Then $g$ is \alslbc iff $f$ is \alslbc.

Proof: Let $V \in \text{RCO}(Y)$, then $X \times V \subseteq \text{RCO}(X \times Y)$. Since $g$ is \alslbc, $f^{-1}(V) = f^{-1}(X \times V) \subseteq \beta O(X)$.

Thus $f$ is \alslbc.

Conversely, let $x$ in $X$ and $F \subseteq \text{RCO}(X \times Y, g(x))$. Then $F \cap \{ \{x\} \times Y \} \subseteq \text{RCO}(\{x\} \times Y, g(x))$. Also $\{x\} \times Y$ is homeomorphic to $Y$. Hence $\{y \in Y : (x, y) \in F \} \subseteq \text{RCO}(Y)$. Since $f$ is \alslbc, $f^{-1}(y) \cap \{x\} \times F \subseteq f^{-1}(F)$ is open in $X$. Further $x \in \cup (f^{-1}(y) \cap \{x\} \times F \subseteq f^{-1}(F))$. Hence $g^{-1}(F)$ is open. Thus $g$ is \alslbc.

Theorem 3.6: \begin{enumerate}
\item $f: \Pi X_\lambda \to \Pi Y_\lambda$ is \alslbc iff $f: X_\lambda \to Y_\lambda$ is \alslbc for each $\lambda \in \Gamma$.
\item If $f: X \to \Pi Y_\lambda$ is \alslbc, then $P_\lambda f: X \to Y_\lambda$ is \alslbc for each $\lambda \in \Gamma$, where $P_\lambda: \Pi Y_\lambda$ onto $Y_\lambda$.
\end{enumerate}

Remark 1: Composition, Algebraic sum, product and the pointwise limit of \alslbc functions is not in general \alslbc. However we can prove the following:

Theorem 3.7: The uniform limit of a sequence of \alslbc functions is \alslbc.

Note 3: Pasting Lemma is not true for \alslbc functions. However we have the following weaker versions.

Theorem 3.8: Let $X$ and $Y$ be topological spaces such that $X = A \cup B$ and let $f_A$ and $g_B$ are \alslrc maps such that $f(x) = g(x)$ for all $x \in A \cap B$. If $A, B \in \text{RO}(X)$ and $\text{RO}(X)$ is closed under finite unions, then the combination $\alpha: X \to Y$ is \alslbc continuous.

Theorem 3.9: Pasting Lemma Let $X$ and $Y$ be spaces such that $X = A \cup B$ and let $f_A$ and $g_B$ are \alslbc maps such that $f(x) = g(x)$ for all $x \in A \cap B$. $A, B \in \text{RO}(X)$ and $\beta O(X)$ is closed under finite unions, then the combination $\alpha: X \to Y$ is \alslbc.

Proof: Let $F \in \text{RO}(Y)$, then $\alpha^{-1}(F) = f^{-1}(F) \cup g^{-1}(F)$, where $f^{-1}(F) \subseteq \beta O(A)$ and $g^{-1}(F) \subseteq \beta O(B) \Rightarrow f^{-1}(F) \cup g^{-1}(F) \subseteq \alpha^{-1}(F) \subseteq \beta O(X) \Rightarrow f^{-1}(F) \cup g^{-1}(F) = \alpha^{-1}(F) \subseteq \beta O(X)$. Hence $\alpha: X \to Y$ is \alslbc.

Definition 3.2: A function $f$ is said to be almost somewhat $\beta$-continuous if for $U \in \text{RO}(\sigma)$ and $f^{-1}(U) \neq \emptyset$, there exists a non-empty $\beta$-open set $V$ in $X$ such that $V \subseteq f^{-1}(U)$.

It is clear that every continuous function is almost somewhat continuous and almost somewhat continuous function is almost somewhat $\beta$-continuous. But the converse is not true.
Example 3.7: Let \( X = \{a, b, c\} \), \( \tau = \sigma = \{\varnothing, \{a\}, \{a, b\}, \{b, c\}, X\} \). The function \( f \) defined by \( f(a) = b, f(b) = c \) and \( f(c) = a \) is almost somewhat \( \beta \)-continuous but not somewhat \( \beta \)-continuous.

Note 4: Every almost somewhat \( \beta \)-continuous function is almost slightly \( \beta \)-continuous.

Theorem 3.10: If \( f \) is almost somewhat \( \beta \)-continuous and \( g \) is continuous, then \( g \circ f \) is almost somewhat \( \beta \)-continuous.

Corollary 3.3: If \( f \) is almost somewhat \( \beta \)-continuous and \( g \) is \( r \)-continuous\(^*\), then \( g \circ f \) is almost somewhat \( \beta \)-continuous.

Theorem 3.11: For a surjective function \( f \), the following statements are equivalent:
(i) \( f \) is almost somewhat \( \beta \)-continuous.
(ii) If \( C \) is a \( r \)-closed subset of \( Y \) such that \( f^{-1}(C) \neq X \), then there is a proper \( \beta \)-closed subset \( D \) of \( X \) such that \( f^{-1}(C) \subset D \).
(iii) If \( M \) is a dense subset of \( X \), then \( f(M) \) is a dense subset of \( Y \).

Proof:
(i) \( \Rightarrow \) (ii): For \( C \in \text{RC}(Y) \) with \( f^{-1}(C) \neq X \), \( Y - C \in \text{RO}(Y) \) such that \( f^{-1}(Y - C) = X - f^{-1}(C) \neq \varnothing \). By (i), there exists a \( \beta \)-open set \( V \) such that \( V \neq \varnothing \) and \( V \subset f^{-1}(Y - C) = X - f^{-1}(C) \). Thus \( X - V = f^{-1}(C) \) and \( X - V = D \) is a proper \( \beta \)-closed set in \( X \).

(ii) \( \Rightarrow \) (i): Let \( U \in \text{RO}(\sigma) \) and \( f^{-1}(U) \neq \varnothing \). Then \( Y - U \) is \( r \)-closed and \( f^{-1}(Y - U) = X - f^{-1}(U) \neq X \). By (ii), there exists a proper \( \beta \)-closed set \( D \) such that \( D \subset f^{-1}(Y - U) \). This implies that \( X - D \subset f^{-1}(U) \) and \( X - D \neq \varnothing \).

(iii) \( \Rightarrow \) (ii): Let \( M \) be dense in \( X \). If \( f(M) \) is not dense in \( Y \). Then there exists a proper \( r \)-closed set \( C \) in \( Y \) such that \( f(M) \subset C \subset Y \). Clearly \( f^{-1}(C) \neq X \). By (ii), there exists a proper \( \beta \)-closed set \( D \) such that \( M \subset f^{-1}(C) \subset D \subset X \). This is a contradiction to the fact that \( M \) is dense in \( X \).

But by (iii), \( f(f^{-1}(C)) = C \) must be dense in \( Y \), which is a contradiction to the choice of \( C \).

Theorem 3.12: Let \( f \) be a function and \( X = A \cup B \), where \( A, B \in \text{RO}(X) \). If \( f_A \) and \( f_B \) are almost somewhat \( \beta \)-continuous, then \( f \) is almost somewhat \( \beta \)-continuous.

Proof: Let \( U \in \text{RO}(\sigma) \) such that \( f_A^{-1}(U) \neq \varnothing \). Then \( f_B^{-1}(f_A^{-1}(U)) \neq \varnothing \). Suppose (iii) is not true, there exists a \( r \)-closed set \( C \) in \( Y \) such that \( f^{-1}(C) \neq X \) and there is no proper \( \beta \)-closed set \( D \) in \( X \) such that \( f^{-1}(C) \subset D \). This means that \( f^{-1}(C) \) is dense in \( X \).

