MORE RESULTS ON EDGE TRIMAGIC LABELING OF GRAPHS

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ABSTRACT

An edge magic total labeling of a (p, q) graph is a bijection \( f: V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p+q\} \) such that for each edge \( xy \in E(G) \), the value of \( f(x)+f(xy)+f(y) \) is a constant \( k \). If there exists three constants \( k_1, k_2 \) and \( k_3 \) such that \( f(x)+f(xy)+f(y) \) is either \( k_1 \) or \( k_2 \) or \( k_3 \), it is said to be an edge trimagic total labeling. In this paper we prove that the ladder \( L_n \) (odd \( n \)), triangular ladder \( TL_n \), generalized Petersen graph \( P(n, \frac{n-1}{2}) \), the helm graph \( H_n \) and the flower graph \( F_n \) are edge trimagic total and super edge trimagic total graphs.

Keywords: Function, Bijection, Magic labeling, Trimagic labeling.

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1. INTRODUCTION

We begin with simple, finite and undirected graph \( G = (V(G), E(G)) \). A graph labeling is an assignment of integers to elements of a graph, the vertices or edges, or both subject to certain conditions. The concept of graph labeling was introduced by Rosa in 1967. In 1970, Kotzig and Rosa[5] defined, a magic labeling of graph \( G = (V(G), E(G)) \) is a bijection \( f: V \cup E \rightarrow \{1, 2, \ldots, p+q\} \) such that for each edge \( xy \in E(G) \), \( f(x)+f(xy)+f(y) \) is a magic constant. In 1996, Ringel and Llado called this labeling as edge magic. In 2001, Wallis [6] introduced this as edge magic total labeling. An edge magic total labeling is called a super edge magic total if the vertices are labeled with smallest positive integers.

In 2004, J.Baskar Babujee[1] introduced the bimagic labeling of graphs. In 2013, C. Jayasekaran, M. Regees and C. Davidraj[3] introduced the edge trimagic total labeling of graphs. An edge trimagic total labeling of a (p, q) graph \( G \) is a bijection \( f: V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p+q\} \) such that for each edge \( xy \in E(G) \), the value of \( f(x)+f(xy)+f(y) \) is equal to any of the distinct constants \( k_1, k_2 \) or \( k_3 \). A graph \( G \) is said to be an edge trimagic total if it admits an edge trimagic total labeling. An edge trimagic total labeling is called super edge trimagic total labeling if \( G \) has the additional property that the vertices are labeled with smallest positive integers. A simple graph in which there exists an edge between every pair of vertices is called a complete graph. The complete graph with \( n \) vertices is denoted by \( K_n \). A walk of a graph \( G \) is an alternating sequence of vertices and edges \( v_0, x_1, v_1, \ldots, v_{n-1}, x_n, v_n \) beginning and ending with vertices, in which each edge is incident with the two vertices immediately preceding and following it. It is closed if \( v_0 = v_n \) and is open otherwise. An open walk in which no vertex appears more than once is called a path. A path with \( n \) vertices is denoted by \( P_n \). A ladder \( L_n \) is a graph \( P_n \times P_2 \) with \( V(L_n) = \{u_i, v_i / 1 \leq i \leq n\} \) and \( E(L_n) = \{u_iu_{i+1}, v_iv_{i+1} / 1 \leq i \leq n-1\} \cup \{u_i v_{i+1} / 1 \leq i \leq n\} \). A triangular ladder \( TL_n \) is a graph obtained from the ladder \( L_n \) by adding the edges \( u_i v_{i+1} \) for \( 1 \leq i \leq n-1 \). The generalized Petersen graph \( P(n, m) \) is a graph that consists of an outer-cycle \( y_0, y_1, y_2, \ldots, y_{n-1} \) and \( n \) edges \( x_{i}x_{i+m} \) for \( 0 \leq i < n \), where all subscripts are taken modulo \( n \). A wheel \( W_n \) with \( n \) spokes is a graph that has a center \( x \) connected to all the \( n \) vertices in cycle \( C_n \). A helm \( H_n \) is constructed from a wheel \( W_n \) by adding \( n \) vertices of degree one adjacent to each terminal vertex. A flower graph \( F_n \) is constructed from a helm \( H_n \) by joining each vertex of degree one to the center.

