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A STUDY ON MATHEMATICAL MODELLING FOR OLDEST-OLD MORTALITY RATES

Pallabi Saikia[1*] and Munindra Borah[2]

[1,2]Department of Mathematical Sciences, Tezpur University, Napaam, Tezpur: 784001, Assam, India.

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ABSTRACT

Five mortality models: Gompertz, Makeham, Logistic, Beard and Kannisto models have been considered for this investigation. An attempt has been made to study certain mathematical properties of these above models. The first derivative of the model describes the slope of the curve or the rate of change of the dependent variable with respect to the independent variable, whereas the second derivative of the models quantifies the rate of mortality. The main objective of this paper is to select the best fit model for projection of oldest-old mortality rates at ages 80-110 based on certain mortality data for a specific period. Using the complete life table of Japan (2005 and 2010) as input, the parameters of these mortality models have been estimated using Levenberg – Marquardt iteration procedure. The estimated parameters of these models have been used for the testing the validity of the models and for the projection of mortality rates at ages 120. Matlab version 7.11.0 has been used for the estimation of the parameters. It is observed from our result that the four parameter logistic model fits oldest-old ages mortality data better than the Gompertz and three other models.

***Key Words:** Force of mortality, Probability of dying, Gompertz, Makeham, Logistic, Beard, Kannisto models, parameter estimation.*

***Subject Classification**: 62-07, 62J02, 62N02, 62Q05.*

1. INTRODUCTION

The description of observed age patterns of mortality with mathematical models is one of the oldest and most important topics in demography. Many attempts have been made to find mathematical formulae that will summarise the way in which the probability of dying depends on age. Such formulae have many potential applications. For example, they may be useful in the projection of population numbers and as aids in actuarial work such as the construction of life tables. The first explanatory model, and the most influential parametric mortality modelling the literature, is that proposed by Benjamin Gompertz [8]. He recognised that an exponential pattern in age captured the behaviour of human mortality for large portions of the life table [9]. Over much of the age range; this model still gives an excellent approximation. At higher ages, however the law does not work so well. The paper Bongaarts [2] discussed that for many purposes the Gompertz model provides a satisfactory fit to adult mortality rates, but this model underestimate of actual mortality at youngest adult ages (under 40) and overestimate at the oldest ages (over 80). Ever since Gompertz, many models have been suggested to mathematically describe survival and mortality curves [13], of which the Gompertz model and the Weibull model are the most generally used at present [6,7]. The law of Makeham [11], which improves on the Gompertz law by adding a further term which does not depend on age.The paper Bongaarts [2] discussed that the Makeham model represents a clear improvement over the Gompertz model at younger ages, but it still overestimates mortality at the oldest ages. Several models which are known under different names but which are essentially the same, because in all of them the force of mortality is a logistic function of age. These include the models by Perks [12], by Beard [1]. We shall describe these models collectively as the logistic model. There is also a special case of the logistic model which holds approximately, at least modern data, which was noted by Kannisto [10].The paper by Doray[4] discussed that logistic type models for the force of mortality provide better fit to mortality data of people aged over 85 than Makeham's models where the force of mortality increases exponentially with age.

In this paper, the following models namely Gompertz, Makeham, Logistic, Beard, and Kannisto models will be considered in our investigation that to find a suitable mortality model for oldest-old mortality rates. The main focus of this paper is to select the best fit model for projection of oldest-old mortality rates at ages 80-110 based on the complete life table of Japan (2005 and 2010).

Corresponding author: Pallabi Saikia[1]*

2. MODELS FOR THE FORCE OF MORTALITY

The very first attempt to develop a parametric model of mortality was that of Gompertz [8]. Gompertz modelled the aging or senescent component of mortality with two parameters: a positive scale parameter a that a varies with level of mortality, and a positive shape parameter b that measures the rate of increase in mortality with age. The force of mortality in the Gompertz model is

$$\mu_x = ae^{bx} \tag{1}$$

The earliest modification to the Gompertz model, proposed by Makeham [11], involves adding a constant term, so that

$$\mu_x = c + ae^{bx} \tag{2}$$

The new parameter $c$represents mortality resulting from causes, such as accidents or sexually transmitted diseases, unrelated to either maturation or senescence, which is the same for all ages.