The proof of other cases are similar.

Definition 3.3: If \( X \) is a set and \( \tau \) and \( \sigma \) are topologies on \( X \), then \( \tau \) is said to be \( \beta \)-equivalent to \( \sigma \) provided if \( U \in \sigma \) and \( U \neq \varnothing \), there is an \( \beta \)-open set \( V \) in \( X \) such that \( V \neq \varnothing \) and \( V \subset U \). If \( U \in \beta \sigma \) and \( U \neq \varnothing \), there is an \( \beta \)-open set \( V \) in \( (X, \sigma) \) such that \( V \neq \varnothing \) and \( V \supset U \).

Definition 3.4: \( A \subset X \) is said to be dense in \( X \) if there is no proper closed set \( C \subset X \) such that \( M \subset C \subset X \).

Now, consider the identity function \( f \) and assume that \( \tau \) and \( \sigma \) are equivalent. Then \( f \) and \( f^{-1} \) are almost somewhat continuous. Conversely, if the identity function \( f \) is almost somewhat continuous in both directions, then \( \tau \) and \( \sigma \) are equivalent.

Theorem 3.13: Let \( f(X, \tau) \to (Y, \sigma) \) be a almost somewhat \( \beta \)-continuous surjection and \( \tau^* \) be a topology for \( X \), which is \( \beta \)-equivalent to \( \tau \). Then \( f(X, \tau^*) \to (Y, \sigma) \) is almost somewhat \( \beta \)-continuous.

Proof: Let \( V \in \text{RO}(\sigma) \) and \( f^{-1}(V) \neq \varnothing \). Since \( f \) is almost somewhat \( \beta \)-continuous, \( \exists \) a nonempty \( U \in \beta \sigma \) and \( U \supset f^{-1}(V) \). For \( U \) is \( \beta \)-equivalent to \( \tau \), \( U^* \in \beta \sigma \) and \( U^* \subset U \). But \( U \subset f^{-1}(V) \). Hence \( f(X, \tau^*) \to (Y, \sigma) \) is almost somewhat \( \beta \)-continuous.

Theorem 3.14: Let \( f(X, \tau) \to (Y, \sigma) \) be a almost somewhat \( \beta \)-continuous surjection and \( \sigma^* \) be a topology for \( Y \), which is \( \beta \)-equivalent to \( \sigma \). Then \( f(X, \tau) \to (Y, \sigma^*) \) is almost somewhat \( \beta \)-continuous.
Proof: Let $V \ast \in RO(\sigma) \ni f^{-1}(V) \neq \varnothing$. Since $\sigma^*$ is $\beta$-equivalent to $\sigma$, $\exists V \varnothing \in \beta O(Y, \sigma) \ni V \subseteq V^*$. 

Now $\varnothing \neq f^{-1}(V) \subseteq f^{-1}(V^*)$. Since $f$ is almost somewhat $\beta$-continuous, $\exists U \varnothing \in \beta O(X, \tau) \ni U \subseteq f^{-1}(V)$. Then $U \subseteq f^{-1}(V^*)$; hence $f(X, \tau) \to (Y, \sigma^*)$ is almost somewhat $\beta$-continuous.

4. SLIGHTLY $\beta$-OPEN MAPPINGS, ALMOST SLIGHTLY $\beta$-OPEN MAPPINGS AND ALMOST SOMEWHAT $\beta$-OPEN FUNCTION

Definition 4.1: A function $f: X \to Y$ is said to be 
(i) slightly $\beta$-open if image of every clopen set in $X$ is $\beta$-open in $Y$ 
(ii) almost slightly $\beta$-open if image of every regular-clopen set in $X$ is $\beta$-open in $Y$

Example 4.1: Let $X = Y = \{a, b, c\}; \tau = \{\varnothing, \{a\}, \{a, b\}, X\}; \sigma = \{\varnothing, \{a, c\}, Y\}$. Let $f: X \to Y$ be defined $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then $f$ is slightly open, slightly pre-open, slightly semi-open, almost $\beta$-open, almost slightly open, almost semi-open, almost slightly pre-open, and almost slightly $\beta$-open.

Example 4.2: Let $X = Y = \{a, b, c\}; \tau = \{\varnothing, \{a\}, \{b, c\}, X\}; \sigma = \{\varnothing, \{a, b\}, \{a, b\}, \{a, b\}, Y\}$. Let $f: X \to Y$ be defined $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then $f$ is not slightly open, slightly pre-open, slightly semi-open, slightly $\beta$-open, almost slightly open, almost semi-open, almost slightly pre-open, and almost slightly $\beta$-open.

Note 5:
(i) If $R \alpha O(Y) = \beta O(Y)$, then $f$ is [almost-] slightly ro-open iff $f$ is [almost-] slightly $\beta$-open.
(ii) If $\beta O(Y) = RO(Y)$, then $f$ is [almost-] slightly-\textit{r}-open iff $f$ is [almost-] slightly $\beta$-open.
(iii) If $\beta O(Y) = \alpha O(Y)$, then $f$ is [almost-] slightly $\alpha$-open iff $f$ is [almost-] slightly $\beta$-open.

Theorem 4.1: 
(i) If $f$ is [almost-] slightly open and $g$ is $\beta$-open[r-\textit{open}] then $g \ast f$ is slightly $\beta$-open 
(ii) If $f$ is [almost-] slightly $\beta$-open and $g$ is $M$- $\beta$-open [M-\textit{r}-\textit{open}] then $g \ast f$ is slightly $\beta$-open

Proof: Let $A$ be clopen[regular clopen] set in $X \ni f(A)$ is open in $Y \Rightarrow g(f(A)) = g \ast f(A)$ is $\beta$-open in $Z$. Hence $g \ast f$ is [almost-] slightly $\beta$-open.

Theorem 4.2: If $f$ and $g$ are r-\textit{open} then $g \ast f$ is [almost-] slightly $\beta$-open

Proof: Let $A$ be clopen[r-clopen] set in $X \ni f(A)$ is r-\textit{open} and so open in $Y \Rightarrow g(f(A))$ is r-\textit{open} in 
$Z \Rightarrow g(f(A)) = g \ast f(A)$ is open in $Z$. Hence $g \ast f$ is [almost-] slightly $\beta$-open.

Theorem 4.3: If $f$ is almost slightly-\textit{r}-open and $g$ is [almost-] $\beta$-open then $g \ast f$ is [almost-] slightly $\beta$-open

Corollary 4.1: 
(i) If $f$ is almost slightly-\textit{r}-open and $g$ is open[r-\textit{open}] then $g \ast f$ is [almost-] slightly $\beta$-open.
(ii) If $f$ is almost slightly-r-\textit{open} and $g$ is [almost-] $\beta$-open then $g \ast f$ is [almost-] slightly $\beta$-open.
(iii) If $f$ and $g$ are almost slightly-r-\textit{open} then $g \ast f$ is [almost-] slightly $\beta$-open.

Theorem 4.4: If $f$ is [almost-] slightly $\beta$-open, then $f(A) \subseteq \beta(f(A))^\#$

Proof: Let $A \subseteq X$ and $f$ is slightly $\beta$-open gives $f(A)$ is $\beta$-open in $Y$ and $f(A) \subseteq f(A)$ which in turn gives 

$f(A)^\# \subseteq \beta(f(A))^\#$ 

(1)

Since $f(A)$ is $\beta$-open in $Y$, $\beta(f(A))^\# = f(A)$ 

(2)

From (1) and (2) we have $f(A) \subseteq \beta(f(A))^\#$ for every subset $A$ of $X$.

Remark 2: converse is not true in general.

