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Fuzzy Bi-Γ-ideals in Γ-semi near-field spaces (F-Bi-I-Γ-SNF-S)

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ABSTRACT

In this paper, we consider the fuzzification of bi-Γ-ideals in Γ-semi near-field spaces, and investigate some of their related properties. Maximal fuzzy bi-Γ-ideals of Γ-semi near-field spaces are introduced and their properties discussed. Finally, chain conditions relating to fuzzy bi-Γ-ideals of Γ-semi near-field spaces are investigated

Key words: *Γ-semi near-field space, fuzzy Γ-ideals, fuzzy bi-Γ-ideals, fuzzy interior,Γ-ideals.*

AMS 2000 Subject classification code: *16D50, 20M99, 20N25, 16P20, 16Y30, 03E72.*

SECTION 1: INTRODUCTION

Fuzzy bi-Γ-ideals in Γ-semi near-field spaces were first introduced in the literature by N V Nagendram in depth study of existing literature of Zadeh [25], the fuzzy set theories developed by zadeh and others found many applications in the domain of mathematics and elsewhere.

Γ-semi near-field spaces defined by N V Nagendram and the ideal theory in Γ-semi near-field spaces studied by N V Nagendram, Dr. T V Pradeep Kumar and Dr. Y Venkateswara Reddy. Fuzzy ideals of rings were introduced by Liu and have been studied by several authors. The notion of fuzzy ideals and its properties were applied to various areas: semi-groups and BCK algebras and semi-rings. The classification of left(resp. right) ideals of Γ-near-rings, and investigated the related properties by Y B Jun. Also he introduced the notion of fuzzy characteristic left (resp. right) ideals and normal fuzzy left (resp. right) ideals of Γ-near-rings, and studied some of their related properties.

In continuation, I state fuzzy bi-Γ-ideals in Γ-semi near-field spaces, and investigate its properties. As we know, Γ-semi groups are generalization of semi-groups, Chinram [4] studied some properties of bi-ideals in semi-groups and it has motivated us to study and to introduce the notion fuzzification of a bi-Γ-ideals in a Γ-semi near-field space as a generalization of a Γ-semi near-field.

In this paper, we consider a fuzzification of the concept of a bi-Γ-ideal in a Γ-semi near-field space and some properties of such bi-Γ-ideals are investigated. The homeomorphic property of fuzzy bi-Γ-ideal is established. The concept of a fuzzy interior Γ-bi-ideal and a fuzzy -Γ-ideal are also introduced and some properties are discussed. Now the notion of maximal fuzzy bi-Γ-ideals of Γ-semi near-field spaces discussed. Finally chain conditions relating to fuzzy bi-Γ-ideals of Γ-semi near-field spaces are discussed.

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SECTION 2: PRELIMINARIES

We first recall some basic concepts for the sake of completeness.

Definition 2.1: A near-ring space N is a system with two binary operations "+" and "·" such that:
(1) (N, +) is a near- ring,(not necessarily abelian) (2) (N, ·) is a semi-near- ring and
(3) $(x + y)z = xz + yz$, for all $x, y, z \in N$.

We will use the word "near-ring" to mean "right distributive near-ring" and write xy instead of x. y.

Note that $0. x = 0$ and $(-x)y = -xy$ but in general $x.0 = 0$ for some $x \in N$.

Definition 2.2: Let (N, +, ·) be a near-ring. A subset I of N is said to be an ideal of N if:
(1) (I, +) is a normal subgroup of (N, +), (2) $IN \in I$,
(3) $n_1 (n_2 + i) - n_1 n_2 \in I$, for all $i \in I$ and $n_1, n_2 \in N$.

If I satisfies (1) and (2), then it is called a right ideal of N. If I satisfies (1) and (3), then it is called a left ideal of N.

Definition 2.3: A Γ-near-ring is a triple (N, +, Γ) where
(i) (N, +) is a group,
(ii) Γ is a non-empty set of binary operators on N such that $\forall \beta \in \Gamma$, (N, +, β) is a near-ring and
(iii) $x\beta(y\alpha z) = (x\beta y)\alpha z$ for all $x, y, z \in N$ and $\beta, \alpha \in \Gamma$.

Definition 2.4: A subset A of a Γ-near-ring N is called a left ideal (or right ideal) of N if (i) (A, +) is a normal divisor of (N, +), (ii) $u\beta(x + v) - u\beta v \in A$ i.e., $u\beta x \in A$ for all $x \in A$, $\beta \in \Gamma$ and $u, v \in N$.