The logistic model is known under a variety of names. It was first discovered by Perks [12], who found empirically that the values of μ_x in a life table which he was examining could be fitted by a certain curve, which was in fact a logistic function (though he did not describe it as such at the time). Here we take the logistic function in the following form:

$$\mu_x = c + \frac{ae^{bx}}{1+de^{bx}} \tag{3}$$

Note that the Makeham model ($d = 0$) is a special case of the logistic model. When $d$is small, any theories which may explain why should follow a logistic function will also help to explain why the Makeham and Gompertz laws work so well over much of the age range.

By assuming that the parameter $c = 0$ in (3), Beard obtained the three-parameter model

$$\mu_x = \frac{ae^{bx}}{1+de^{bx}} \tag{4}$$

Beard, who was a colleague of Perks, wrote several papers on this subject that were summarised in a paper published in 1971. He identified (3) as a logistic curve and showed how it could arise in a simple model of a heterogeneous population. If the members of the population are subject to hazards of the Makeham form (2), but with the parameter a varying from individual to individual in such a way that they have a gamma distribution at birth, then the average value of for the survivors who reach age will have the logistic form (3). Beard also showed how the logistic curve could arise from a very simple type of stochastic process which assumed that individuals accumulate "shots" from random firings and is assumed to be dead when the total reaches a given figure. Special assumptions were about initial conditions.

Kannisto**[10],** a demographer, used the simple 2-parameter model

$$\mu_x = \frac{ae^{bx}}{1+ae^{bx}} \tag{5}$$

Note that the Gompertz($c = 0, d = 0$), Makeham ($d = 0$), Beard ($c = 0$) and Kannisto ($c = 0, d = a$) models are all special cases of the logistic model.

3. MATERIAL AND METHODS

The six mortality models i.e. Gompertz, Makeham, Logistic, Beard and Kannisto have been considered for this study. These nonlinear models can be written in the form as

$$\mu_i = f(x_i, \mathbf{B}) + \varepsilon_i, \tag{6}$$

$i = 1,2,\cdots,n$, where μ is the response variables, x is the independent variable, **B**is the vector of parameters $\beta_j$to be estimated $(\beta_1, \beta_2, \cdots, \beta_p)$, ε_i is a random error term,p is the number of unknown parameters, and n is the number of observations[3]. The estimators ofβ_j's are found by minimizing the sum of squares residual(SS_{Res}) function

$$(SS_{Res}) = \sum_{i=1}^{n}[\mu_i - f(x_i, \mathbf{B})]^2 \tag{7}$$

under the assumption that the ε_i are normal and independent with mean zero and common variance σ^2. Since μ_i and x_i are fixed observations, the sum of squares residual is a function of **B**. Least squares estimates of **B**are values which

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when substitutedinto equations (7) will make the (SS_{Res}) a minimum and are found by differentiating equations (7) with respect to each parameter and setting the result to zero. This provides the p normal equations that must be solved for $\widehat{\mathbf{B}}$. These normal equations take the form

$$\sum_{i=1}^{n}\{\mu_i - f(x_i,\mathbf{B})\}\left[\frac{\partial f(x_i,\mathbf{B})}{\partial\beta_j}\right] = 0 \qquad (8)$$

for $j = 1,2,\cdots,p$[5]. Here the parameters are estimated using the Levenverg - Marquardt iteration method based on an empirical data set of complete life table males, Canada 2007 to 2009. The Levenberg–Marquardt algorithmalso known as the damped least-squaresmethod, provides a numerical solution to the problem of minimizing a function, generally nonlinear, over a space of parameters of the function. The Levenberg–Marquardt algorithm is more robust than the Gauss–Newton, which means that in many cases it finds a solution even if it starts very far off the final minimum. Matlab version 7.11.0 has been used for the estimation of the parameters. The best fit model has been selected on the basis of root mean square error (RMSE).log is the natural logarithms and exp (e) is the base of the natural logarithms.