Theorem 4.5: If $f$ is slightly $\beta$-open and $A \subseteq X$ is r-\textit{open}, then $f(A)$ is $\tau_\text{p}$-\textit{open} in $Y$.

Proof: Let $A \subseteq X$ and $f$ is slightly $\beta$-open implies $f(A) \subseteq \beta(f(A))^\#$ which in turn implies $\beta(f(A))^\# \subseteq f(A)$, since $f(A) = f(A)^\#$. But $f(A) \subseteq \beta(f(A))^\#$. Combining we get $f(A) = \beta(f(A))^\#$. Hence $f(A)$ is $\tau_\text{p}$-\textit{open} in $Y$.  

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Corollary 4.2:
(i) If \( f \) is \([\text{almost-}]\) slightly \( r\)-open, then \( f(A^o) \subset \beta(f(A))^o \)
(ii) If \( f \) is \([\text{almost-}]\) slightly \( r\)-open, then \( f(A) \) is \( \tau_r \)-open in \( Y \) if \( A \) is \( r\)-open set in \( X \).
(iii) If \( f \) is almost slightly \( r\)-open and \( A \subset X \) is \( r\)-open, then \( f(A) \) is \( \tau_r \)-open in \( Y \).

Theorem 4.6: If \( \beta(A)^o = r(A^o) \) for every \( A \subset Y \), then the following are equivalent:
(i) \( f \) is \([\text{almost-}]\) slightly \( r\)-open map
(ii) \( f(A^o) \subset \beta(f(A))^o \)

Proof:
(i) \( \Rightarrow \) (ii): follows from theorem 4.4
(ii) \( \Rightarrow \) (i): Let \( A \) be any \( r\)-open set in \( X \), then \( f(A) = \beta(f(A))^o \supset f(A^o) \) by hypothesis. We have \( f(A) \subset f(A^o) \).
Combining we get \( f(A) = \beta(A)^o = r(A^o) \) [by given condition] which implies \( f(A) \) is \( r\)-open and hence open. Thus \( f \) is slightly \( r\)-open.

Theorem 4.7: \( f \) is \([\text{almost-}]\) slightly \( r\)-open iff for each subset \( S \) of \( Y \) and each \( r\)-clopen set \( U \) containing \( f^{-1}(S) \), there is a \( r\)-open set \( V \) of \( Y \) such that \( S \subset V \) and \( f^{-1}(V) \subset U \).

Remark 3: composition of two \([\text{almost-}]\) slightly \( r\)-open maps is not \([\text{almost-}]\) slightly \( r\)-open in general.

Theorem 4.8: Let \( X, Y, Z \) be topological spaces and every open set is \( r\)-clopen in \( Y \), then the composition of two \([\text{almost-}]\) slightly \( r\)-open maps is \([\text{almost-}]\) slightly \( r\)-open.

Proof: Let \( A \) be \( r\)-clopen in \( X \) \( \Rightarrow f(A) \) is open and so \( r\)-clopen in \( Y \) [by assumption]
\[ \Rightarrow g(f(A)) = g\circ f(A) \text{ is open in } Z. \]
Hence \( g\circ f \) is almost slightly \( r\)-open.

Theorem 4.9: If \( f \) is \([\text{almost-}]\) slightly \( g\)-open; \( g \) is open[\( r\)-open] and \( Y \) is \( T_{\text{rg}} \), then \( g\circ f \) is \([\text{almost-}]\) slightly \( r\)-open.

Proof: (i) Let \( A \) be regular clopen in \( X \) \( \Rightarrow A \) be clopen in \( X \) \( \Rightarrow f(A) \) is \( g\)-open and open in \( Y \) [since \( Y \) is \( T_{\text{rg}} \)]
\[ \Rightarrow g(f(A)) = g\circ f(A) \text{ is open in } Z. \]
Hence \( g\circ f \) is \([\text{almost-}]\) slightly \( r\)-open.

Corollary 4.3:
(i) If \( f \) is \([\text{almost-}]\) slightly \( g\)-open; \( g \) is open[\( r\)-open] and \( Y \) is \( T_{\text{rg}} \) then \( g\circ f \) is \([\text{almost-}]\) slightly \( r\)-open.
(ii) If \( f \) is \([\text{almost-}]\) slightly \( g\)-open; \( g \) is \([\text{almost-}]\) \( \beta\)-open \([\text{almost-}]\) \( r\)-open and \( Y \) is \( T_{\text{rg}} \) then \( g\circ f \) is \([\text{almost-}]\) slightly \( r\)-open.

Theorem 4.10: If \( f \) is \([\text{almost-}]\) slightly \( r\)-open; \( g \) is open[\( r\)-open] and \( Y \) is \( r\)-\( T_{\text{rg}} \), then \( g\circ f \) is \([\text{almost-}]\) slightly \( r\)-open.

Proof: Let \( A \) be \( r\)-clopen in \( X \) \( \Rightarrow A \) be clopen in \( X \) \( \Rightarrow f(A) \) is \( r\)-open and \( r\)-open in \( Y \) [since \( Y \) is \( r\)-\( T_{\text{rg}} \)]
\[ \Rightarrow g(f(A)) = g\circ f(A) \text{ is open in } Z. \]
Hence \( g\circ f \) is almost slightly \( r\)-open.

Theorem 4.11: If \( f \) is \([\text{almost-}]\) slightly \( r\)-open; \( g \) is \([\text{almost-}]\) \( \beta\)-open \([\text{almost-}]\) \( r\)-open and \( Y \) is \( r\)-\( T_{\text{rg}} \), then \( g\circ f \) is \([\text{almost-}]\) slightly \( r\)-open.

Proof: Let \( A \) be \( r\)-clopen in \( X \) \( \Rightarrow A \) be clopen in \( X \) \( \Rightarrow f(A) \) is \( r\)-open in \( Y \) \( \Rightarrow f(A) \) is \( r\)-open in \( Y \) [since \( Y \) is \( r\)-\( T_{\text{rg}} \)]
\[ \Rightarrow g(f(A)) = g\circ f(A) \text{ is open in } Z. \]
Hence \( g\circ f \) is almost slightly \( r\)-open.

Corollary 4.4:
(i) If \( f \) is \([\text{almost-}]\) slightly \( r\)-open; \( g \) is open[\( r\)-open] and \( Y \) is \( r\)-\( T_{\text{rg}} \), then \( g\circ f \) is \([\text{almost-}]\) slightly \( r\)-open.
(ii) If \( f \) is \([\text{almost-}]\) slightly \( r\)-open; \( g \) is \([\text{almost-}]\) \( \beta\)-open \([\text{almost-}]\) \( r\)-open and \( Y \) is \( r\)-\( T_{\text{rg}} \), then \( g\circ f \) is \([\text{almost-}]\) slightly \( r\)-open.

Theorem 4.12: If \( f, g \) be two mappings such that \( g\circ f \) is \([\text{almost-}]\) slightly \( r\)-open \([\text{almost-}]\) \( r\)-open. Then the following are true
(i) If \( f \) is continuous[\( r\)-continuous] and surjective, then \( g \) is \([\text{almost-}]\) slightly \( r\)-open
(ii) If \( f \) is \( g\)-continuous, surjective and \( X \) is \( T_{\text{rg}} \), then \( g \) is \([\text{almost-}]\) slightly \( r\)-open
(iii) If \( f \) is \( r\)-continuous, surjective and \( X \) is \( r\)-\( T_{\text{rg}} \), then \( g \) is \([\text{almost-}]\) slightly \( r\)-open.
Proof: Let A be regular clopen in Y ⇒ A be clopen in Y ⇒ \( f^{-1}(A) \) is open in X ⇒ \( g(f^{-1}(A)) = g(A) \) is open in Z. Hence g is almost slightly β-open.

