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MORE RESULTS ON EDGE TRIMAGIC LABELING OF GRAPHS

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ABSTRACT

An edge magic total labeling of a (p, q) graph is a bijection f: $V(G) \cup E(G) \to \{1, 2, \ldots, p+q\}$ such that FOR each edge $xy \in E(G)$, the value of $f(x)+f(xy)+f(y)$ is a constant k. If there exists three constants k_1, k_2 and k_3 such that $f(x)+f(xy)+f(y)$ is either k_1 or k_2 or k_3, it is said to be an edge trimagic total labeling. In this paper we prove that the ladder L_n (odd n), triangular ladder TL_n, generalized Petersen graph $P(n, \frac{n-1}{2})$, the helm graph H_n and the flower graph Fl_n are edge trimagic total and super edge trimagic total graphs.

Keywords: *Function, Bijection, Magic labeling, Trimagic labeling.*

AMS Subject Classification: *05C78.*

1. INTRODUCTION

We begin with simple, finite and undirected graph G = (V(G), E(G)). A graph labeling is an assignment of integers to elements of a graph, the vertices or edges, or both subject to certain conditions. The concept of graph labeling was introduced by Rosa in 1967. In 1970, Kotzig and Rosa[5] defined, a magic labeling of graph G = (V(G), E(G)) is a bijection f: $V \cup E \to \{1, 2, \ldots, p+q\}$ such that for each edge $xy \in E(G)$, $f(x)+f(xy)+f(y)$ is a magic constant. In 1996, Ringel and Llado called this labeling as edge magic. In 2001, Wallis [6] introduced this as edge magic total labeling. An edge magic total labeling is called a super edge magic total if the vertices are labeled with smallest positive integers.

In 2004, J.Baskar Babujee[1] introduced the bimagic labeling of graphs. In 2013, C. Jayasekaran, M. Regees and C. Davidraj[3] introduced the edge trimagic total labeling of graphs. An **edge trimagic total labeling** of a (p, q) graph G is a bijection $f:V(G) \cup E(G) \to \{1, 2, \ldots, p+q\}$ such that for each edge $xy \in E(G)$, the value of $f(x)+f(xy)+f(y)$ is equal to any of the distinct constants k_1 or k_2 or k_3. A graph G is said to be an edge trimagic total if it admits an edge trimagic total labeling. An edge trimagic total labeling is called **super edge trimagic total labeling** if G has the additional property that the vertices are labeled with smallest positive integers. A simple graph in which there exists an edge between every pair of vertices is called a complete graph. The complete graph with n vertices is denoted by K_n. A walk of a graph G is an alternating sequence of vertices and edges $v_0, x_1, v_1, \ldots, v_{n-1}, x_n, v_n$ beginning and ending with vertices, in which each edge is incident with the two vertices immediately preceding and following it. It is closed if $v_0 = v_n$ and is open otherwise. An open walk in which no vertex appears more than once is called a path. A path with n vertices is denoted by P_n. A ladder L_n is a graph $P_n \times P_2$ with $V(L_n) = \{u_i, v_i \,/\, 1 \le i \le n\}$ and $E(L_n) = \{u_iu_{i+1}, v_iv_{i+1} \,/\, 1 \le i \le n-1\} \cup \{u_iv_i \,/\, 1 \le i \le n\}$. A triangular ladder TL_n, $n \ge 2$, is a graph obtained from the ladder $L_n \approx P_n \times P_2$ by adding the edges u_iv_{i+1} for $1 \le i \le n-1$. The generalized Petersen graph P(n, m) is a graph that consists of an outer-cycle $y_0, y_1, y_2, \ldots, y_{n-1}$ a set of n spokes y_ix_i, $0 \le i \le n-1$, and n edges x_ix_{i+m}, $0 \le i \le n-1$, where all subscripts are taken modulo n. A wheel W_n with n spokes is a graph that has a centre x connected to all the n vertices in cycle C_n. A helm H_n is constructed from a wheel W_n by adding n vertices of degree one adjacent to each terminal vertex. A flower graph Fl_n is constructed from a helm H_n by joining each vertex of degree one to the center.

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For further references, we use Dynamic survey of graph labeling by J. A. Galian[5]. We follow the notations and terminology of [2]. In [4], we introduced the concept edge trimagic and super edge trimagic total labeling and proved, the pyramid graph Py(n), K_4 snake graph, wheel snake nW_4 and a fan graph F_n are edge trimagic total and super edge trimagic total graphs. In this paper, we prove the ladder L_n, triangular ladder TL_n, generalized Petersen graph P(n, $\frac{n-1}{2}$), the helm graph H_n and the flower graph Fl_n are edge trimagic total and super edge trimagic total graphs.