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For further references, we use Dynamic survey of graph labeling by J. A. Galian[5]. We follow the notations and terminology of [2]. In [4], we introduced the concept edge trimagic and super edge trimagic total labeling and proved, the pyramid graph Py(n), K₄ snake graph, wheel snake nW₄ and a fan graph F₆ are edge trimagic total and super edge trimagic total graphs. In this paper, we prove the ladder Lᵣ, triangular ladder TLᵣ, generalized Petersen graph P(n, \( \frac{n-1}{2} \)), the helm graph Hᵣ and the flower graph Flᵣ are edge trimagic total and super edge trimagic total graphs.

2. EDGE TRIMAGIC LABELING FOR SOME FAMILIES OF GRAPHS

In this section, we prove edge trimagic total and super edge trimagic total labeling for the families of graphs like Ladder, Triangular Ladder, generalized Petersen graph, Helm and Flower graphs and give examples for edge trimagic labeling for each of the above graphs.

Theorem: 2.1 The Ladder Lᵣ = PₓPᵧ admits an edge trimagic total labeling for all n ≥ 2.

Proof: Let V = \{uᵢ, vᵢ /1 ≤ i ≤ n\} be the vertex set and E = \{uᵢuᵢ₊₁, vᵢvᵢ₊₁ /1 ≤ i ≤ n-1\} ∪ \{uᵢvᵢ /1 ≤ i ≤ n\} be the edge set of the ladder Lᵣ. Then Lᵣ has 2n vertices and 3n-2 edges.

Case: 1 n is odd.

Define a bijection f: V ∪ E → {1, 2, ..., 2n, 2n+1, ..., 5n-2} such that

\[
\begin{align*}
  f(uᵢ) &= \begin{cases} 
    i+1 \quad, i \text{ is odd} \\
    n+i \quad, i \text{ is even}
  \end{cases} \\
  f(vᵢ) &= \begin{cases} 
    3n+i \quad, i \text{ is odd} \\
    2n+i \quad, i \text{ is even}
  \end{cases}
\]

\[
\begin{align*}
  f(uᵢuᵢ₊₁) &= 3n- i, 1 ≤ i ≤ n-1; \quad f(vᵢvᵢ₊₁) = 5n- i-1, 1 ≤ i ≤ n-1 \quad \text{and} \quad f(uᵢvᵢ) = 4n- i, 1 ≤ i ≤ n.
\end{align*}
\]

Now we prove this labeling is an edge trimagic total labeling.

Consider the edges uᵢuᵢ₊₁, 1 ≤ i ≤ n-1.

For odd i, \( f(uᵢ)+f(uᵢuᵢ₊₁)+f(uᵢ₊₁) = \frac{i+1}{2} + 3n-i + \frac{n+i+1}{2} = \frac{7n+3}{2} = \lambda₁ \) (say).

For even i, \( f(uᵢ)+f(uᵢuᵢ₊₁)+f(uᵢ₊₁) = \frac{n+i+1}{2} + 3n-i + \frac{i+1}{2} = \frac{7n+3}{2} = \lambda₁ \).

Next we consider the edges vᵢvᵢ₊₁, 1 ≤ i ≤ n-1.

For odd i, \( f(vᵢ)+f(vᵢvᵢ₊₁)+f(vᵢ₊₁) = \frac{3n+i}{2} + 5n-i-1 + \frac{2n+i+1}{2} = \frac{15n-1}{2} = \lambda₂ \) (Say).

For even i, \( f(vᵢ)+f(vᵢvᵢ₊₁)+f(vᵢ₊₁) = \frac{2n+i}{2} + 5n-i-1 + \frac{3n+i+1}{2} = \frac{15n-1}{2} = \lambda₂ \).

Finally we consider the edges uᵢvᵢ, 1 ≤ i ≤ n.