Definition 2.5.: A near-field space N is a system with two binary operations "+" and "·" such that:
(1) (N, +) is a near-field (N not necessarily abelian) (2) (N, ·) is a semi-near-field and
(3) $(x + y)z = xz + yz$, for all $x, y, z \in N$.

We will use the word "near-field" to mean "right distributive near-field" and write xy instead of x. y.

Note that $0. x = 0$ and $(-x)y = -xy$ but in general $x.0 = 0$ for some $x \in N$.

Definition 2.5: Let S be s semi near-field. By a sub semi near-field of S we mean a non-empty sub near-field space A of S such that $A^2 \subseteq A$.

For the sake of completeness, we now recall some concepts of fuzzy theory.

Definition 2.6: A mapping $\lambda: N \to [0, 1]$ is called fuzzy near-field space of N and complement of a near-field space λ, denoted by λ' is the fuzzy near-field space in N given by $\lambda'(x) = 1 - \lambda(x)$ for all $x \in N$. the level near-field space of a fuzzy near-field space λ of N is defined as $U(\lambda; t) = \{x \in N / \lambda(x) \geq t\}$.

Definition 2.7: A fuzzy near-field space λ in N is called a fuzzy sub semi near-field space of N if $\lambda(xy) \geq \min\{\lambda(x), \lambda(y)\}\ \forall x, y \in N$.

Definition 2.8: A sub semi near-field space A of a semi near-field space N is called a bi-ideal of N if $ANA \subseteq A$.

Definition 2.9: A fuzzy sub semi near-field space λ of a semi near-field space N is called a fuzzy bi-ideal of N if $\lambda(xyz) \geq \min \{\lambda(x), \lambda(z)\}\ \forall x, y$ and $z \in N$.

Definition 2.10: Let $N = \{x, y, z, . . .\}$ and $\Gamma = \{\alpha,\beta,\gamma,. . .\}$ be two non-empty semi near-field spaces. Then N is called a Γ-semi near-field space if it satisfies the axioms
(i) $x\gamma y \in N$,
(ii) $(x\alpha y)\gamma z = x\alpha(y\gamma z)$, $\forall x, y, z \in N$ and $\alpha, \gamma \in \Gamma$.

Definition 2.11: Let N be a Γ-semi near-field space. A non-empty Γ-semi near-field sub space A of a Γ-semi near-field space N is said to be a Γ-sub semi near-field space if $A\Gamma A \subseteq A$.

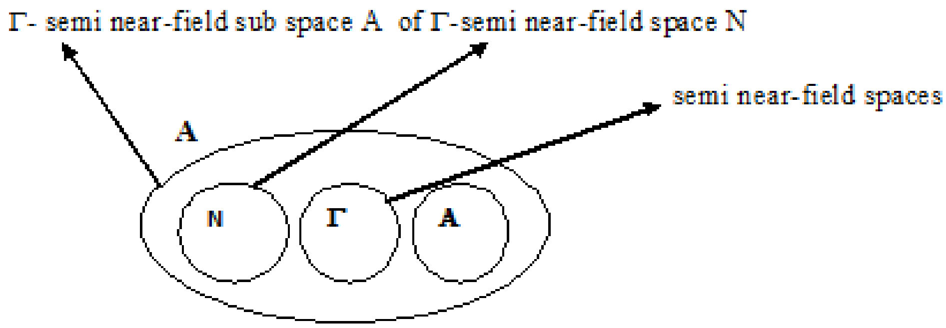

Fig.01

Definition 2.12: A left (right) ideal of a Γ-semi near-field space N is a non-empty Γ-semi near-field sub space A of a Γ-semi near-field space N such that NΓA ⊆ A(AΓM ⊆ A).