4.1 Mathematical properties of the models

The Gompertz curve can be written as

$$y = ae^{bx} \qquad (9)$$

where $a > 0, b > 0$

From (9) it is clear that as x becomes positively infinite y will approach infinite. Differentiating (10) w.r.t x we have

$$\frac{dy}{dx} = abe^{bx} > 0 \qquad (10)$$

From (10) we see that the slope is always positive. Differentiating (10) we have

$$\frac{d^2y}{dx^2} = ab^2e^{bx} > 0 \qquad (11)$$

From (11) we see that there will be no point of inflection and the curve is concave up. $\frac{dy}{dx}$ is an increasing function and the function itself must be concave up. The mathematical properties of the other models have been summarized in the following table 1.

It is observed from the table 1 that $\frac{dy}{dx}$ is an increasing function for all the models. From the table 1 we also see that those models (Logistic, Beard, and Kannisto) follow a logistic-type curve for the force of mortality, i.e., as age increases, y tends asymptotically to a constant. This asymptote is equal to 1 for the Kannisto model and $\frac{a}{d}$ for the Beard model and $c+\frac{a}{d}$ for the logistic models.

The Beard and Kannisto models possess same number of parameters as the Makeham model, but have the point of inflections.

Table 1: Summarizes some mathematical properties of the models

Models	Equation	No. of parameters	Asymptote	Point of Inflection	First order derivative
Gompertz	$y = ae^{bx}$	2	Non-asymptotic	No point of inflection	$\frac{dy}{dx} = abe^{bx}$
Makeham	$y = c + ae^{bx}$	3	Non-asymptotic	No point of inflection	$\frac{dy}{dx} = abe^{bx}$
Logistic	$y = c + \frac{ae^{bx}}{1+de^{bx}}$	4	$c + \frac{a}{d}$	$\begin{cases} x = \frac{1}{b}\ln\left(\frac{1}{d}\right) \\ y = c + \frac{a}{2d} \end{cases}$	$\frac{dy}{dx} = \frac{abe^{bx}}{(1+de^{bx})^2}$
Beard	$y = \frac{ae^{bx}}{1+de^{bx}}$	3	$\frac{a}{d}$	$\begin{cases} x = \frac{1}{b}\ln\left(\frac{1}{d}\right) \\ y = \frac{a}{2d} \end{cases}$	$\frac{dy}{dx} = \frac{abe^{bx}}{(1+de^{bx})^2}$
Kannisto	$y = \frac{ae^{bx}}{1+ae^{bx}}$	3	1	$\begin{cases} x = \frac{1}{b}\ln\left(\frac{1}{a}\right) \\ y = \frac{1}{2} \end{cases}$	$\frac{dy}{dx} = \frac{abe^{bx}}{(1+ae^{bx})^2}$

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4.2 Estimated parameters

Table 2 and table 3 contain the values of the parameters for Male people estimated with the Levenverg - Marquardt iteration method for the period 2005 and 2010 respectively. Similarly table 4 and table 5 represents the values of the parameters for Female peoplefor the period 2005 and 2010 respectively.

Table 2: Estimated parameters of the models along with confidence bounds for Males at ages 80-110 for **2005**

Parameters along with errors	Confidence bounds of parameters (95%)	Gompertz (a,b)	Makeham (a,b,c)	Logistic (a,b,c,d)	Beard (a,b,d)	Kannisto (a,b)
a	$\hat{a}$	0.2653	0.3491	0.3381	0.3016	0.3793
	L.B	0.259	0.3423	0.3323	0.2962	0.3499
	U.B	0.2717	0.3559	0.344	0.307	0.4088
b	$\hat{b}$	0.7361	0.6063	0.7104	0.9177	1.337
	L.B	0.7164	0.5971	0.6765	0.8916	1.248
	U.B	0.7557	0.6154	0.7444	0.9438	1.427
c	$\hat{c}$		-0.07665	-0.04819		
	L.B		-0.08277	-0.05712		
	U.B		-0.07053	-0.03925		
d	$\hat{d}$			0.05927	0.13	
	L.B			0.04256	0.1138	
	U.B			0.07598	0.1462	
RMSE		0.01368	0.00238	0.00157	0.00457	0.02942
SSE		0.00543	0.00016	0.00007	0.00058	0.02509
R-square		0.9969	0.9998	0.9999	0.9997	0.9858