2. EDGE TRIMAGIC LABELING FOR SOME FAMILIES OF GRAPHS

In this section, we prove edge trimagic total and super edge trimagic total labeling for the families of graphs like Ladder, Triangular Ladder, generalized Petersen graph, Helm and Flower graphs and give examples for edge trimagic labeling for each of the above graphs.

Theorem: 2.1 The Ladder $L_n = P_n x P_2$ admits an edge trimagic total labeling for all $n \geq 2$.

Proof: Let V = $\{u_i, v_i / 1 \leq i \leq n\}$ be the vertex set and E = $\{u_iu_{i+1}, v_iv_{i+1} / 1 \leq i \leq n-1\} \cup \{u_iv_i / 1 \leq i \leq n\}$ be the edge set of the ladder L_n. Then L_n has 2n vertices and 3n-2 edges.

Case: 1 n is odd.
Define a bijection f: $V \cup E \rightarrow \{1, 2, \ldots, 2n, 2n+1, \ldots, 5n-2\}$ such that

$$f(u_i) = \begin{cases} \frac{i+1}{2}, \text{ i is odd} \\ \frac{n+i+1}{2}, \text{ i is even} \end{cases}$$

$$f(v_i) = \begin{cases} \frac{3n+i}{2}, \text{ i is odd} \\ \frac{2n+i}{2}, \text{ i is even} \end{cases}$$

$f(u_iu_{i+1}) = 3n - i$, $1 \leq i \leq n-1$; $f(v_iv_{i+1}) = 5n - i - 1$, $1 \leq i \leq n-1$ and $f(u_iv_i) = 4n - i$, $1 \leq i \leq n$.

Now we prove this labeling is an edge trimagic total labeling.

Consider the edges u_iu_{i+1}, $1 \leq i \leq n-1$.

For odd i, $f(u_i)+f(u_iu_{i+1})+f(u_{i+1}) = \frac{i+1}{2} + 3n - i + \frac{n+i+1+1}{2} = \frac{7n+3}{2} = \lambda_1$(say).

For even i, $f(u_i)+f(u_iu_{i+1})+f(u_{i+1}) = \frac{n+i+1}{2} + 3n - i + \frac{i+1+1}{2} = \frac{7n+3}{2} = \lambda_1$.

Next we consider the edges v_iv_{i+1}, $1 \leq i \leq n-1$.

For odd i, $f(v_i)+f(v_iv_{i+1})+f(v_{i+1}) = \frac{3n+i}{2} + 5n - i - 1 + \frac{2n+i+1}{2} = \frac{15n-1}{2} = \lambda_2$(Say).

For even i, $f(v_i)+f(v_iv_{i+1})+f(v_{i+1}) = \frac{2n+i}{2} + 5n - i - 1 + \frac{3n+i+1}{2} = \frac{15n-1}{2} = \lambda_2$.

Finally we consider the edges u_iv_i, $1 \leq i \leq n$.

For odd i, $f(u_i)+f(u_iv_i)+f(v_i) = \frac{i+1}{2} + 4n - i + \frac{3n+i}{2} = \frac{11n+1}{2} = \lambda_3$ (say).

For even i, $f(u_i)+f(u_iv_i)+f(v_i) = \frac{n+i+1}{2} + 4n - i + \frac{2n+i}{2} = \frac{11n+1}{2} = \lambda_3$.

Hence for each edge $uv \in E$, f(u)+f(uv)+f(v) yields any one of the magic constant $\lambda_1 = \frac{7n+3}{2}$, $\lambda_2 = \frac{15n-1}{2}$ and $\lambda_3 = \frac{11n+1}{2}$.

Hence the Ladder L_n is an edge trimagic total when n is odd.

Case: 2 n is even.
Define a bijection f: $V \cup E \rightarrow \{1, 2, \ldots, 2n, 2n+1, \ldots, 5n-2\}$ such that

$$f(u_i) = \begin{cases} \frac{i+1}{2}, \text{ i is odd} \\ \frac{n+i}{2}, \text{ i is even} \end{cases}$$

$$f(v_i) = \begin{cases} \frac{3n+i+1}{2}, & i \text{ is odd} \\ \frac{2n+i}{2}, & i \text{ is even} \end{cases}$$

$f(u_iu_{i+1}) = 3n - i$, $1 \le i \le n-1$; $f(v_iv_{i+1}) = 4n - i - 1$, $1 \le i \le n-1$ and $f(u_iv_i) = 5n - i - 1$, $1 \le i \le n$.