Similarly we can prove the remaining parts and so omitted.

**Corollary 4.5:** If \( f, g \) be two mappings such that \( g \circ f \) is [almost-] slightly β-open [[almost-] slightly r-open]. Then the following are true
(i) If \( f \) is continuous[r-continuous] and surjective, then g is [almost-] slightly β-open.
(ii) If \( f \) is g-continuous, surjective and X is \( T_{\beta} \), then g is [almost-] slightly β-open.
(iii) If \( f \) is rg-continuous, surjective and X is \( r-T_{\beta} \), then g is [almost-] slightly β-open.

**Theorem 4.13:** If X is regular, \( f \) is r-open, nearly-continuous, open surjection and \( \tilde{A} = A \) for every open[r-open] set in Y, then Y is regular.

**Theorem 4.14:** If \( f \) is [almost-]slightly β-open and A is r-clopen[clopen] set of X, then \( f_A \) is [almost-]slightly β-open.

**Proof:** Let F be r-open set in A. Then \( F = \Lambda \cap E \) is r-open in X for some r-open set E of X which implies \( f(A) \) is open in Y. But \( f(F) = f_A(F) \). Therefore \( f_A \) is [almost-] slightly β-open.

**Theorem 4.15:** If \( f \) is [almost-] slightly β-open, X is \( T_{\beta} \) and A is g-open set of X, then \( f_A \) is [almost-] slightly β-open.

**Corollary 4.6:** If \( f \) is [almost-] slightly open, X is \( T_{\beta} \) and A is g-open set of X, then \( f_A \) is [almost-] slightly β-open.

**Theorem 4.16:** If \( f_i \colon X_i \rightarrow Y_i \) be [almost-] slightly β-open for \( i = 1, 2 \). Let \( f \colon X_1 \times X_2 \rightarrow Y_1 \times Y_2 \) be defined as \( f(x_1, x_2) = (f_1(x_1), f_2(x_2)) \). Then \( f \colon X_1 \times X_2 \rightarrow Y_1 \times Y_2 \) is [almost-] slightly β-open.

**Proof:** Let \( U_1 \times U_2 \subset X_1 \times X_2 \) where \( U_i \) is r-clopen in \( X_i \) for \( i = 1, 2 \). Then \( f(U_1 \times U_2) = f_1(U_1) \times f_2(U_2) \) a open set in \( Y_1 \times Y_2 \). Thus \( f(U_1 \times U_2) \) is open and hence f is [almost-] slightly β-open.

**Corollary 4.7:** If \( f_i \colon X_i \rightarrow Y_i \) be [almost-] slightly open for \( i = 1, 2 \). Let \( f \colon X_1 \times X_2 \rightarrow Y_1 \times Y_2 \) be defined as \( f(x_1, x_2) = (f_1(x_1), f_2(x_2)) \). Then \( f \colon X_1 \times X_2 \rightarrow Y_1 \times Y_2 \) is [almost-] slightly β-open.

**Theorem 4.17:** Let \( h \colon X \rightarrow X_1 \times X_2 \) be [almost-] slightly β-open. Let \( f_i \colon X \rightarrow X_i \) be defined as \( h(x) = (x_1, x_2) \) and \( f(x) = x_i \). Then \( f_i \colon X \rightarrow X_i \) is [almost-] slightly β-open for \( i = 1, 2 \).

**Proof:** Let \( U_i \) be r-clopen in \( X_i \), then \( U_i \times X_2 \) is r-clopen in \( X_i \times X_2 \), and \( h(U_i \times X_2) \) is open in X. But \( f_i(U_i) = h(U_i \times X_2) \), therefore \( f_i \) is [almost-] slightly β-open. Similarly we can show that \( f_2 \) is [almost-] slightly β-open and thus \( f_i \colon X \rightarrow X_i \) is [almost-] slightly β-open for \( i = 1, 2 \).

**Corollary 4.8:** Let \( h \colon X \rightarrow X_1 \times X_2 \) be [almost-] slightly open. Let \( f_i \colon X \rightarrow X_i \) be defined as \( h(x) = (x_1, x_2) \) and \( f(x) = x_i \). Then \( f_i \colon X \rightarrow X_i \) is [almost-] slightly β-open for \( i = 1, 2 \).

**Definition 4.2:** A function \( f \) is said to be almost somewhat β-open provided that if \( U \in \text{RO}(\tau) \) and \( U \neq \varnothing \), then there exists a non-empty β-open set \( V \) in \( Y \) such that \( V \subset f(U) \).

**Example 4.3:** Let \( X = \{a, b, c\} \). \( \tau = \{\varnothing, \{a\}, \{b, c\}, X\} \) and \( \sigma = \{\varnothing, \{a\}, \{b, c\}, X\} \). The function \( f \) defined by \( f(a) = a, f(b) = c \) and \( f(c) = b \) is almost somewhat open and almost somewhat β-open.

**Example 4.4:** Let \( X = \{a, b, c\} \). \( \tau = \{\varnothing, \{a, b\}, \{a, c\}, \{a, b, c\}, X\} \) and \( \sigma = \{\varnothing, \{a\}, \{b, c\}, \{a, b\}, \{a, c\}, X\} \). The function \( f \) defined by \( f(a) = c, f(b) = a \) and \( f(c) = b \) is not almost somewhat β-open.

**Theorem 4.18:** Let \( f \) be an r-open function and \( g \) almost somewhat β-open. Then \( g \circ f \) is almost somewhat β-open.

**Theorem 4.19:** For a bijective function \( f \), the following are equivalent:
(i) \( f \) is almost somewhat β-open.
(ii) If C is an r-closed subset of X, such that \( f(C) \neq Y \), then there is a β-closed subset D of Y such that \( D \neq Y \) and \( D \supseteq f(C) \).

**Proof:** (i) \( \Rightarrow \) (ii): Let C be any r-closed subset of X such that \( f(C) \neq Y \). Then X-C is r-open in X and X-C \( \neq \varnothing \). Since \( f \) is almost somewhat β-open, there exists a β-open set \( V \neq \varnothing \) in Y such that \( V \subset f(X-C) \). Put \( D = Y - V \). Clearly D is β-closed.
in Y and we claim D ≠ Y. If D = Y, then V = φ, which is a contradiction. Since V ⊆ f(X-C), D = Y - V ⇒ (Y - f(X-C)) = f(C).

(ii) ⇒(i): For U ≠ φ an r-open in X, C = X - U is r-closed in X and f(X-U) = f(C) = Y - f(U) implies f(C) ≠ Y. Therefore, by (ii), there is a β-closed set D of Y such that D ≠ Y and f(C) ⊆ D. Clearly V = Y - D is a β-open set and V ≠ φ. Also, V = Y - D ⊆ Y - f(C) = Y - f(X-U) = f(U).

Theorem 4.20: The following statements are equivalent:
(i) f is almost somewhat β-open.
(ii) If A is a dense subset of Y, then f⁻¹(A) is a dense subset of X.

Proof:
(i) ⇒(ii): If A is dense in set X. If f⁻¹(A) is not dense in X, then there exists a r-closed set B in X such that f⁻¹(A) ⊆ B ⊆ X. Since f is almost somewhat β-open and X-B is open, there exists a nonempty β-open set C in Y such that C ⊆ f(X-B). Therefore, C ⊆ f(X-B) ⊆ f(f⁻¹(Y-A)) ⊆ Y-A. That is, A ⊆ Y-C ⊆ Y. Now, Y-C is a β-closed set and A ⊆ Y-C ⊆ Y. This implies that A is not a dense set in Y, which is a contradiction. Therefore, f⁻¹(A) is a dense subset in X.

