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FLOW OF VISCO-ELASTIC [OLDROYD (1958) MODEL] LIQUID OF FIRST ORDER THROUGH POROUS MEDIA BETWEEN TWO PARALLEL FLAT PLATES

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ABSTRACT

Present paper is concerned with the oscillatory motion of visco-elastic [Oldroyd (1958) model] liquid of first order through porous medium between two infinite flat plates both of which execute simple harmonic motion in the plane of plates with different amplitudes and frequencies. Some particular cases have also been discussed.

INTRODUCTION:

Visco-elastic fluids are particular cases of non-Newtonian fluids which exhibit appreciable elastic behaviour and stress-strain velocity relations and are time dependent. The subject of Rheology is of great technological importance in many branches of industry, the problem arises of designing apparatus to transport or to process substances which can not be governed the classical stress-strain velocity relations. Examples of such substances and the process are many, in the extrussion of plastics, in the manufacture of rayon, nylon or other textiles, fibres, visco-elastic effects are encountered when the spinning solutions are transported or forced through spinnerts and in the manufacture of lubricating greases and rubbers.

The study of the physics of flow through porous medium has become the basis of many scientific and engineering applications. This type of flow is of great importance in the petroleum engineering concerned with the movement of oil, gas and water through reservoir of an oil or gas field to the hydrologist in the study of the migration of underground water and to the chemical engineer in the filtration process.

Seth (1977) has discussed the flow of visco-elastic fluid past a flat plate. Ravi Kant (1980) studied the unsteady flow of an elastico-viscous fluid over a porous flat plate. Many research workers have paid their attention towards the application of visco-elastic fluid flow of different category through porous medium in channels of various cross-section such as Sudhakar and Venkataramana (1988); Kumar and Singh (1989); Gupta and Sharma (1990); Singh, Shankar and Singh (1995); Gupta and Gupta (1996); Singh and Kumar (1998); Hayat, Asghar and Siddiqui (2001); Kundu and Sengupta (2001); Hassanien (2002); Sengupta and Basak (2002); Pundhir and Pundhir (2003); Rehman and Alam Sarkar (2004); Agarwal and Agarwal (2006); Saroa (2006); Sharma and Pareek (2006); Singh, Kumar and Sharma (2008); Singh, Mishra and Sharma (2008) and Kumar, Singh and Sharma (2009) etc.

The aim of the present paper is to study the oscillatory motion of visco-elastic Oldroyd liquid of first order through porous medium between two infinite parallel flat plates. Both the plates are assumed to be oscillating harmonically with different amplitudes and frequencies. Some particular cases have also been discussed.

GOVERNING EQUATIONS OF MOTION:

The rheological equations satisfied by visco-elastic [Oldroyd (1958) model] liquid are:

$$P_{ik} = -p\delta_{ik} + P_{ik}^{'}$$

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$$p'_{ik} + \lambda_1 \frac{D}{Dt} p'_{ik} + \mu_0 p'_{jj} e_{ik} - \mu_1 \left(p'_{ij} e_{jk} + p'_{jk} e_{ij} \right) - \upsilon_1 p'_{jl} e_{jl} \delta_{ik}$$
$$= 2\eta_0 \left(e_{ik} + \lambda_2 \frac{D}{Dt} e_{ik} - 2\mu_2 e_{ij} e_{jk} + \upsilon_2 e_{jl} \delta_{ik} \right)$$

with the equation of incompressibility

$$e_{ii} = 0$$

where

$$\frac{D}{Dt} b_{ik} = \frac{\partial}{\partial t} b_{ik} + \upsilon_{ij} b_{ik \cdot j} + w_{ij} b_{jk} + w_{kj} b_{ij}$$

$$e_{ij} = \frac{1}{2}\left(v_{k \cdot i} + v_{i \cdot k} \right)$$

$$w_{ik} = \frac{1}{2}\left(v_{k \cdot i} - v_{i \cdot k} \right)$$

$e_{ik} =$ rate of strain tensor

$p_{ik} =$ stress tensor

$\lambda_1 =$ relaxation time

$\lambda_2 =$ retardation time

$\eta_0 =$ coefficient of viscosity

and $\mu_0, \mu_1, \mu_2, \upsilon_1$ *and* υ_2 are the material constants, each being of the dimension of time.