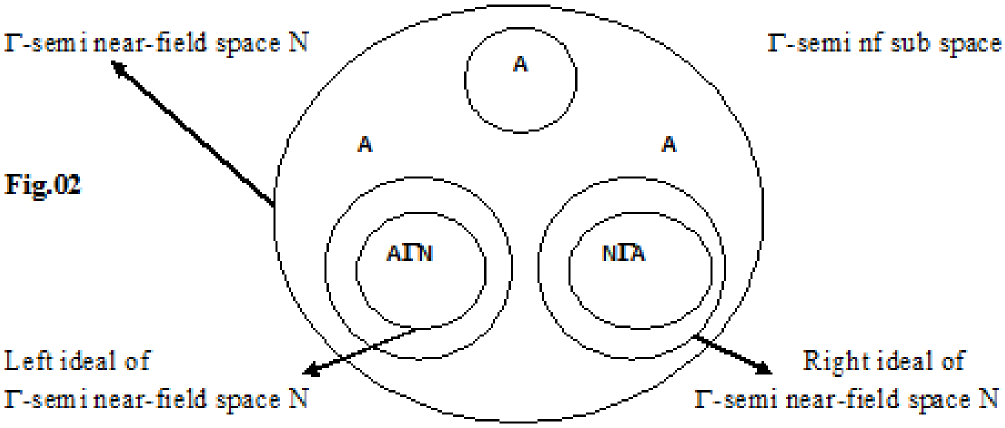

Fig.02

Definition 2.13: If A is both a left and right ideal of a Γ-semi near-field space N, then we say that A is a Γ-ideal of (Γ-semi near-field space) N.

Definition 2.14: A Γ-semi near-field space N is called left-zero (or right-zero) if xγy = x (or xγy = y) for all x, y ∈ N and γ ∈ Γ.

Definition 2.15: An element e in a Γ-semi near-field space N is called an idempotent if eγe = e for some γ ∈ Γ.

Definition 2.16: A Γ-sub semi near-field space A of Γ-semi near-field space N is called an interior Γ-ideal of N if MΓAΓM⊆ A.

Definition 2.17: Let N be a Γ-semi near-field space. A sub Γ-semi near-field space A of N is called a bi-Γ-ideal of N if AΓMΓA⊆ A.

Definition 2.18: Let N be a Γ-semi near-field space and N_1 a Γ_1-semi near-field space. A pair of mappings $f_1 : N \to N_1$ and $f_2 : \Gamma \to \Gamma_1$ is said to be a homomorphism from (N, Γ) to (N_1, Γ_1) if $f_1(axb) = f_1(a) f_2(x) f_1(b)$ for all a, b ∈ N and x ∈ Γ.

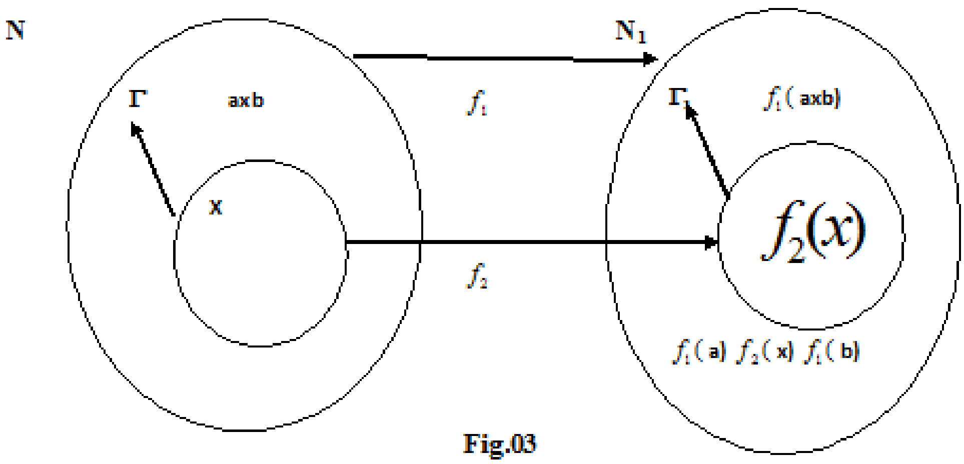

Fig.03

(f_1, f_2) **pair is a Homomorphism from (N, Γ) to (N_1, Γ_1)**

Definition 2.19: A fuzzy set λ of a Γ-semi near-field space is called a fuzzy Γ-semi near-field space of N if and only if $\lambda(x\ \gamma\ y) \geq \min\{\lambda(x), \lambda(y)\}$ ∀ x, y ∈ N and γ ∈ Γ.

Definition 2.20: A Γ-semi near-field space N is said to satisfy the left (or right) ascending chain condition a.c.c. of left (or right) ideals if every strictly increasing sequence $U_1 \subset U_2 \subset U_3 \subset \cdots$of left (right) ideals of N is of finite length.

Definition 2.21: A Γ-semi near-field space N is said to satisfy the left (or right) Notherian if N satisfies the left (or right) ascending chain condition a.c.c. of left (or right) ideals.

Note 2.22: Let E_N denote the set of all idempotent elements in a Γ-semi near-field space N and χ_A the characteristic function of a sub Γ-semi near-field space A of N. And Let N denote Γ-semi near-field space otherwise specified and λ of a near-field space N is a notion of fuzzy bi-Γ-ideal of N.