Now we prove this labeling is an edge trimagic total.

Consider the edges u_iu_{i+1}, $1 \le i \le n-1$.

For odd i, $f(u_i)+f(u_iu_{i+1})+f(u_{i+1}) = \frac{i+1}{2}+3n - i+\frac{n+i+1}{2} = \frac{7n+2}{2} = \lambda_1$(say).

For even i, $f(u_i)+f(u_iu_{i+1})+f(u_{i+1}) = \frac{n+i}{2} +3n - i + \frac{i+1+1}{2} = \frac{7n+2}{2} = \lambda_1$.

Consider the edges v_iv_{i+1}, $1 \le i \le n-1$.

For odd i, $f(v_i)+f(v_iv_{i+1})+f(v_{i+1}) = \frac{3n+i+1}{2} +4n - i - 1 + \frac{2n+i+1}{2} = \frac{13n}{2} = \lambda_2$(say).

For even i, $f(v_i)+f(v_iv_{i+1})+f(v_{i+1}) = \frac{2n+i}{2} +4n-i-1+\frac{3n+i+1+1}{2} = \frac{13n}{2} = \lambda_2$.

Consider the edges u_iv_i, $1 \le i \le n$.

For odd i, $f(u_i)+f(u_iv_i)+f(v_i) = \frac{i+1}{2} +5n - i - 1+ \frac{3n+i+1}{2} = \frac{13n}{2}= \lambda_2$.

For even i, $f(u_i)+f(u_iv_i)+f(v_i) = \frac{n+i}{2} +5n-i-1+ \frac{2n+i}{2} = \frac{13n-2}{2} = \lambda_3$(say).

Hence for each edge $uv \in E$, $f(u)+f(uv)+f(v)$ yields any one of the magic constant

$\lambda_1 = \frac{7n+2}{2}$, $\lambda_2 = \frac{13n}{2}$ and $\lambda_3 = \frac{13n-2}{2}$.

Hence the Ladder L_n is an edge trimagic total when n is even.

Therefore, by case1 and case 2 the ladder L_n admits an edge trimagic total labeling.

Theorem: 2.2 The Ladder $L_n = P_n x P_2$ is a super edge trimagic total for all $n \ge 2$.

Proof: We proved that the Ladder $L_n = P_n x P_2$ is an edge trimagic total graph for all n with 2n vertices. The labeling given in Theorem 2.1 is as follows:

When n is odd,

$$f(u_i) = \begin{cases} \frac{i+1}{2}, & i \text{ is odd} \\ \frac{n+i+1}{2}, & i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} \frac{3n+i}{2}, & i \text{ is odd} \\ \frac{2n+i}{2}, & i \text{ is even}. \end{cases}$$

When n is even,

$$f(u_i) = \begin{cases} \frac{i+1}{2}, & i \text{ is odd} \\ \frac{n+i}{2}, & i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} \frac{3n+i+1}{2}, & i \text{ is odd} \\ \frac{2n+i}{2}, & i \text{ is even} \end{cases} .$$

Hence the 2n vertices get labels 1, 2, …, 2n. Therefore, the ladder L_n is a super edge trimagic total for all n.

Example: 2.3 An edge trimagic total labeling of the Ladders L_7 and L_6 are given in figure 1 and figure 2, respectively.

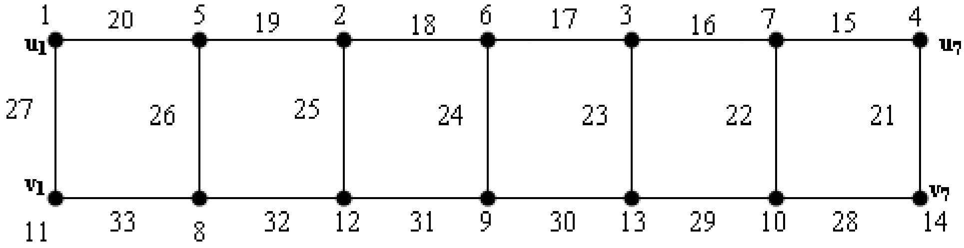

Figure 1: L_7 with $\lambda_1 = 26$, $\lambda_2 = 39$ and $\lambda_3 = 52$.