(ii) ⇒(i): If A ≠ φ is an r-open set in X. We want to show that (f(A))⁻¹φ. Suppose (f(A))⁻¹ φ. Then, cl(f(A)) = Y. By (ii), f⁻¹((Y - f(A)) is dense in X. But f⁻¹((Y - f(A))) ⊆ X-A. Now, X-A is r-closed. Therefore, f⁻¹((Y - f(A))) ⊆ X-A gives X = cl(f⁻¹((Y - f(A)))) ⊆ X-A. This implies that A = φ, which is contrary to A ≠ φ. Therefore, (f(A))⁻¹ φ. Hence f is almost somewhat β-open.

Theorem 4.21: Let f be almost somewhat β-open and A be any r-open subset of X. Then f⁻¹(A) is almost somewhat β-open.

Proof: Let U ∈ RO(τ_A) such that U ≠ φ. Since U ∈ RO(τ_A); A ∈ RO(X); U ∈ RO(X) and f is almost somewhat β-open, ∃ V ∈ BO(Y), such that V ⊆ f(U). Thus f⁻¹(A) is almost somewhat β-open.

Theorem 4.22: Let f be a function and X = A ∪ B, where A,B ∈ τ(X). If the restriction functions f_A and f_B are almost somewhat β-open, then f is almost somewhat β-open.

Proof: Let U be any r-open subset of X such that U ≠ φ. Since X = A ∪ B, either A ∩ U ≠ φ or B ∩ U ≠ φ or both A ∩ U ≠ φ and B ∩ U ≠ φ. Since U is open in X, U is open in both A and B.

Case (i): If A ∩ U ≠ φ, where U ∩ A ∈ RO(τ_A). Since f_A is almost somewhat β-open, ∃ V ∈ BO(Y) such that V ⊆ f(U ∩ A) ⊆ f(U), which implies that f is almost somewhat β-open.

Case (ii): If B ∩ U ≠ φ, where U ∩ B ∈ RO(τ_B). Since f_B is almost somewhat β-open, ∃ V ∈ BO(Y) such that V ⊆ f(U ∩ B) ⊆ f(U), which implies that f is almost somewhat β-open.

Case (iii): If both A ∩ U ≠ φ and B ∩ U ≠ φ. Then by cases (i) and (ii) f is almost somewhat β-open.

Remark 4: Two topologies τ and σ for X are said to be β-equivalent if and only if the identity function f: (X, τ) → (Y, σ) is almost somewhat β-open in both directions.

Theorem 4.23: Let f: (X, τ) → (Y, σ) be a almost somewhat almost β-open function. Let τ* and σ* be topologies for X and Y, respectively such that τ* is β-equivalent to τ and σ* is β-equivalent to σ. Then f: (X, τ*) → (Y, σ*) is almost somewhat β-open.

5. SLIGHTLY β-CLOSED MAPPINGS AND ALMOST SLIGHTLY β-CLOSED MAPPINGS

Definition 5.1: A function f: X → Y is said to be
(i) slightly β-closed if image of every clopen set in X is β-closed in Y
(ii) almost slightly β-closed if image of every regular-clopen set in X is β-closed in Y

Example 5.1: Let X = Y = {a, b, c}; τ = {φ, {a}, {a, b}, X}; σ = {φ, {a, c}, Y}. Let f: X → Y be defined f(a) = c, f(b) = b and f(c) = a. Then f is slightly closed, slightly pre-closed, slightly semi-closed, slightly β-closed, almost slightly closed, almost slightly semi-closed, almost slightly pre-closed, and almost slightly β-closed.
Example 5.2: Let $X = Y = \{a, b, c\}; \tau = \{\emptyset, \{a\}, \{b, c\}, X\}; \sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f: X \rightarrow Y$ be defined $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then $f$ is not slightly closed, slightly pre-closed, slightly semi-closed, slightly $\beta$-closed, almost slightly closed, almost slightly semi-closed, almost slightly pre-closed, and almost slightly $\beta$-closed.

Note 6:
(i) If $R\alpha C(Y) = \beta C(Y)$, then $f$ is [almost-] slightly $\alpha$-closed iff $f$ is [almost-] slightly $\beta$-closed.
(ii) If $\beta C(Y) = RC(Y)$, then $f$ is [almost-] slightly $r$-closed iff $f$ is [almost-] slightly $\beta$-closed.
(iii) If $\beta C(Y) = \alpha C(Y)$, then $f$ is [almost-] slightly $\alpha$-closed iff $f$ is [almost-] slightly $\beta$-closed.

Theorem 5.1: (i) If $f$ is [almost-] slightly closed and $g$ is $\beta$-closed then $g \circ f$ is [almost-] slightly $\beta$-closed.
(ii) If $f$ is [almost-] slightly $\beta$-closed and $g$ is $M$- $\beta$-closed then $g \circ f$ is [almost-] slightly $\beta$-closed.

Proof: Let $A$ be clopen[regular clopen] set in $X$ $\Rightarrow f(A)$ is closed in $Y$ $\Rightarrow g(f(A)) = g \circ f(A)$ is $\beta$-closed in $Z$. Hence $g \circ f$ is [almost-] slightly $\beta$-closed.

Theorem 5.2: If $f$ and $g$ are $r$-closed then $g \circ f$ is [almost-] slightly $\beta$-closed.

Proof: Let $A$ be clopen[regular clopen] set in $X$ $\Rightarrow f(A)$ is $r$-closed and so closed in $Y$ $\Rightarrow g(f(A))$ is $r$-closed in $Z$ $\Rightarrow g(\gamma f(A)) = g \circ f(A)$ is closed in $Z$. Hence $g \circ f$ is [almost-] slightly $\beta$-closed.

Theorem 5.3: If $f$ is almost slightly $r$-closed and $g$ is [almost-] $\beta$-closed then $g \circ f$ is [almost-] slightly $\beta$-closed.

Corollary 5.1:
(i) If $f$ is almost slightly $\beta$-closed and $g$ is [almost-] $r$-closed then $g \circ f$ is [almost-] slightly $\beta$-closed.
(ii) If $f$ and $g$ are almost slightly $\beta$-closed then $g \circ f$ is [almost-] slightly $\beta$-closed.
(iii) If $f$ is almost slightly $r$-closed and $g$ is [almost-] $\beta$-closed then $g \circ f$ is [almost-] slightly $\beta$-closed.

Theorem 5.4: If $f$ is [almost-] slightly $\beta$-closed, then $\beta cl(\{f(A)\}) \subseteq f(cl\{A\})$

Proof: Let $A \subseteq X$ and $f$ is slightly $\beta$-closed gives $f(cl\{A\})$ is $\beta$-closed in $Y$ and $f(A) \subseteq f(cl\{A\})$ which in turn gives
\[
\beta cl(\{f(A)\}) \subseteq \beta cl(\{f(cl\{A\})\})
\] (1)
Since $f(cl\{A\}$) is $\beta$-closed in $Y$, $\beta cl(\{f(cl\{A\})\}) = f(cl\{A\}$) (2)
From (1) and (2) we have ($\beta cl(\{f(A)\}) \subseteq f(cl\{A\}$) for every subset $A$ of $X$.

Remark 5: converse is not true in general.

Theorem 5.5: If $f$ is slightly $\beta$-closed and $A \subseteq X$ is $r$-closed, then $f(A)$ is $\tau_p$-closed in $Y$.