Definition 2.23: A fuzzy set λ of N is called a fuzzy bi-Γ-ideal of N if satisfies following axioms:
(i) $\lambda(x\,\gamma\,y) \geq \min\{\lambda(x), \lambda(y)\}$ ∀ x, y ∈ N and γ ∈ Γ,
(ii) $\lambda(x\beta y\alpha z) \geq \min\{\lambda(x), \lambda(z)\}$ ∀ x, y, z ∈ N and β, α ∈ Γ.

Example 2.24: Let N = {0, p, q, r} and Γ = {γ, β, α} be the non-empty near-field space of binary operations defined and shown in composition tables as below:

β	0	p	q	r
0	0	0	0	0
p	p	p	p	p
q	0	0	0	q
r	0	0	0	r

α	0	p	q	r
0	0	0	0	0
p	0	p	0	0
q	0	0	q	0
r	0	0	0	r

γ	0	p	q	r
0	0	0	0	0
p	0	p	0	p
q	0	q	0	r
r	0	0	0	q

Fig.04

Clearly N is a Γ-semi near-field space. Moreover the fuzzy near-field space map defined λ: N → [0,1] such as λ(0) = 0.6, λ(p) = 0.7, λ(q) = 0.8, λ(r) = 0.9 is a fuzzy bi-Γ-ideal of a Γ-semi near-field space N.

SECTION: 3 some results on fuzzy bi-Γ-ideals in a Γ-semi near-field space

In this section we obtain some results on fuzzy bi-Γ-ideals in a Γ-semi near-field space N.

Lemma 3.1: If W is a bi-Γ-ideal of N then ∀ 0 < q < 1, ∃ a fuzzy bi-Γ-ideal λ of Γ-semi near-field space N ∍ λ_q = W.

Proof: Let $\lambda : N \to [0, 1]$ defined by $\lambda(x) = \begin{matrix} \langle q, if x \in W, \\ \langle 0, if x \notin W, \end{matrix}$ where q is a fixed number in (0, 1). Then, clearly λ_q = W.

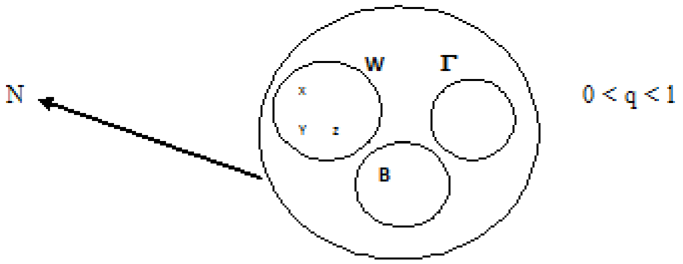

Fig.05

Now suppose that W is a bi-Γ-ideal of N. ∀ x, y ∈ W and γ ∈ Γ ∍ xγy ∈ W, we have λ(x γ y) ≥ q = min {λ(x), λ(y)} ∀ x, y ∈ N and γ ∈ Γ. Also for all x, y, z ∈ W and β, α ∈ Γ such that xβyαz ∈ W, we have λ(xβyαz) ≥ q = min {λ(x), λ(z)}. Thus λ is a fuzzy bi--Γ-ideal of Γ-semi near-field space N. This completes the proof of Lemma.

Lemma 3.2: Let W be a non-empty near-field subspace of Γ-semi near-field space N. Then W is a bi-Γ-ideal λ of Γ-semi near-field space N if and only if χ_W is a fuzzy bi-Γ-ideal λ of N.

Proof: (⇒if) Let x, y ∈ W and γ ∈ Γ. From hypothesis, xγy∈W.

(a) If x, y ∈ W and γ ∈ Γ, then χ_W(x) = 1 and χ_W(y) = 1.

In this case χ_W(xγy) =1 ≥ min {χ_W (x), χ_W (y)}.

(b) If x ∈ W, y ∉ W and γ ∈ Γ, then χ_W(x) = 1 and χ_W(y) = 0.

So, χ_W(xγy) =0 ≥ min {χ_W (x), χ_W (y)}.

(c) If x ∉ W, y ∈ W and γ ∈ Γ, then χ_W(x) = 0 and χ_W(y) = 1.

So, χ_W(xγy) =0 ≥ min {χ_W (x), χ_W (y)}.

(d) If x ∉ W, y ∉ W and γ ∈ Γ, then χ_W(x) = 0 and χ_W(y) = 0.

So, χ_W(xγy) ≥0= min {χ_W (x), χ_W (y)}.

Thus by (i) of definition 2.23 holds good.

Let x, y, z ∈ W and β,α ∈ Γ. From hypothesis, xβyαz ∈ W.