For $\eta_0 > 0,\ \lambda_1 = \mu_1 = \mu_2 = \lambda_2 = 0,\ \ \mu_0 = \upsilon_1 = \upsilon_2 = 0$ the liquid will behave as ordinary viscous liquid.

FORMULATION OF THE PROBLEM:

Let *d* be the distance between parallel plates, x-axis along the lower plate in the direction of flow of liquid and y-axis perpendicular to the plates. Supposing that the lower and upper plates execute oscillations with different amplitudes v_1, v_2 and frequencies ω_1, ω_2 respectively.

Following Ghosh (1968) the equation of motion for visco-elastic Oldroyd liquid of first order through porous medium between oscillating plates is given by

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = -\frac{1}{\rho}\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial p}{\partial x} + \upsilon\left(1 + \lambda_2 \frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\upsilon}{K}\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)u \tag{1}$$

where $\upsilon = \dfrac{\mu}{\rho} =$ kinetic viscosity, μ the coefficient of viscosity, *u* the velocity of liquid in the direction of oscillation, *p* the fluid pressure, *t* the time and *K* is the permeability of porous medium.

Assuming the pressure gradient to be zero, the equation (1) becomes

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = \upsilon\left(1 + \lambda_2 \frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\mu}{K}\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)u \tag{2}$$

The boundary conditions are:

$$\left.\begin{array}{lll} u = v_1 e^{-i\omega_1 t} & when & y = 0 \\ u = v_2 e^{-i\omega_2 t} & when & y = d \end{array}\right\} \tag{3}$$

Introducing the following non-dimensional quantities:

$$y^* = \frac{1}{d} y, \quad t^* = \frac{\upsilon}{d^2} t, \quad u^* = \frac{d}{\upsilon} u, \quad \lambda_1^* = \frac{\upsilon}{d^2} \lambda_1, \quad \lambda_2^* = \frac{\upsilon}{d^2} \lambda_2,$$
$$v_1^* = \frac{d}{\upsilon} v_1, \quad v_2^* = \frac{d}{\upsilon} v_2, \quad \omega_1^* = \frac{d^2}{\upsilon} \omega_1, \quad \omega_2^* = \frac{d^2}{\upsilon} \omega_2, \quad K^* = \frac{1}{d^2} K$$

in (2) and (3) and then dropping stars, we get

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \frac{1}{K}\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) u \tag{4}$$

and

$$\left.\begin{array}{lll} u = v_1 e^{-i\omega_1 t} & when & y = 0 \\ u = v_2 e^{-i\omega_2 t} & when & y = 1 \end{array}\right\} \tag{5}$$

SOLUTION OF THE PROBLEM:

We look for a solution of equation (4) in the form

$$u = v_1 f(y) e^{-i\omega_1 t} + v_2 g(y) e^{-i\omega_2 t} \tag{6}$$

which is evidently periodic in *t*.

Substituting (6) in (4), we get

$$\begin{aligned} & v_1 e^{-i\omega_1 t}\left[(1 - i\omega_1 \lambda_2) \frac{d^2 f}{dy^2} + \left\{\lambda_1 \omega_1^2 + i\omega_1 - \frac{1}{K}(1 - i\omega_1 \lambda_1)\right\} f\right] \\ & \quad + v_2 e^{-i\omega_2 t}\left[(1 - i\omega_2 \lambda_2) \frac{d^2 g}{dy^2} + \left\{\lambda_1 \omega_2^2 + i\omega_2 - \frac{1}{K}(1 - i\omega_2 \lambda_1)\right\} g\right] = 0 \end{aligned} \tag{7}$$