Table 3: Estimated parameters of the models along with confidence bounds for Males at ages 80-110 for **2010**

Parameters along with errors	Confidence bounds of parameters (95%)	Gompertz (a,b)	Makeham (a,b,c)	Logistic (a,b,c,d)	Beard (a,b,d)	Kannisto (a,b)
a	$\hat{a}$	0.2668	0.3099	0.3079	0.2827	0.3728
	L.B	0.2626	0.3082	0.3059	0.2792	0.3161
	U.B	0.271	0.3115	0.3098	0.2862	0.4295
b	$\hat{b}$	0.82	0.7441	0.7636	0.9241	1.621
	L.B	0.8073	0.7413	0.7509	0.9039	1.432
	U.B	0.8327	0.7469	0.7764	0.9444	1.81
c	$\hat{c}$		-0.03938	-0.03506		
	L.B		-0.04084	-0.03804		
	U.B		-0.03792	-0.03208		
d	$\hat{d}$			0.008915	0.06183	
	L.B			0.003354	0.05111	
	U.B			0.01448	0.07254	
RMSE		0.00902	0.00081	0.00070	0.00392	0.05275
SSE		0.002357	0.00002	0.00001	0.00043	0.08069
R-square		0.9990	0.9997	0.9999	0.9998	0.9665

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Table 4: Estimated parameters of the models along with confidence bounds for Females at ages 80-110 for 2005

Parameters along with errors	Confidence bounds of parameters (95%)	Gompertz (a,b)	Makeham (a,b,c)	Logistic (a,b,c,d)	Beard (a,b,d)	Kannisto (a,b)
a	$\hat{a}$	0.185	0.225	0.2191	0.1958	0.2171
	L.B	0.1803	0.2217	0.2159	0.1929	0.1967
	U.B	0.1896	0.2284	0.2223	0.1987	0.2374
b	$\hat{b}$	0.9054	0.803	0.8751	1.063	1.512
	L.B	0.8855	0.795	0.8483	1.033	1.416
	U.B	0.9252	0.811	0.9019	1.093	1.608
c	$\hat{c}$		-0.03718	-0.02678		
	L.B		-0.04011	-0.03081		
	U.B		-0.03424	-0.02274		
d	$\hat{d}$			0.02713	0.07148	
	L.B			0.01807	0.05997	
	U.B			0.03619	0.08298	
RMSE		0.00995	0.00184	0.00128	0.00418	0.02482
SSE		0.00287	0.00009	0.00004	0.00049	0.01786
R-square		0.9982	0.9999	0.9999	0.9997	0.9887

Table 5: Estimated parameters of the models along with confidence bounds for Females at ages 80-110 for 2010

Parameters along with errors	Confidence bounds of parameters (95%)	Gompertz (a,b)	Makeham (a,b,c)	Logistic (a,b,c,d)	Beard (a,b,d)	Kannisto (a,b)
a	$\hat{a}$	0.192	0.242	0.2074	0.2027	0.2105
	L.B	0.184	0.2244	0.1931	0.1986	0.1827
	U.B	0.2001	0.2596	0.2218	0.2069	0.2382
b	$\hat{b}$	0.9645	0.8423	1.192	1.225	1.795
	L.B	0.9321	0.8022	1.084	1.179	1.653
	U.B	0.997	0.8823	1.3	1.272	1.937
c	$\hat{c}$		-0.04723	-0.004079		
	L.B		-0.06269	-0.01619		
	U.B		-0.03177	0.008037		
d	$\hat{d}$			0.08843	0.09201	
	L.B			0.0709	0.07931	
	U.B			0.106	0.1047	
RMSE		0.01713	0.01045	0.00673	0.00667	0.03252
SSE		0.00851	0.00306	0.00122	0.00125	0.03066
R-square		0.996	0.9985	0.9994	0.9994	0.9854