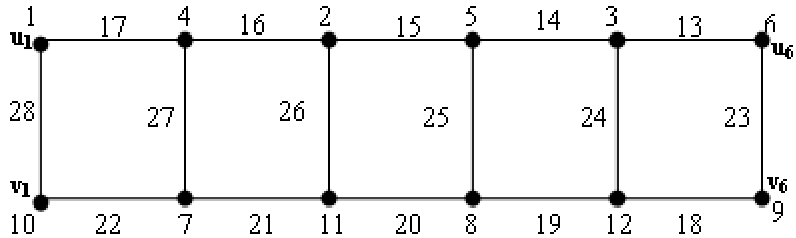

Figure 2: L_6 with $\lambda_1 = 22$, $\lambda_2 = 39$ and $\lambda_3 = 38$.

Theorem: 2.4 The triangular Ladder TL_n admits an edge trimagic total labeling for all $n \geq 2$.

Proof: Let V= $\{v_i, u_i / 1 \leq i \leq n\}$ be the vertex set and E = $\{v_i v_{i+1}, u_i u_{i+1} / 1 \leq i \leq n–1\} \cup \{u_i v_i / 1 \leq i \leq n\} \cup \{ u_i v_{i+1} / 1 \leq i \leq n–1\}$ be the edge set of the triangular Ladder TL_n. Then TL_n has 2n vertices and 4n–3 edges.

Case: 1 n is odd.
Define a bijection f: $V \cup E \rightarrow \{1, 2, …, 6n–3\}$ such that $f(u_i) = 2i$, $1 \leq i \leq n$; $f(v_i) = 2i–1$, $1 \leq i \leq n$; $f(u_i u_{i+1}) = 6n–4i$, $1 \leq i \leq n–1$; $f(v_i v_{i+1}) = 6n–4i–2$, $1 \leq i \leq n–1$; $f(u_i v_i) = 6n4i+1$, $1 \leq i \leq n$ and $f(u_i v_{i+1}) = 6n–4i–1$, $1 \leq i \leq n–1$.

Now, we prove this labeling is an edge trimagic total.

For the edge $u_i u_{i+1}$, $1 \leq i \leq n–1$,

$f(u_i)+ f(u_i u_{i+1})+f(u_{i+1}) = 2i+6n– 4i +2(i+1) = 6n+2 = \lambda_1$ (say).

For the edges $v_i v_{i+1}$, $1 \leq i \leq n–1$,

$f(v_i)+f(v_i v_{i+1})+f(v_{i+1}) = 2i–1+6n – 4i–2+2(i+1) –1 = 6n–2 = \lambda_2$ (say).

For the edges $u_i v_i$, $1 \leq i \leq n$,

$f(u_i)+f(u_i v_i)+f(v_i) = 2i+6n–4i+1+2i–1 = 6n = \lambda_3$ (say).

Also, for the edges, $u_i v_{i+1}$, $1 \leq i \leq n–1$,

$f(u_i)+f(u_i v_{i+1})+f(v_{i+1}) = 2i+6n–4i–1+2(i+1) –1 = 6n = \lambda_3$.

Hence for each edge $uv \in E$, f(u)+f(uv)+f(v) yields any one of the magic constant $\lambda_1 = 6n+2$, $\lambda_2 = 6n–2$ and $\lambda_3 = 6n$.

Therefore, the triangular Ladder TL_n admits an edge trimagic total labeling when n is odd.

Case: 2 n is even.

Define a bijection f: $V \cup E \rightarrow \{1, 2, …, 6n–3\}$such that

$f(u_i) = 2i–1$, $1 \leq i \leq n$; $f(v_i) = 2i$, $1 \leq i \leq n$; $f(u_i u_{i+1}) = 6n– 4i–1$, $1 \leq i \leq n–1$; $f(v_i v_{i+1}) = 6n– 4i$, $1 \leq i \leq n–1$; $f(u_i v_i) = 6n–4i+1$, $1 \leq i \leq n$ and $f(u_i v_{i+1}) = 6n–4i–2$, $1 \leq i \leq n–1$.

Now we prove this labeling is an edge trimagic total.

For the edges u_iu_{i+1}, $1\leq i \leq n-1$,

$f(u_i)+f(u_iu_{i+1})+f(u_{i+1}) = 2i-1+6n-4i-1+2(i+1) -1 = 6n-1= \lambda_1$(say).

For the edges v_iv_{i+1}, $1\leq i \leq n-1$,

$f(v_i)+f(v_iv_{i+1})+f(v_{i+1}) = 2i+6n- 4i- 2(i+1) = 6n +2 = \lambda_2$(say).

For the edges u_iv_i, $1\leq i \leq n$,

$f(u_i)+f(u_iv_i)+f(v_i) = 2i-1+6n- 4i+1+2i = 6n = \lambda_3$ (say).

Also, for the edges u_iv_{i+1}, $1\leq i \leq n-1$,

$f(u_i)+f(u_iv_{i+1})+f(v_{i+1}) = 2i-1+6n- 4i-2+2(i+1) = 6n-1 = \lambda_1$.