Proof: Let $A \subseteq X$ and $f$ is slightly $\beta$-closed implies $\beta cl(\{f(A)\}) \subseteq f(cl\{A\})$ which in turn implies $\beta cl(\{f(A)\}) \subseteq f(A)$, since $f(A) = f(cl\{A\})$. But $f(A) \subseteq f(cl\{f(A)\})$. Combining we get $f(A) = (\beta cl(\{f(A)\})$. Hence $f(A)$ is $\tau_p$-closed in $Y$.

Corollary 5.2:
(i) If $f$ is [almost-] slightly $r$-closed then $\beta cl(\{f(A)\}) \subseteq f(cl\{A\})$
(ii) If $f$ is [almost-] slightly $r$-closed, then $f(A)$ is closed in $Y$ if $A$ is $r$-closed set in $X$.
(iii) If $f$ is almost slightly $\beta$-closed and $A \subseteq X$ is $r$-closed, then $f(A)$ is $\tau_p$-closed in $Y$.

Theorem 5.6: If $\beta cl\{A\}) = r(cl\{A\})$ for every $A \subseteq Y$, then the following are equivalent:
(i) $f$ is [almost-] slightly $\beta$-closed map
(ii) $\beta cl(\{f(A)\}) \subseteq f(cl\{A\})$

Proof: (i) $\Rightarrow$ (ii) follows from theorem 5.4.
(ii) $\Rightarrow$ (i) Let $A$ be any $r$-closed set in $X$, then $f(A) = f(cl\{A\}) \supseteq (\beta cl(\{f(A)\})$ by hypothesis. We have $f(A) \subseteq (\beta cl(\{f(A)\})$. Combining we get $f(A) = (\beta cl(\{f(A)\}) = r(cl\{f(A)\})$ by given condition which implies $f(A)$ is $r$-closed and hence closed. Thus $f$ is slightly $\beta$-closed.
**Theorem 5.7**\[5.14\]: \( f \) is [almost]-slightly \( \beta \)-closed iff for each subset \( S \) of \( Y \) and each \( r \)-clopen set \( U \) containing \( f^{-1}(S) \), there is a \( \beta \)-closed set \( V \) of \( Y \) such that \( S \subseteq V \) and \( f^{-1}(V) \subseteq U \).

**Remark 6**: composition of two [almost-] slightly \( \beta \)-closed maps is not [almost-] slightly \( \beta \)-closed in general.

**Theorem 5.8**: Let \( X, Y, Z \) be topological spaces and every closed set is \( r \)-clopen in \( Y \), then the composition of two [almost-] slightly \( \beta \)-closed maps is [almost-] slightly \( \beta \)-closed.

**Proof**: Let \( A \) be \( r \)-clopen in \( X \Rightarrow f(A) \) is closed and so \( r \)-clopen in \( Y \) [by assumption] \( \Rightarrow g(f(A)) \) is closed in \( Z \). Hence \( g \circ f \) is almost slightly \( \beta \)-closed.

**Theorem 5.9**: If \( f \) is [almost-]slightly \( g \)-closed; \( g \) is closed[\( r \)-closed] and \( Y \) is \( T_{\beta}[r-T_{\beta}] \), then \( g \circ f \) is [almost-]slightly \( \beta \)-closed.

**Proof**: (i) Let \( A \) be \( r \)-clopen in \( X \Rightarrow A \) be clopen in \( X \Rightarrow f(A) \) is \( g \)-closed in \( Y \Rightarrow f(A) \) is closed in \( Y \) [since \( Y \) is \( T_{\beta} \)] \( \Rightarrow g(f(A)) = g \circ f(A) \) is closed in \( Z \). Hence \( g \circ f \) is [almost-] slightly \( \beta \)-closed.

**Corollary 5.3**: (i) If \( f \) is [almost-]slightly \( g \)-closed; \( g \) is closed[\( r \)-closed] and \( Y \) is \( T_{\beta}[r-T_{\beta}] \) then \( g \circ f \) is [almost-] slightly \( \beta \)-closed.

(ii) If \( f \) is [almost-] slightly \( g \)-closed; \( g \) is [almost-] \( \beta \)-closed [\( \beta \)-closed] and \( Y \) is \( T_{\beta}[r-T_{\beta}] \) then \( g \circ f \) is [almost-] slightly \( \beta \)-closed.

**Theorem 5.10**: If \( f \) is [almost-]slightly \( r \)-\( g \)-closed; \( g \) is closed[\( r \)-closed] and \( Y \) is \( r-T_{\beta} \), then \( g \circ f \) is [almost-] slightly \( \beta \)-closed.

**Proof**: Let \( A \) be \( r \)-clopen in \( X \Rightarrow A \) be clopen in \( X \Rightarrow f(A) \) is \( r \)-\( g \)-closed and so \( r \)-closed in \( Y \) [since \( Y \) is \( r-T_{\beta} \)] \( \Rightarrow g(f(A)) = g \circ f(A) \) is closed in \( Z \). Hence \( g \circ f \) is almost slightly \( \beta \)-closed.

**Theorem 5.11**: If \( f \) is [almost-]slightly \( r \)-\( g \)-closed; \( g \) is [almost-] \( \beta \)-closed [\( \beta \)-closed] and \( Y \) is \( r-T_{\beta} \), then \( g \circ f \) is [almost-] slightly \( \beta \)-closed.

**Proof**: Let \( A \) be \( r \)-clopen in \( X \Rightarrow A \) be clopen in \( X \Rightarrow f(A) \) is \( r \)-\( g \)-closed and so \( r \)-closed in \( Y \) [since \( Y \) is \( r-T_{\beta} \)] \( \Rightarrow g(f(A)) = g \circ f(A) \) is closed in \( Z \). Hence \( g \circ f \) is almost slightly \( \beta \)-closed.

**Corollary 5.4**: (i) If \( f \) is [almost-] slightly \( r \)-\( g \)-closed; \( g \) is closed[\( r \)-closed] and \( Y \) is \( r-T_{\beta} \), then \( g \circ f \) is [almost-] slightly \( \beta \)-closed.

(ii) If \( f \) is [almost-] slightly \( r \)-\( g \)-closed; \( g \) is [almost-] \( \beta \)-closed [\( \beta \)-closed] and \( Y \) is \( r-T_{\beta} \), then \( g \circ f \) is [almost-] slightly \( \beta \)-closed.

**Theorem 5.12**: If \( f, g \) be two mappings such that \( g \circ f \) is [almost-] slightly \( \beta \)-closed [\( \beta \)-closed] then the following are true

(i) If \( f \) is continuous[\( r \)-continuous] and surjective, then \( g \) is [almost-] slightly \( \beta \)-closed

(ii) If \( f \) is [almost-] slightly \( g \)-continuous, surjective and \( X \) is \( T_{\beta} \), then \( g \) is [almost-] slightly \( \beta \)-closed

(iii) If \( f \) is \( r \)-\( g \)-continuous, surjective and \( X \) is \( r-T_{\beta} \), then \( g \) is [almost-] slightly \( \beta \)-closed

**Proof**: Let \( A \) be regular clopen in \( Y \Rightarrow A \) be clopen in \( Y \Rightarrow f^{-1}(A) \) is closed in \( X \Rightarrow g \circ f^{-1}(A) = g(A) \) is closed in \( Z \). Hence \( g \) is almost slightly \( \beta \)-closed.

Similarly we can prove the remaining parts and so omitted.

**Corollary 5.5**: If \( f, g \) be two mappings such that \( g \circ f \) is [almost-] slightly \( \beta \)-closed [\( \beta \)-closed] then the following are true

(i) If \( f \) is continuous[\( r \)-continuous] and surjective, then \( g \) is [almost-] slightly \( \beta \)-closed.