(a) If x, z ∈ W, then χ_W(x) = 1 and χ_W(z) = 1.

In this case χ_W(xβyαz) =1 ≥ min {χ_W (x), χ_W (z)}.

(b) If x ∈ W, z ∉ W, then χ_W(x) = 1 and χ_W(z) = 0.

So, χ_W(xβyαz) =0 ≥ min {χ_W (x), χ_W (z)}.

(c) If x ∉ W, z ∈ W, then χ_W(x) = 0 and χ_W(z) = 1.

So, χ_W(xβyαz) =0 ≥ min {χ_W (x), χ_W (z)}.

(d) If x ∉ W, y ∉ W, then χ_W(x) = 0 and χ_W(z) = 0.

So, χ_W(xβyαz) ≥ 0 = min {χ_W (x), χ_W (y)}.

Thus (ii) of definition 2.23 holds good.

(⇐iff) Suppose, that χ_W is a fuzzy bi-Γ-ideal of N. Then by Lemma 3.1, χ_W is two valued, hence W is a bi-Γ-ideal of N. this completes the proof of Lemma.

The following theorem proves that an intersection of a family of fuzzy bi--Γ-ideals of Γ-semi near-field space N also a fuzzy bi--Γ-idea.

Theorem 3.3: If $\{W_i\}_{i\in\Lambda}$ is a family of fuzzy bi-Γ-ideals of Γ-semi near-field space N. Then $\bigcap W_i$ is a fuzzy bi-Γ-ideals of N, where $\bigcap W_i = \{\Lambda\lambda_i\}$ and $\Lambda\lambda_i(x) = \inf.\{\lambda_i(x)/i\in\Lambda, x\in W\}$.

Proof: Let x, y ∈ N, where N is a Γ-semi near-field space.

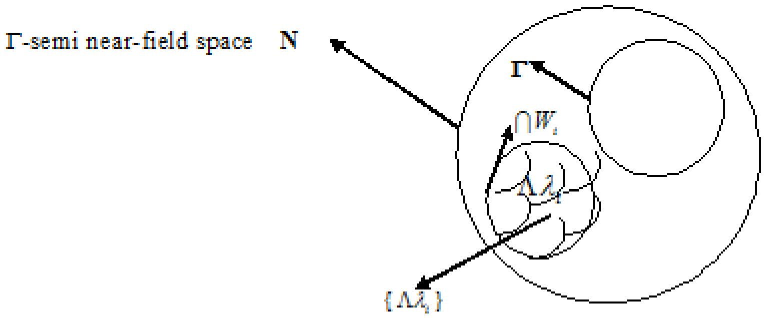

Fig.06

Let x, y ∈ N. Then we have,

(a) $\Lambda\lambda_i(x\gamma y) = \inf.\{\min.\{\lambda_i(x),\lambda_i(y)\} \ni i\in\Lambda, x,y\in W\}$
$= \min.\{\inf.\{(\lambda_i(x)),\inf(\lambda_i(y))\} \ni i\in\Lambda, x,y\in W\}$
$= \min.\{\inf.(\lambda_i(x) \ni i\in\Lambda, x\in W)\},\inf(\lambda_i(y))\} \ni i\in\Lambda, y\in W)\}$
$= \min.\{\Lambda\lambda_i(x),\Lambda\lambda_i(y)\}$.

Let x, y, z ∈ N and β,α ∈ Γ. Then we have,

(b) $\Lambda\lambda_i(x\beta y\alpha z) = \inf.\{\min.\{\lambda_i(x),\lambda_i(z)\} \ni i\in\Lambda, x,z\in W\}$
$= \min.\{\inf.\{(\lambda_i(x)),\inf(\lambda_i(z))\} \ni i\in\Lambda, x,z\in W\}$
$= \min.\{\inf.(\lambda_i(x) \ni i\in\Lambda, x\in W)\},\inf(\lambda_i(z))\} \ni i\in\Lambda, z\in W)\}$
$= \min.\{\Lambda\lambda_i(x),\Lambda\lambda_i(z)\}$.

Therefore, $\bigcap W_i$ is a fuzzy bi-Γ-ideals of Γ-semi near-field space N. This completes the proof of theorem.

Theorem 3.4: A fuzzy near-field sub space W in a bi-Γ-ideals of Γ-semi near-field space N is a fuzzy bi-Γ-ideal of N if and only if the level fuzzy near-field space $U(\lambda;q) = \{x\in N/\lambda(x)\geq q\}$ is a bi-Γ-ideal of N when it is non-empty.