Hence for each edge $uv\in E$, $f(u)+f(uv)+f(v)$ yields any one of the magic constant $\lambda_1 = 6n-1$, $\lambda_2 = 6n+2$, and $\lambda_3 = 6n$.

Therefore, the triangular Ladder TL_n admits an edge trimagic total labeling for even n.

Hence by case 1 and case 2, the triangular Ladder TL_n admits an edge trimagic total labeling.

Theorem: 2.5 The triangular ladder TL_n admits a super edge trimagic total labeling.

Proof: We have proved that the triangular ladder TL_n has an edge trimagic total labeling with 2n vertices. The labeling given in the proof of Theorem 2.4, is as follows:

For odd n, $f(u_i) = 2i$, $1\leq i \leq n$ and $f(v_i) = 2i-1$, $1\leq i \leq n$.

For even n, $f(u_i) = 2i-1$, $1\leq i \leq n$ and $f(v_i) = 2i$, $1\leq i \leq n$.

Hence the 2n vertices get labels 1, 2, …, 2n. Therefore, the triangular ladder TL_n admits a super edge trimagic total labeling for all $n \geq 2$.

Example: 2.6 An super edge trimagic total labeling of the triangular ladders TL_7 and TL_6 are given in figure 3 and figure 4, respectively.

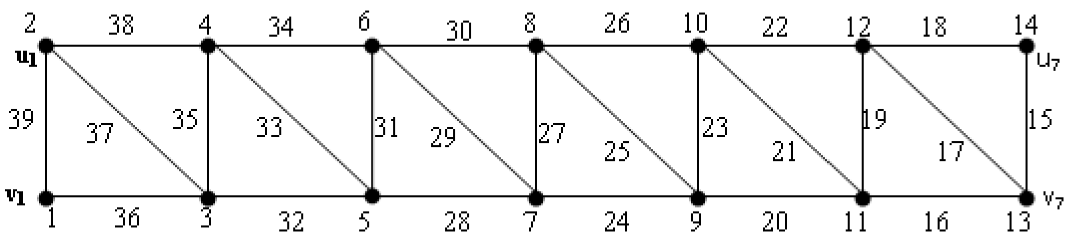

Figure 3: TL_7 with magic constants $\lambda_1 = 40$, $\lambda_2 = 42$ and $\lambda_3 = 44$.

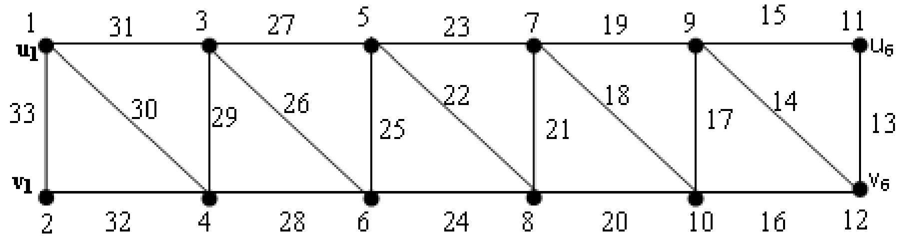

Figure 4: TL_6 with magic constants $\lambda_1 = 35$, $\lambda_2 = 36$ and $\lambda_3 = 38$.

Theorem: 2.7 The generalized Petersen graph P(n, $\frac{n-1}{2}$) admits an edge trimagic total labeling (n is odd).

Proof: Consider a generalized Petersen graph P(n, $\frac{n-1}{2}$) with the vertex set
$V = \{x_i, y_i / 0 \le i \le n-1\}$ and the edge set
$E = \{x_iy_i / 0 \le i \le n-1\} \cup \{y_iy_{i+1} / 0 \le i \le n-2\} \cup \{x_ix_{i+\frac{n-1}{2}} / 0 \le i \le n-1\} \cup \{y_0y_{n-1}\}$, where the subscripts taken modulo n.

Then P(n, $\frac{n-1}{2}$) has 2n vertices and 3n edges.

Define a bijection $f: V \cup E \to \{1, 2, \ldots, 5n\}$ such that $f(x_i) = 2n-i$, $0 \le i \le n-1$;

$f(y_i) = n-i$, $0 \le i \le n-1$; $f(y_iy_{i+1}) = 3n+2i+2$, $0 \le i \le n-2$; $f(y_0y_{n-1}) = 5n$; $f(x_iy_i) = 3n+2i+1$,

$0 \le i \le n-1$ and $f(x_ix_{i+\frac{n-1}{2}}) = 2n+2i+1$, $0 \le i \le n-1$.