By assumption v_1 *and* v_2 are not zero, we have

$$\frac{d^2 f}{dy^2} + \frac{i\omega_1 (1 - i\omega_1 \lambda_1) - \frac{1}{K}(1 - i\omega_1 \lambda_1)}{(1 - i\omega_1 \lambda_2)} f = 0$$

or

$$\frac{d^2 f}{dy^2} + \frac{(1 - i\omega_1 \lambda_1)\left(i\omega_1 - \frac{1}{K}\right)}{(1 - i\omega_1 \lambda_2)} f = 0$$

or

$$\frac{d^2 f}{dy^2} + m^2 f = 0 \tag{8}$$

and

$$\frac{d^2 g}{dy^2}+\frac{i\omega_2(1-i\omega_2\lambda_1)-\frac{1}{K}(1-i\omega_2\lambda_1)}{(1-i\omega_2\lambda_2)}g=0$$

or

$$\frac{d^2 g}{dy^2}+\frac{(1-i\omega_2\lambda_1)\left(i\omega_2-\frac{1}{K}\right)}{(1-i\omega_2\lambda_2)}g=0$$

or

$$\frac{d^2 g}{dy^2}+n^2 g=0 \tag{9}$$

where

$$m^2=\frac{(1-i\omega_1\lambda_1)\left(i\omega_1-\frac{1}{K}\right)}{(1-i\omega_1\lambda_2)} \quad \text{and} \quad n^2=\frac{(1-i\omega_2\lambda_1)\left(i\omega_2-\frac{1}{K}\right)}{(1-i\omega_2\lambda_2)}$$

The solution of (8) is

$$f(y)=A\cos m\,y+B\sin m\,y \tag{10}$$

and corresponding boundary conditions for $f(y)$ are

$$\left.\begin{aligned} f(y)=1 \quad when \quad y=0\\ f(y)=0 \quad when \quad y=1 \end{aligned}\right\} \tag{11}$$

$\therefore$ From (10) and (11), we get

$$f(y)=\cos\,m\,y-\frac{\cos m}{\sin m}\sin m\,y \tag{12}$$

Similarly from (9) with boundary conditions $g(0)=0$ *and* $g(1)=1$, we get

$$g(y)=\frac{\sin n\,y}{\sin n} \tag{13}$$

Now putting the values of $f(y)$ *and* $g(y)$ in (6), we get the velocity of visco-elastic Oldroyd (1958) type liquid through porous medium between two parallel flate plates.

$$u=v_1\frac{\sin m\,(1-y)}{\sin m}e^{-i\omega_1 t}+v_2\frac{\sin n\,y}{\sin n}e^{-i\omega_2 t} \tag{14}$$

PARTICULAR CASES:

Case I: If both the plates oscillate with same amplitudes and different frequencies then $v_1=v_2=v$ (say) and from (14)

$$u=v\left\{\frac{\sin m\,(1-y)}{\sin m}e^{-i\omega_1 t}+\frac{\sin n\,y}{\sin n}e^{-i\omega_2 t}\right\} \tag{15}$$

Case II: If both the plates oscillate with same frequencies and different amplitudes then $\omega_1 = \omega_2 = \omega$ (say) and from (14)

$$u = \left\{ v_1 \frac{\sin m\,(1-y)}{\sin m} + v_2 \frac{\sin n\, y}{\sin n} \right\} e^{-i\omega t} \tag{16}$$

where $m^2 = \dfrac{(1-i\omega\lambda_1)\left(i\omega - \dfrac{1}{K}\right)}{(1-i\omega\lambda_2)} = n^2$

Case III: If both the plates oscillate with same amplitudes and frequencies then $v_1 = v_2 = v$ (say) and $\omega_1 = \omega_2 = \omega$ (say) and from (14)

$$u = v\{(1-\cot m)\sin m\, y + \cos m\, y\} e^{-i\omega t} \tag{17}$$

where $m^2 = \dfrac{(1-i\omega\lambda_1)\left(i\omega - \dfrac{1}{K}\right)}{(1-i\omega\lambda_2)}$

Case IV: If porous medium is withdrawn then $K = \infty$ and from (14)

$$u = v_1 \frac{\sin m\,(1-y)}{\sin m} e^{-i\omega_1 t} + v_2 \frac{\sin n\, y}{\sin n} e^{-i\omega_2 t} \tag{18}$$

where $m^2 = \dfrac{i\omega_1(1-i\omega_1\lambda_1)}{(1-i\omega_1\lambda_2)}$ and $n^2 = \dfrac{i\omega_2(1-i\omega_2\lambda_1)}{(1-i\omega_2\lambda_2)}$.

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