Proof: (IF Part) Let λ be a fuzzy bi-Γ-ideal of N. Then $\lambda(x\gamma y)\geq \min\{\lambda(x),\lambda(y)\}$.
$x,y\in U(\lambda;q), \gamma\in\Gamma \Rightarrow \lambda(x)\geq q, \lambda(y)\geq q$
$\Rightarrow \lambda(x\gamma y)\geq \min\{\lambda(x),\lambda(y)\}\geq q$
$\Rightarrow \lambda(x\gamma y)\geq q$
$\Rightarrow x\gamma y\in U(\lambda;q)$.

Also, $\lambda(x\beta y\alpha z) \geq \min\{\lambda(x), \lambda(z)\}$

$$\begin{aligned} x, y \in U(\lambda; q), \beta, \alpha \in \Gamma &\Rightarrow \lambda(x) \geq q, \lambda(y) \geq q, \lambda(z) \geq q \\ &\Rightarrow \lambda(x\beta y\alpha z) \geq \min\{\lambda(x), \lambda(z)\} \geq q \\ &\Rightarrow \lambda(x\beta y\alpha z) \geq q \\ &\Rightarrow x\beta y\alpha z \in U(\lambda; q) \end{aligned}$$

Thus $U(\lambda; q)$ is a bi-Γ-ideal of N.

(⇐IFF part) If $U(\lambda; q)$ is a bi-Γ-ideal of N.

let $q = \min\{\lambda(x), \lambda(y)\}$. Then $x, y \in U(\lambda; q), \gamma \in \Gamma \Rightarrow x\gamma y \in U(\lambda; q)$

$$\begin{aligned} &\Rightarrow \lambda(x\gamma y) \geq q \\ &\Rightarrow \lambda(x\gamma y) \geq \min\{\lambda(x), \lambda(y)\} \end{aligned}$$

Let us define, $q = \min\{\lambda(x), \lambda(z)\}$. Then $x, y, z \in U(\lambda; q), \beta, \alpha \in \Gamma \Rightarrow x\beta y\alpha z \in U(\lambda; q)$

$$\begin{aligned} &\Rightarrow \lambda(x\beta y\alpha z) \geq q \\ &\Rightarrow \lambda(x\beta y\alpha z) \geq \min\{\lambda(x), \lambda(z)\}. \end{aligned}$$

Consequently, λ is a fuzzy bi-Γ-ideal of N. This completes the proof of theorem.

Note 3.5: Let λ be a fuzzy bi-Γ-ideal of N. If N is completely regular, then $\lambda(a) = \lambda(a\beta a)$ for all $a \in N$ and $\beta \in \Gamma$.

Theorem 3.6: Let the pair of mappings $f : N \to N_1, h : \Gamma \to \Gamma_1$ be a homomorphism of Γ-semi near-field spaces. If λ is a fuzzy bi-Γ-ideal of M_1, then the pre image $f^{-1}(\lambda)$ of λ under f is a fuzzy bi-Γ-ideal of N.

Proof: (i) Let x, y ∈ N and γ ∈ Γ. Then we have, $f^{-1}(\lambda)(x\gamma y) = \lambda(f(x\gamma y))$

$$\begin{aligned} &= \lambda(f(x)h(\gamma)f(z)) \\ &= \min\{\lambda(f(x), \lambda(f(y))\} \\ &= \min\{f^{-1}(\lambda(x)), f^{-1}(\lambda(y))\} \end{aligned}$$

(ii) Let x, y, z ∈ N and β, α ∈ Γ. Then,

$$\begin{aligned} f^{-1}(\lambda)(x\beta y\alpha z) &= \lambda(f(x\beta y\alpha z)) \\ &= \lambda(f(x)h(\beta)f(y)h(\alpha)f(z)) \\ &\geq \min\{\lambda(f(x), \lambda(f(z))\} \\ &= \min\{f^{-1}(\lambda(x)), f^{-1}(\lambda(y))\} \end{aligned}$$

Therefore $f^{-1}(\lambda)$ is a fuzzy bi-Γ-ideal of N. This completes the proof of the theorem.

Proposition 3.7: If W is a left-zero fuzzy Γ- sub semi near-field space in a bi-Γ-ideals of Γ-semi near-field space N and λ a fuzzy Γ-left ideal of N, then $\lambda(x) = \lambda(z)$ for all x, y, z ∈ N.

Proof: Let x, y, z ∈ W. Since W is left-zero fuzzy Γ- sub semi near-field space of N and λ a fuzzy Γ-left ideal of N, $x\beta y\alpha z = x$ and $z\alpha y\beta x = z$ for all β, α ∈ Γ.