Now we have to prove that the generalized Petersen graph P(n, $\frac{n-1}{2}$) admits an edge trimagic total labeling.

For the edges y_iy_{i+1}, $0 \le i \le n-2$;

$f(y_i)+f(y_iy_{i+1})+f(y_{i+1}) = n-i+3n+2i+2+n-(i+1) = 5n+1 = \lambda_1$(say).

For the edge y_0y_{n-1};

$f(y_0)+f(y_0y_{n-1})+f(y_{n-1}) = n-0+5n+n-(n-1) = 6n+1 = \lambda_2$(say).

For the edges x_iy_i, $0 \le i \le n-1$;

$f(x_i)+f(x_iy_i)+f(y_i) = 2n-i+3n+2i+1+n-i = 6n+1 = \lambda_2$.

For the edges $x_i\, x_{i+\frac{n-1}{2}}$, $0 \le i \le n-1$ with i taken modulo n.

$f(x_i)+f(x_i\, x_{i+\frac{n-1}{2}})+f(x_{i+\frac{n-1}{2}}) = 2n-i+2n+2i+1+2n-(i+\frac{n-1}{2}) = \frac{11n+3}{2} = \lambda_3$(say).

Hence for each edge $uv \in E$, $f(u)+f(uv)+f(u)$ yields any one of the magic constants $\lambda_1 = 5n+1$, $\lambda_2 = 6n+1$ and $\lambda_3 = \frac{11n+3}{2}$.

Therefore, the generalized Petersen graph P(n, $\frac{n-1}{2}$) admits an edge trimagic total labeling.

Theorem: 2.8 The generalized Petersen graph P(n, $\frac{n-1}{2}$) admits a super edge trimagic total labeling.

Proof: We have proved that the generalized Petersen graph P(n, $\frac{n-1}{2}$) admits edge trimagic total labeling. The labeling given in the Theorem 2.7 for the vertices is, $f(x_i) = 2n-i$, $0 \le i \le n-1$ and $f(y_i) = n-i$, $0 \le i \le n-1$. Since the vertices get labels 1, 2, …, 2n the generalized Petersen graph P(n, $\frac{n-1}{2}$) is a super edge trimagic total.

Example: 2.9 Generalized Petersen graph P(9, 4) is super edge trimagic total.

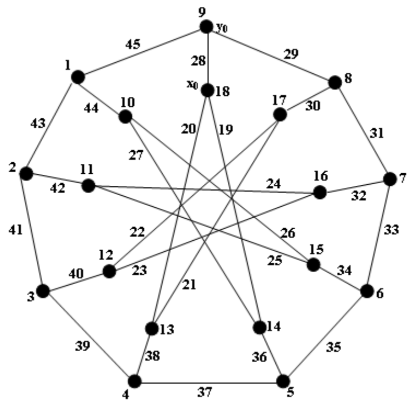

Figure 5: Petersen graph P(9, 4) with $\lambda_1 = 46$, $\lambda_2 = 55$ and $\lambda_3 = 51$.

Theorem: 2.10 The Helm graph H_n has an edge trimagic total labeling for every positive even integer n.

Proof: Let $V = \{u\}\cup\{v_i /1 \le i \le n\}\cup\{w_i /1 \le i \le n\}$ be the vertex set and $E = \{uv_i, v_iw_i / 1 \le i \le n\} \cup \{v_iv_{i+1} / 1 \le i \le n-1\}\cup\{v_1v_n\}$ be the edge set of the helm graph H_n. Then H_n has 2n+1 vertices and 3n edges.

Define a bijection $f: V\cup E \to \{1, 2, \ldots, 5n+1\}$ such that $f(u) = 1$,

$$f(v_i) = \begin{cases} \frac{i+1}{2}+1, & i \text{ is odd} \\ \frac{i+n}{2}+1, & i \text{ is even} \end{cases}$$

$$f(w_i) = \begin{cases} n+\frac{i+1}{2}+1, & i \text{ is odd} \\ n+\frac{i+n}{2}+1, & i \text{ is even} \end{cases}$$

$$f(uv_i) = \begin{cases} 5n-\frac{i+1}{2}+2, & i \text{ is odd} \\ 5n-\frac{n+i}{2}+2, & i \text{ is even} \end{cases}$$

$f(v_1v_n) = 4n+1$, $f(v_iv_{i+1}) = 4n-i+1$, $1 \le i \le n-1$ and $f(v_iw_i) = 3n-i+2$, $1 \le i \le n$.

Now we prove this labeling is an edge trimagic total.