(ii) If \( f \) is \( r \)-\( g \)-continuous, surjective and \( X \) is \( T_{\beta} \), then \( g \) is [almost-] slightly \( \beta \)-closed.

(iii) If \( f \) is \( r \)-\( g \)-continuous, surjective and \( X \) is \( r-T_{\beta} \), then \( g \) is [almost-] slightly \( \beta \)-closed.

**Theorem 13**: If \( X \) is regular, \( f \) is \( r \)-closed, nearly-continuous, closed surjection and \( \tilde{A} = A \) for every closed \( r \)-closed set in \( Y \), then \( Y \) is regular.

**Theorem 14**: If \( f \) is [almost-] slightly \( \beta \)-closed and \( A \) is \( r \)-clopenclopen set of \( X \), then \( f_{A} \) is [almost-] slightly \( \beta \)-closed.
Proof: For F, r-closed in A, then F = A ∩ E is r-closed in X for some r-closed set E of X which implies f(A) is closed in Y. But f(F) = f_α(F). Therefore f_α is [almost-] slightly β-closed.

**Theorem 5.15:** If f is [almost-] slightly β-closed, X is Tᵢ, and A is g-closed set of X, then f_α is [almost-] slightly β-closed.

**Corollary 5.6:** If f is [almost-] slightly β-closed, X is Tᵢ, and A is g-closed set of X, then f_α is [almost-] slightly β-closed.

**Theorem 5.16:** If f: Xᵢ → Yᵢ be [almost-] slightly β-closed for i = 1, 2. Let f': Xᵢ × Xᵢ → Yᵢ × Yᵢ be defined as f'(xᵢ, xᵢ') = (f(xᵢ), f(xᵢ')). Then f: Xᵢ × Xᵢ → Yᵢ × Yᵢ is [almost-] slightly β-closed.

**Proof:** Let Uᵢ× Uᵢ ⊂ Xᵢ × Xᵢ where Uᵢ ∈ RCO(Xᵢ) for i = 1, 2. Then f(Uᵢ× Uᵢ) = f_1(Uᵢ) × f_2(Uᵢ) a closed set in Yᵢ × Yᵢ. Thus f(Uᵢ× Uᵢ) is closed and hence f is [almost-] slightly β-closed.

**Corollary 5.7:** If f: Xᵢ → Yᵢ be [almost-] slightly β-closed for i = 1, 2. Let f: Xᵢ × Xᵢ → Yᵢ × Yᵢ be defined as f'(xᵢ, xᵢ') = (f(xᵢ), f(xᵢ')). Then f: Xᵢ × Xᵢ is [almost-] slightly β-closed for i = 1, 2.

**Theorem 5.17:** Let h: X → Xᵢ × Xᵢ be [almost-] slightly β-closed. Let f': X → Xᵢ be defined as h(x) = (xᵢ, xᵢ) and f(x) = xᵢ. Then f: X → Xᵢ is [almost-] slightly β-closed for i = 1, 2.

**Proof:** Let Uᵢ be r-clopen in Xᵢ, then Uᵢ × Xᵢ is r-clopen in Xᵢ × Xᵢ, and h(Uᵢ) is closed in Xᵢ. But f(Uᵢ) = h(Uᵢ), therefore f is [almost-] slightly β-closed. Similarly we can show that f_2 is [almost-] slightly β-closed and thus f: X → Xᵢ is [almost-] slightly β-closed for i = 1, 2.

**Corollary 5.8:** Let h: X → Xᵢ × Xᵢ be [almost-] slightly β-closed. Let f': X → Xᵢ be defined as h(x) = (xᵢ, xᵢ') and f'(x) = xᵢ. Then f': X → Xᵢ is [almost-] slightly β-closed for i = 1, 2.

### 6. COVERING AND SEPARATION PROPERTIES OF al.sl.β.c. and al.swt.β.c. FUNCTIONS

**Theorem 6.1:** If f is al.sl.β.c.[resp: al.sl.r.c] surjection and X is β-compact, then Y is compact.

**Proof:** Let {Gᵢ ∋ i} be any r-clopen cover for Y. Then each Gᵢ is r-clopen in Y and f is al.sl.β.c., f⁻¹(Gᵢ) is β-open in X. Thus {f⁻¹(Gᵢ)} forms a β-open cover for X with a finite subcover, since X is β-compact. Since f is surjection, Y = f(X) = ∪ᵢ Gᵢ. Therefore Y is compact.

**Theorem 6.2:** If f is al.sl.β.c., surjection and X is β-compact[β-Lindeloff] then Y is mildly compact[β-Lindeloff].

**Proof:** Let {Uᵢ ∋ i} be r-clopen cover for Y. For each x in X, ∃ αₓ ∈ I such that f(x) ∈ Uₓ and ∃ Vₓ ∈ βO(X, x) ∋ f(Vₓ). Since {Vₓ ∋ i} is a β-open cover of X, ∃ a finite subset I₀ of I such that X ⊂ {Vₓ | x ∈ I₀}. Thus Y ⊂ ∪ᵢ(f(Vₓ) ∋ i) ⊂ ∪ₓ(Uₓ ∋ i). Hence Y is mildly compact.

**Corollary 6.1:**
(i) If f is al.sl.r.c. surjection and X is β-compact, then Y is compact.
(ii) If f is al.sl.β.c.[resp: al.sl.r.c] surjection and X is locally β-compact[resp: β-Lindeloff, locally β-Lindeloff], then Y is locally compact[resp: Lindeloff, locally Lindeloff, locally mildly compact; locally mildly Lindeloff].
(iii) If f is al.sl.β.c., [resp: al.sl.r.c] surjection and X is β-compact[β-Lindeloff] then Y is mildly compact[β-Lindeloff].

**Theorem 6.3:** If f is al.sl.β.c., surjection and X is s-closed then Y is mildly compact[β-Lindeloff].

**Proof:** Let {Vᵢ | Vᵢ ∈ RCO(Y); i ∈ I} be a cover of Y, then {f⁻¹(Vᵢ) ∋ i} is β-open cover of X and so there is finite subset I₀ of I, such that {f⁻¹(Vᵢ) | i ∈ I₀} covers X. Therefore {Vᵢ | i ∈ I₀} covers Y since f is surjection. Hence Y is mildly compact.

**Theorem 6.4:** If f is al.sl.β.c.[al.sl.r.c.] surjection and X is β-connected, then Y is connected.

**Proof:** If Y is disconnected, then Y = A ∪ B where A and B are disjoint r-clopen sets in Y. Since f is al.sl.β.c. surjection, X = f⁻¹(Y) = f⁻¹(A) ∪ f⁻¹(B) where f⁻¹(A) and f⁻¹(B) are disjoint β-open sets in X, which is a contradiction for X is β-connected. Hence Y is connected.
Corollary 6.2: 
(i) If $f$ is al.sl.c[resp: al.sl.r.c] surjection and $X$ is $s$-closed then $Y$ is mildly compact[mildly lindeloff].
(ii)The inverse image of a disconnected space under an al.sl.$\beta$.c.[resp: al.sl.r.c.] surjection is $\beta$-disconnected.

Theorem 6.5: If$f$ is al.sl.$\beta$.c.[resp: al.sl.r.c.], injection and $Y$ is $\beta T_i$, then $X$ is $\beta T_i$, $i = 0, 1, 2$.

Proof: Let $x_i \neq x_i \in X$. Then $f(x_i) \neq f(x_i) \in Y$ since $f$ is injective. For $Y$ is $\beta T_i \exists V_j \in \text{RCO}(Y)$ such that $f(x_i) \in V_j$ and $\cap V_j = \emptyset$ for $j = 1, 2$. By Theorem 3.1, $x_j \in f^{-1}(V_j) \in \beta O(X)$ for $j = 1, 2$ and $\cap f^{-1}(V_j) = \emptyset$ for $j = 1, 2$. Thus $X$ is $\beta T_i$.