In this case, by hypothesis, we have that $\lambda(x) = \lambda(x\beta y\alpha z) \geq \lambda(z), \lambda(z) = \lambda(z\alpha y\beta x) \geq \lambda(x)$.

Thus we obtain $\lambda(x) = \lambda(z)\ \forall$ x, y, z ∈ N. This completes the proof of theorem.

Lemma 3.8: If W is a left-zero fuzzy Γ- sub semi near-field space in a bi-Γ-ideals of Γ-semi near-field space N, then χ_W is a fuzzy Γ-left ideal of N.

Proof: Let x, y, z $\in$ W and β, α $\in$ Γ. Since W is left-zero fuzzy Γ- sub semi near-field space of N, $x\beta y\alpha z \in$ N.
(i) If z $\in$ N, then $\chi_W(z) = 1 \Rightarrow \chi_W(x\beta y\alpha z) = 1 = \chi_W(z)$
(ii) If z $\notin$ N, then $\chi_W(z) = 0$, hence $\chi_W(x\beta y\alpha z) \geq 0 = \chi_W(z)$.

Consequently, χ_W is a fuzzy Γ-left ideal of Γ-semi near-field space N. This completes the proof of lemma.

Theorem 3.9: Let λ be a fuzzy Γ-left ideal of Γ-semi near-field space N. If E_W is a left – zero fuzzy Γ- sub semi near-field space in a bi-Γ-ideals of Γ-semi near-field space N, then $\lambda(e) = \lambda(e_2)$ for all e, $e_2 \in E_W$.

Proof: Let e, $e_2 \in E_W$. From the hypothesis, $e\beta e_1\alpha e_2 = e$ and $e_2\alpha e_1\beta e = e_2$ for all $e_1 \in E_W$ and β, α∈Γ. Thus, since λ be a fuzzy Γ-left ideal of Γ-semi near-field space N, we get that
$\lambda(e) = \lambda(e\beta e_{1\alpha} e_2) \geq \lambda(e_2)$, $\lambda(e_2) = \lambda(e_2\alpha e_1\beta e) \geq \lambda(e)$.

Hence we have $\lambda(e) = \lambda(e_2)$ for all e, $e_2 \in E_W$. This completes the proof of theorem.

Theorem 3.10: If W is an interior Γ-left ideal of Γ-semi near-field space N, Then χ_W is a fuzzy interior Γ-left ideal of Γ-semi near-field space N.

Proof: Since W is a none-empty fuzzy Γ- sub semi near-field space in a bi-Γ-ideals of Γ-semi near-field space N. Let x, y, z $\in$ W andβ, α ∈Γ. From the hypothesis, xβyαz $\in$ W
(i) If y $\in$ W, then $\chi_W(y) = 1$ thus, $\chi_W(x\beta y\alpha z) = 1 = \chi_W(y)$
(ii) If y $\notin$ W, then $\chi_W(x\beta y\alpha z) \geq 0 = \chi_W(y)$.

Hence χ_W is a fuzzy interior Γ-left ideal of Γ-semi near-field space N. This completes the proof of the theorem.

Note 3.11: Every fuzzy Γ-left ideal of Γ-semi near-field space N is a fuzzy interior Γ-left ideal of Γ-semi near-field space N.

Note 3.12: If N is regular, then every fuzzy interior Γ-left ideal of Γ-semi near-field space N is a fuzzy Γ-left ideal of Γ-semi near-field space N.

SECTION 4: MAIN RESULTS ON CHAIN CONDITIONS OF A Γ-SEMI NEAR-FIELD SPACE N

In this section I derived main results on chain conditions of a Γ-semi near-field space N. we know that the intersection of all bi-Γ-ideals of Γ-semi near-field space N. Let ∧ be a totally ordered set of a Γ-semi near-field space N and let $\{A_j / j \in \wedge\}$ be a collection of all bi-Γ-ideals of Γ-semi near-field space N such that for all j < I if and only if $A_i \subset A_j$. Then $\cap A_j$, j < I is a bi-Γ-ideal of Γ-semi near-field space N.

Theorem 4.1: Let $\{A_j / j \in \wedge \subseteq [0, 1]\}$ be a collection of bi-Γ-ideals of Γ-semi near-field space N such that (a) $N = \bigcup_{j\in\Lambda} A_j$ (b) j < I if and only if $A_i \subset A_j$ ∀ i, j∈∧. Then the fuzzy set λ $\in$ M defined by $\lambda(x) = \sup\{ j \in \wedge / x \in A_j\}$ ∀ x $\in$ M is a fuzzy bi-Γ-ideal of Γ-semi near-field space N.