Table 6: Projected value of the force of mortality at age 120 for males

Model	Based on Period 2005	Period 2010
	$\mu(120)$	$\mu(120)$
Logistic	1.682	2.308
Makeham	1.773	2.359
Beard	1.435	2.011

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Table 7: Projected value of the force of mortality at age 120 for females

Model	Period 2005	Period 2010
	$\mu(120)$	$\mu(120)$
Logistic	1.842	1.640
Makeham	2.010	2.406
Beard	1.563	1.603

5. RESULTS AND DISCUSSION

Based on complete life table of Japan (2005 and 2010), we have fitted Gompertz, Makeham, Logistic, Beard and Kannisto mortality models to force of mortality from ages 80 to 110,by estimating the parameters of the models using Levenverg-Marquardt iteration method.It is observed fromour result that the four parameter Logistic model is the most successful in describing the trajectory of oldest old mortality for both male and female for the year 2005. For the year 2010, both Beard and logistic model give almost same values of RMSE, SSE and R^2. The fitting of Logistic and Beard model to data at ages 80-110 are almost identical for both the year.Hence it is not easy to choose which one is better.However, Makeham and Beard model fit the data much better than the exponential Gompertz model Kannisto model. It is also to be noted that allother models namely Gompertz, Makeham, Logistic, Beard give better result than the two parameter Kannisto model for the estimation of force of mortality.Three models Logistic, Makeham and Beard have been selected for the projection of oldest-old mortality rates at ages 80-110 based on mortality data of Japan.

Data source: Vital, Health and Social Statistics Division, Statistics and Information Department, Minister's Secretariat, Ministry of Health, Labour and Welfare, Japan.

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Appendix A
Estimated values of force of mortality

Table 8: Estimated values of force of mortality for male people for the year 2005

Age	Observed values	Gompertz	Makeham	Kannisto	Beard	Logistic
80	0.0586	0.0788	0.05174	0.04011	0.06451	0.05465
81	0.06516	0.0854	0.0606	0.04617	0.07116	0.06284
82	0.07241	0.0926	0.07006	0.0531	0.07846	0.07168
83	0.0807	0.1004	0.08018	0.06099	0.08648	0.0812

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84	0.09007	0.1089	0.09099	0.06998	0.09529	0.09145
85	0.10047	0.1181	0.10255	0.08018	0.10495	0.1025
86	0.11202	0.128	0.11491	0.09171	0.11554	0.1144
87	0.12495	0.1388	0.12812	0.10472	0.12714	0.12721
88	0.13916	0.1505	0.14224	0.11933	0.13983	0.14099
89	0.15401	0.1632	0.15733	0.13567	0.15369	0.15581
90	0.17097	0.177	0.17347	0.15386	0.16883	0.17175
91	0.18873	0.1919	0.19072	0.17399	0.18533	0.18888
92	0.20767	0.2081	0.20915	0.19614	0.20328	0.20728
93	0.22787	0.2256	0.22886	0.22037	0.2228	0.22704
94	0.2494	0.2447	0.24993	0.24667	0.24397	0.24825
95	0.27236	0.2653	0.27245	0.27499	0.2669	0.27099
96	0.29684	0.2877	0.29652	0.30526	0.29168	0.29537
97	0.32294	0.3119	0.32226	0.33731	0.31841	0.32148
98	0.35077	0.3382	0.34976	0.37092	0.34717	0.34943
99	0.38044	0.3668	0.37917	0.40583	0.37803	0.37932
100	0.41208	0.3977	0.4106	0.44172	0.41106	0.41126
101	0.44582	0.4312	0.4442	0.47823	0.44632	0.44537
102	0.48179	0.4676	0.48012	0.51498	0.48384	0.48175
103	0.52014	0.507	0.51851	0.55156	0.52362	0.52053
104	0.56104	0.5498	0.55955	0.58759	0.56568	0.5618
105	0.60464	0.5961	0.60342	0.62271	0.60996	0.60569
106	0.65114	0.6464	0.65032	0.65659	0.65641	0.65231
107	0.70071	0.7009	0.70045	0.68894	0.70494	0.70175
108	0.75357	0.76	0.75404	0.71955	0.75543	0.75412
109	0.80992	0.8241	0.81132	0.74825	0.80772	0.80952
110	0.87002	0.8936	0.87255	0.77492	0.86165	0.86803