Theorem 6.6: If $f$ is al.sl.$\beta$.c.[resp: al.sl.r.c.] injection; $r$-closed and $Y$ is $\beta T_i$, then $X$ is $\beta T_i$, $i = 3, 4$.

Proof: 
(i) Let $x$ in $X$ and $F$ be disjoint $r$-closed subset of $X$ not containing $x$, then $f(x)$ and $f(F)$ be disjoint $r$-closed subset of $Y$ not containing $f(x)$, since $f$ is $r$-closed and injection. Since $Y$ is ultraregular, $f(x)$ and $f(F)$ are separated by disjoint $r$-closed sets $U$ and $V$ respectively.

Hence $x \in f^{-1}(U)$; $f^{-1}(V)$, $f^{-1}(U)$; $f^{-1}(V) \in \beta O(X)$ and $f^{-1}(U) \cap f^{-1}(V) = \emptyset$. Thus $X$ is $\beta T_i$.

(ii) Let $F_j$ and $f(F_j)$ are disjoint $r$-closed sets in $X$ and $Y$ respectively for $j = 1, 2$, since $f$ is $r$-closed and injection. For $Y$ is ultranormal, $f(F_j)$ are separated by disjoint $r$-closed sets $V_j$ respectively for $j = 1, 2$. Hence $F_j \subseteq f^{-1}(V_j)$ and $f^{-1}(V_j) \in \beta O(X)$ and $\cap f^{-1}(V_j) = \emptyset$. Thus $X$ is $\beta T_i$.

Theorem 6.7: If$f$ is al.sl.$\beta$.c.[resp: al.sl.r.c.], injection and 
(i) $Y$ is $\beta R_i[C[resp: \beta D_i], i = 0, 1, 2$.
(ii)$Y$ is $\beta R_i$, then $X$ is $\beta R_i$, $i = 1, 0$.

Theorem 6.8: If $f$ is al.sl.$\beta$.c.[resp: al.sl.r.c. and $Y$ is $\beta T_2$, then the graph $G(f)$ is $\beta$-closed in $X \times Y$.

Proof: Let $(x_1, x_2) \in G(f)$ implies $\forall x \in X \times Y$ such that $f(x_1) \in V$ and $x \in V$. Since $f$ is al.sl.$\beta$.c., $\exists U \in \beta O(X)$ such that $x \in U$ and $f(U) \subseteq W$ and $(x, y) \in U \times V \subseteq X \times Y - G(f)$. Hence $G(f)$ is $\beta$-closed in $X \times Y$.

Theorem 6.9: If $f$ is al.sl.$\beta$.c.[resp: al.sl.r.c.] and $Y$ is $\beta T_2$, then $A = \{(x_1, x_2) | f(x_1) = f(x_2)\}$ is $\beta$-closed in $X \times X$.

Proof: If $(x_1, x_2) \in X \times X - A$, then $f(x_1) \neq f(x_2)$ implies $\exists$ disjoint $V_j \in \text{RCO}(Y)$ such that $(x_1) \in V_j$, and since $f$ is al.sl.$\beta$.c., $f^{-1}(V_j) \in \beta O(X, x_1)$ for $j = 1, 2$. Thus $(x_1, x_2) \notin f^{-1}(V_1) \times f^{-1}(V_2) \subseteq \beta O(\times X)$ and $f^{-1}(V_1) \times f^{-1}(V_2) \subseteq X \times X - A$. Hence $A$ is $\beta$-closed.

Theorem 6.10: If $f$ is al.sl.r.c.[resp: al.sl.$\beta$.c.]; $g$ is al.sl.c.[resp: al.sl.r.c.]; and $Y$ is $\beta T_2$, then $E = \{x \in X : f(x) = g(x)\}$ is $\beta$-closed in $X$.

We have the following consequences of theorems 6.1 to 6.10:

Theorem 6.11: If $f$ is al.swt.$\beta$.c.[resp: al.swt.r.c.] surjection and $X$ is $\beta$-compact, then $Y$ is compact.

Theorem 6.12: If $f$ is al.swt.$\beta$.c., surjection and $X$ is $\beta$-compact[resp: $\beta$-Lindeloff] then $Y$ is mildly compact[mildly lindeloff].

Corollary 6.3 : 
(i) If $f$ is al.swt.r.c. surjection and $X$ is $\beta$-compact, then $Y$ is compact.
(ii) If $f$ is al.swt.$\beta$.c.[resp: al.swt.r.c.] surjection and $X$ is $\beta$-compact[resp: $\beta$-Lindeloff] then $Y$ is mildly compact[mildly lindeloff].
(iii) If $f$ is al.swt.$\beta$.c.[resp: al.swt.r.c.] surjection and $X$ is locally $\beta$-compact[resp: $\beta$-Lindeloff; locally $\beta$-Lindeloff], then $Y$ is locally compact[resp: Lindeloff; locally lindeloff; locally mildly compact; locally mildly lindeloff].

Theorem 6.13: If $f$ is al.swt.$\beta$.c., surjection and $X$ is $s$-closed then $Y$ is mildly compact[mildly lindeloff].

Theorem 6.14: If $f$ is al.swt.$\beta$.c.[resp: al.swt.r.c.] surjection and $X$ is $\beta$-connected, then $Y$ is connected.

Corollary 6.4: 
(i) If $f$ is al.swt.c.[resp: al.swt.r.c.] surjection and $X$ is $s$-closed then $Y$ is mildly compact[mildly lindeloff].
(ii) The inverse image of a disconnected space under an al.swt.$\beta$.c.[resp: al.swt.r.c.] surjection is $\beta$-disconnected.
Theorem 6.15: 
(i) If $f$ is al.swt.$\beta$.c.[al.swt.r.c.], injection and Y is UrT$_i$, then X is $\beta$T$_i$ i = 0, 1, 2. 
(ii) If $f$ is al.swt.$\beta$.c.[al.swt.r.c.] injection; r-closed and Y is UrT$_i$, then X is $\beta$T$_i$ i = 3, 4.

Theorem 6.16: If $f$ is al.swt.$\beta$.c.[resp: al.swt.r.c.], injection and 
(i) Y is UrCi[resp: UrDi] then X is $\beta$C$_i$[resp: $\beta$D$_i$] i = 0, 1, 2. 
(ii) Y is UrRi, then X is $\beta$Ri i = 0, 1.

Theorem 6.17: If $f$ is al.swt.$\beta$.c.[resp: al.swt.r.c.] and Y is UrT$_2$, then 
(i) the graph G($f$) is $\beta$-closed in X$\times$Y. 
(ii) $A = \{(x_1, x_2)\mid f(x_1) = f(x_2)\}$ is $\beta$-closed in X$\times$X.

Theorem 6.18: If $f$ is al.swt.r.c.[resp: al.swt.$\beta$.c.]; g: X$\rightarrow$ Y is al.swt.c[resp: al.swt.r.c]; and Y is UrT$_2$, then 
E = \{x in X: f(x) = g(x)\} is $\beta$-closed in X.

CONCLUSION

In this paper we introduced the concept of almost slightly $\beta$-continuous functions, almost somewhat $\beta$-continuous functions, somewhat $\beta$-open mappings, slightly $\beta$-open mappings, almost slightly $\beta$-open mappings, slightly $\beta$-closed mappings, almost slightly $\beta$-closed mappings, studied their basic properties and the interrelationship between other such maps.

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