Consider the edges uv_i, $1 \le i \le n$.

For odd i, $f(u)+f(uv_i)+f(v_i) = 1+\frac{i+1}{2}+1+5n-\frac{i+1}{2}+2 = 5n+4 = \lambda_1$ (say).

For even i, $f(u)+f(uv_i)+f(v_i) = 1 + 5n-\frac{n+i}{2}+2+\frac{i+n}{2}+1 = 5n+4 = \lambda_1$.

Consider the edges v_iv_{i+1}, $1 \le i \le n-1$.

For odd i, $f(v_i)+f(v_iv_{i+1})+f(v_{i+1}) = \frac{i+1}{2}+1+4n-i+1+\frac{i+1+n}{2}+1 = 4n+\frac{n}{2}+4 = \lambda_2$ (say).

For even i, $f(v_i)+f(v_iv_{i+1})+f(v_{i+1}) = \frac{i+n}{2}+1+4n-i+1+\frac{i+1+1}{2}+1 = 4n+\frac{n}{2}+4 = \lambda_2$

Consider the edges v_iw_i, $1 \le i \le n$.

For odd i, $f(v_i)+f(v_i w_i)+f(w_i) = \frac{i+1}{2}+1+3n-i+2+n+\frac{i+1}{2}+1 = 4n+5 = \lambda_3$ (say).

For even i, $f(v_i)+f(v_iw_i)+f(w_i) = \frac{i+n}{2}+1+3n-i+2+n+\frac{i+n}{2}+1 = 5n+4 = \lambda_1$.

For the edge v_1v_n, $f(v_1)+f(v_1v_n)+f(v_n) = \frac{1+1}{2}+1+4n+1+\frac{n+n}{2}+1 = 5n+4 = \lambda_1$.

Hence for each edge $uv\in E$, $f(u)+f(uv)+f(v)$ yields any one of the constants $\lambda_1= 5n+4$, $\lambda_2 = 4n+\frac{n}{2}+4$ and $\lambda_3 = 4n + 5$.

Therefore, the helm graph H_n admits an edge trimagic total labeling for every positive even integer n.

Theorem: 2.11 The helm graph H_n is a super edge trimagic total for even n.

Proof: We have proved that the helm graph H_n is an edge trimagic total for even n. The labeling given in the proof of Theorem 2.10, the labeling for the vertices are $f(u) = 1$,

$$f(v_i) = \begin{cases} \frac{i+1}{2}+1, & i \text{ is odd} \\ \frac{i+n}{2}+1, & i \text{ is even} \end{cases}$$

and $$f(w_i) = \begin{cases} n+\frac{i+1}{2}+1, & i \text{ is odd} \\ n+\frac{i+n}{2}+1, & i \text{ is even.} \end{cases}$$

Since the helm graph H_n has 2n+1 vertices and get labels 1, 2, …, 2n+1, the helm graph H_n is a super edge trimagic total labeling.

Example: 2.12 The helm graph H_6 given in figure 6 admits a super edge trimagic total labeling with magic constants 29, 31and 34.

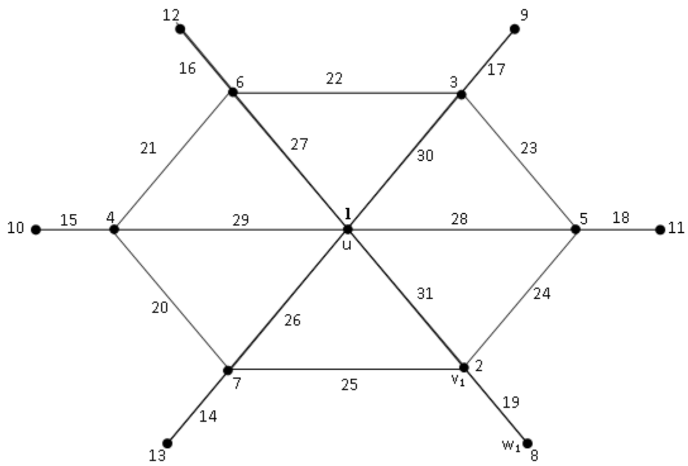

Figure 6: Helm graph H_6 with $\lambda_1 = 29$, $\lambda_2 = 31$ and $\lambda_3 = 34$.

Theorem: 2.13 The flower graph Fl_n has an edge trimagic total labeling for all n.

Proof: Let $V = \{v_i, w_i /1 \le i \le n\} \cup \{u\}$ be the vertex set and $E = \{uv_i, v_iw_i, uw_i / 1 \le i \le n\} \cup \{v_iv_{i+1} /1 \le i \le n-1\} \cup \{v_1v_n\}$ be the edge set of the flower graph Fl_n. Then the flower graph Fl_n has 2n+1 vertices and 4n edges.