Table 9: Estimated values of force of mortality for male people for the period 2010

Age	Observed values	Gompertz	Makeham	Kannisto	Beard	Logistic
80	0.05407	0.0690	0.0514	0.0251	0.0607	0.0521
81	0.06056	0.0755	0.0592	0.0298	0.0671	0.0597
82	0.0678	0.0826	0.0676	0.0354	0.0742	0.0680
83	0.07629	0.0904	0.0767	0.0420	0.0820	0.0770
84	0.08618	0.0989	0.0866	0.0498	0.0906	0.0867
85	0.09717	0.1083	0.0973	0.0590	0.1001	0.0974
86	0.10871	0.1185	0.1090	0.0697	0.1105	0.1089
87	0.12062	0.1297	0.1216	0.0822	0.1220	0.1215
88	0.13362	0.1419	0.1354	0.0967	0.1347	0.1351
89	0.14839	0.1553	0.1503	0.1134	0.1486	0.1500
90	0.16615	0.1700	0.1664	0.1326	0.1640	0.1661
91	0.18378	0.1860	0.1840	0.1545	0.1808	0.1836
92	0.2029	0.2036	0.2031	0.1792	0.1993	0.2026
93	0.22364	0.2228	0.2237	0.2070	0.2196	0.2233
94	0.24614	0.2438	0.2462	0.2378	0.2419	0.2457
95	0.27056	0.2668	0.2705	0.2716	0.2662	0.2701
96	0.29704	0.2920	0.2969	0.3082	0.2929	0.2966
97	0.32578	0.3195	0.3256	0.3475	0.3220	0.3254
98	0.35695	0.3497	0.3568	0.3889	0.3538	0.3566
99	0.39077	0.3827	0.3905	0.4320	0.3884	0.3905
100	0.42746	0.4188	0.4272	0.4762	0.4261	0.4272
101	0.46726	0.4583	0.4670	0.5207	0.4671	0.4672
102	0.51044	0.5016	0.5102	0.5650	0.5114	0.5105
103	0.55729	0.5489	0.5571	0.6082	0.5595	0.5574
104	0.60811	0.6008	0.6079	0.6497	0.6113	0.6084
105	0.66325	0.6575	0.6631	0.6892	0.6672	0.6636
106	0.72307	0.7195	0.7230	0.7260	0.7272	0.7235
107	0.78796	0.7874	0.7881	0.7600	0.7915	0.7884
108	0.85837	0.8617	0.8586	0.7910	0.8603	0.8586
109	0.93474	0.9431	0.9352	0.8190	0.9335	0.9347
110	1.01761	1.0321	1.0183	0.8439	1.0113	1.0171

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Table 10: Estimated values of force of mortality for female people for the period 2005