Proof: For any i $\in$ [0, 1], we consider the following two cases:

$$i = \sup\{j \in \Lambda / j < i\},\ i \neq \sup\{j \in \Lambda / j < i\}.$$

Case (i): ⇒ we know that for all x, y∈ A_j and γ $\in$ Γ, x γ y $\in$ U(λ ; i)
(⇔) x γ y $\in A_j$ for all j < i,

(⇔) x γ y∈ $\bigcap_{j<i} A_j$. Also, ∀ x, y, z $\in A_j$ and β, α $\in$ Γ, xβyαz $\in$ U(λ ; i)
(⇔) xβyαz $\in A_j$ for all j < i

($\Leftrightarrow$) $x\beta y\alpha z \in \bigcap_{j<i} A_j$.

Hence, $U(\lambda\,;\,i) = \bigcap_{j<i} A_j$, which is a bi-Γ-ideal of Γ-semi near-field space N.

Case (ii): $\Rightarrow \exists$ e > 0 such that $(i - e, i) \cap \wedge = \phi$.

We claim that $U(\lambda\,;\,i) = \bigcap_{j<i} A_j$. If $x\,\gamma\,y \in \bigcap_{j<i} A_j$, for some, j < i. $\gamma \in \Gamma$.

$\Rightarrow \lambda(x\,\gamma\,y) \geq j \geq i$.

Hence $x\,\gamma\,y \in U(\lambda\,;\,i)$, showing that if $x\,\gamma\,y \in A_j$, for $j \leq i - e$ and so $x\,\gamma\,y \notin U(\lambda\,;\,i)$.

Also, if $x\beta y\alpha z \in \bigcap_{j<i} A_j$ then $x\beta y\alpha z \in A_j$ for some j < i, $\beta, \alpha \in \Gamma$.

$\Rightarrow \lambda(x\beta y\alpha z) \geq j \geq i$. Hence, $x\beta y\alpha z \in U(\lambda\,;\,i)$, and

$\therefore x\beta y\alpha z \notin U(\lambda\,;\,i)$.

$\therefore U(\lambda\,;\,i). \subseteq \bigcap_{j<i} A_j$ and so $U(\lambda\,;\,i). = \bigcap_{j<i} A_j$.

Hence λ is a fuzzy bi-Γ-ideal of Γ-semi near-field space N.

This completes the proof of theorem.

Note 4.2: Let $\{A_n / n \in \wedge\}$ be a family of bi-Γ-ideals of Γ-semi near-field space N which is nested, that is $N = A_1 \supset A_2 \supset A_3 \supset \ldots An \supset A_{n+1} \supset \ldots\ldots$ Let λ be the fuzzy in N defined by

$$\lambda(x) = \begin{cases} \dfrac{n}{n+1} if\ x \in A_n \setminus A_{n+1,} \forall n = 1,2,3,\ldots \\ 1 \quad if\ x \in \bigcap_{n=1 to} A_n \end{cases} \text{ for all } x \in N.$$

Then λ is a fuzzy bi-Γ-ideal of Γ-semi near-field space N.

Note 4.3: A fuzzy bi-Γ-ideal λ of Γ-semi near-field space N is finite – valued if and only if it is generated by a finite – values fuzzy bi-Γ-ideal λ in N.

Theorem 4.4: If N is a Γ-Notherian, then every fuzzy bi-Γ-ideal λ Γ-semi near-field space N is finite - valued.

Proof: Let, $\lambda: N \to [0, 1]$ be a fuzzy bi-Γ-ideal of Γ-semi near-field space N which is not finite – valued. Then, $\exists$ an infinite sequence of distinct numbers $\lambda(0) = t_1 > t_2\ t_3 > t_4 > \ldots . > t_n > \ldots .$ where $t_i = \lambda(x_i)$ for some $x_i \in N$. This sequence induces an infinite sequence of distinct bi--Γ-ideal of Γ-semi near-field space N. so that we have the following conclusion $U(\lambda; i) \subset U(\lambda; t_2) \subset \ldots.. \subset U(\lambda; t_n) \subset \ldots.$ which is a contradiction. This completes the proof of the theorem.

Note 4.5: By combining note 4.2 and Note 4.3 we have the following corollary.

Corollary 4.6: If N is a Γ-Notherian, then every fuzzy bi-Γ-ideal of Γ-semi near-field space Nis generated by a fuzzy set in N.

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