Define a bijection $f: V \cup E \to \{1, 2, …, 6n+1\}$ such that $f(u) = 1$, $f(v_i) = i+1$, $1 \le i \le n$; $f(w_i) = n+i+1$, $1 \le i \le n$; $f(uv_i) = 5n - i + 2$, $1 \le i \le n$; $f(v_iv_{i+1}) = 4n-2i+1$, $1 \le i \le n - 1$; $f(v_iw_i) = 4n - 2i + 2$, $1 \le i \le n$; $f(uw_i) = 6n - i+2$, $1 \le i \le n$ and $f(v_1v_n) = 4n+1$.

Now, we prove the above labeling is an edge trimagic total.

For the edges uv_i, $1 \le i \le n$,

$f(u)+f(uv_i)+f(v_i) = 1+5n - i+2+i+1 = 5n+4 = \lambda_1$ (say).

For all the edges v_iv_{i+1}, $1 \le i \le n-1$,

$f(v_i)+f(v_iv_{i+1})+f(v_{i+1}) = i+1+4n-2i+1+i+1+1 = 4n+4 = \lambda_2$ (say).

For the edges v_iw_i, $1 \le i \le n$,

$f(v_i)+f(v_iw_i)+f(w_i) = i+1+4n-2i+2+n+i+1 = 5n+4 = \lambda_1$.

For the edge uw_i, $1 \le i \le n$,

$f(u)+f(uw_i)+f(w_i) = 1+6n - i+2+n+i+1 = 7n+4 = \lambda_2$ (say).

And for the edge v_1v_n, $f(v_1)+f(v_1v_n)+f(v_n) = 1+1+4n+1+n+1 = 5n+4 = \lambda_1$.

Hence for each edge uv, $f(u)+f(uv)+f(v)$ yields any one of the magic constant $\lambda_1 = 5n + 4$, $\lambda_2 = 4n+4$ and $\lambda_3 = 7n + 4$.

Therefore, the flower graph Fl_n admits an edge trimagic total labeling for all n.

Theorem: 2.14 The flower graph Fl_n is a super edge trimagic total for all $n \ge 3$.

Proof: We have proved that the flower graph Fl_n admits an edge trimagic total labeling for $n \ge 3$. The labeling given in the proof of the Theorem 2.13, the labeling for the vertices are, $f(u) = 1$, $f(v_i) = i+1$, $1 \le i \le n$; $f(w_i) = n+i+1$, $1 \le i \le n$. Since the flower graph Fl_n has 2n+1 vertices and get labels 1, 2, …, 2n+1, the flower graph Fl_n is a super edge trimagic total labeling.

Example: 2.15 The flower graph Fl_6 given in figure 7 is a super edge trimagic total graph with magic constants 34, 28 and 46.

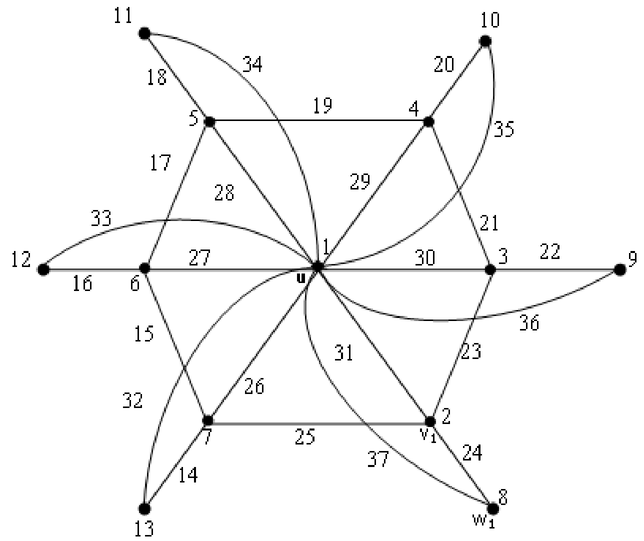

Figure 7: Flower graph Fl_6 with $\lambda_1 = 34$, $\lambda_2 = 28$ and $\lambda_3 = 46$.

CONCLUSION

In this paper, we have proved some classes of graphs namely, the ladder L_n, triangular ladder TL_n, generalized Petersen graph $P(n, \frac{n-1}{2})$, the helm graph H_n and the flower graph Fl_n are edge trimagic total and super edge trimagic total graphs. There will be many trimagic graphs can be constructed in future.

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