Age	Observed values	Gompertz	Makeham	Kannisto	Beard	Logistic
80	0.0274	0.0415	0.0226	0.0176	0.0335	0.0246
81	0.03152	0.0459	0.0282	0.0207	0.0376	0.0298
82	0.03643	0.0507	0.0342	0.0244	0.0422	0.0354
83	0.04206	0.0560	0.0408	0.0287	0.0473	0.0417
84	0.04822	0.0619	0.0480	0.0337	0.0531	0.0485
85	0.05495	0.0683	0.0558	0.0395	0.0595	0.0560
86	0.06256	0.0755	0.0644	0.0463	0.0667	0.0643
87	0.07139	0.0834	0.0738	0.0543	0.0747	0.0734
88	0.08164	0.0921	0.0841	0.0635	0.0837	0.0834
89	0.09303	0.1018	0.0953	0.0741	0.0938	0.0944
90	0.10517	0.1124	0.1075	0.0864	0.1049	0.1064
91	0.11833	0.1242	0.1209	0.1004	0.1174	0.1196
92	0.13321	0.1372	0.1354	0.1165	0.1313	0.1341
93	0.15024	0.1516	0.1514	0.1347	0.1467	0.1500
94	0.16922	0.1675	0.1688	0.1553	0.1638	0.1674
95	0.1874	0.1850	0.1878	0.1784	0.1827	0.1865
96	0.20851	0.2044	0.2086	0.2041	0.2037	0.2075
97	0.23151	0.2258	0.2313	0.2324	0.2269	0.2304
98	0.25659	0.2494	0.2561	0.2634	0.2524	0.2554
99	0.28393	0.2755	0.2832	0.2969	0.2805	0.2829
100	0.31372	0.3044	0.3127	0.3327	0.3114	0.3128
101	0.3462	0.3362	0.3450	0.3706	0.3451	0.3456
102	0.38161	0.3715	0.3803	0.4102	0.3820	0.3813
103	0.4202	0.4103	0.4189	0.4509	0.4220	0.4202
104	0.46227	0.4533	0.4610	0.4923	0.4655	0.4627
105	0.50812	0.5008	0.5070	0.5338	0.5124	0.5088
106	0.55811	0.5532	0.5572	0.5749	0.5629	0.5590
107	0.6126	0.6111	0.6121	0.6150	0.6170	0.6135
108	0.67199	0.6751	0.6721	0.6535	0.6747	0.6726
109	0.73673	0.7458	0.7376	0.6901	0.7359	0.7366
110	0.8073	0.8239	0.8091	0.7245	0.8005	0.8057

Table 11: Estimated values of force of mortality for female people for the period 2010

Age	Observed values	Gompertz	Makeham	Kannisto	Beard	Logistic
80	0.02465	0.0391	0.0131	0.0108	0.0265	0.0246
81	0.02809	0.0435	0.0189	0.0131	0.0303	0.0286
82	0.03203	0.0483	0.0253	0.0159	0.0346	0.0330
83	0.03671	0.0538	0.0324	0.0193	0.0395	0.0382
84	0.04234	0.0598	0.0401	0.0234	0.0451	0.0440
85	0.04909	0.0665	0.0486	0.0284	0.0515	0.0505
86	0.05688	0.0739	0.0579	0.0344	0.0587	0.0580
87	0.0658	0.0822	0.0681	0.0416	0.0669	0.0664
88	0.07604	0.0914	0.0793	0.0502	0.0762	0.0759
89	0.0875	0.1016	0.0916	0.0605	0.0868	0.0867
90	0.10021	0.1130	0.1051	0.0727	0.0987	0.0989
91	0.11432	0.1256	0.1198	0.0872	0.1122	0.1126
92	0.13009	0.1397	0.1361	0.1043	0.1275	0.1280
93	0.14808	0.1553	0.1538	0.1242	0.1447	0.1453
94	0.16863	0.1727	0.1734	0.1473	0.1640	0.1647
95	0.19138	0.1920	0.1948	0.1739	0.1856	0.1865
96	0.21479	0.2135	0.2183	0.2041	0.2098	0.2107
97	0.24009	0.2374	0.2440	0.2380	0.2369	0.2377
98	0.26743	0.2639	0.2723	0.2757	0.2669	0.2677
99	0.29696	0.2935	0.3033	0.3168	0.3001	0.3008
100	0.32888	0.3263	0.3373	0.3610	0.3368	0.3373
101	0.36337	0.3628	0.3747	0.4076	0.3771	0.3773
102	0.40089	0.4035	0.4156	0.4560	0.4210	0.4210
103	0.4844	0.4486	0.4606	0.5053	0.4689	0.4686

104	0.53141	0.4988	0.5098	0.5544	0.5205	0.5200
105	0.58221	0.5546	0.5639	0.6025	0.5759	0.5753
106	0.6371	0.6167	0.6232	0.6487	0.6351	0.6344
107	0.69641	0.6857	0.6883	0.6923	0.6977	0.6971
108	0.7605	0.7625	0.7597	0.7327	0.7634	0.7631
109	0.82975	0.8478	0.8381	0.7695	0.8319	0.8323
110	0.90457	0.9427	0.9240	0.8027	0.9028	0.9041

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