For odd i, \( f(uᵢ)+f(uᵢvᵢ)+f(vᵢ) = \frac{i+1}{2} + 4n-i + \frac{3n+i}{2} = \frac{11n+1}{2} = \lambda₃ \) (say).

For even i, \( f(uᵢ)+f(uᵢvᵢ)+f(vᵢ) = \frac{n+i+1}{2} + 4n-i + \frac{2n+i}{2} = \frac{11n+1}{2} = \lambda₃ \).

Hence for each edge uv ∈ E, \( f(u)+f(uv)+f(v) \) yields any one of the magic constant \( \lambda₁ = \frac{7n+3}{2} \), \( \lambda₂ = \frac{15n-1}{2} \) and \( \lambda₃ = \frac{11n+1}{2} \).

Hence the Ladder Lᵣ is an edge trimagic total when n is odd.

Case: 2 n is even.

Define a bijection f: V ∪ E → {1, 2, ..., 2n, 2n+1, ..., 5n-2} such that

\[
\begin{align*}
  f(uᵢ) &= \begin{cases} 
    i+1 \quad, i \text{ is odd} \\
    n+i \quad, i \text{ is even}
  \end{cases} \\
  f(vᵢ) &= \begin{cases} 
    3n+i \quad, i \text{ is odd} \\
    2n+i \quad, i \text{ is even}
  \end{cases}
\]

\[
\begin{align*}
  f(uᵢuᵢ₊₁) &= 3n- i, 1 ≤ i ≤ n-1; \quad f(vᵢvᵢ₊₁) = 5n- i-1, 1 ≤ i ≤ n-1 \quad \text{and} \quad f(uᵢvᵢ) = 4n- i, 1 ≤ i ≤ n.
\end{align*}
\]
\[ f(v_i) = \begin{cases} 
\frac{3n+i+1}{2}, & \text{i is odd} \\
\frac{2n+i}{2}, & \text{i is even}
\end{cases} \]

\[ f(u_{i+1}) = 3n - i, \ 1 \leq i \leq n-1; \ f(v_{i+1}) = 4n - i - 1, \ 1 \leq i \leq n-1 \text{ and } f(u_{n+i}) = 5n - i - 1, \ 1 \leq i \leq n. \]

Now we prove this labeling is an edge trimagic total.

Consider the edges \(u_{i+1}u_i\), \(1 \leq i \leq n-1\).

For odd \(i\), \(f(u_i) + f(u_{i+1}) + f(u_{i+1}) = \frac{i+1}{2} + 3n - i + \frac{n+i+1}{2} = \frac{7n+2}{2} = \lambda_1\) (say).

For even \(i\), \(f(u_i) + f(u_{i+1}) + f(u_{i+1}) = \frac{n+i}{2} + 3n - i + \frac{i+1+1}{2} = \frac{7n+2}{2} = \lambda_1\).

Consider the edges \(v_{i+1}v_i\), \(1 \leq i \leq n-1\).

For odd \(i\), \(f(v_i) + f(v_{i+1}) + f(v_{i+1}) = \frac{3n+i+1}{2} + 4n - i - 1 + \frac{2n+i+1}{2} = \frac{13n}{2} = \lambda_2\) (say).

For even \(i\), \(f(v_i) + f(v_{i+1}) + f(v_{i+1}) = \frac{2n+i}{2} + 4n - i - 1 + \frac{3n+i+1}{2} = \frac{13n}{2} = \lambda_2\).

Consider the edges \(u_{i+1}v_i\), \(1 \leq i \leq n\).

For odd \(i\), \(f(u_i) + f(u_{i+1}) + f(v_i) = \frac{i+1}{2} + 5n - i - 1 + \frac{n+i+1}{2} = \frac{13n}{2} = \lambda_3\).

For even \(i\), \(f(u_i) + f(u_{i+1}) + f(v_i) = \frac{n+i}{2} + 5n - i - 1 + \frac{2n+i}{2} = \frac{13n-2}{2} = \lambda_3\) (say).

Hence for each edge \(u_iv \in E\), \(f(u) + f(uv) + f(v)\) yields any one of the magic constant \(\lambda_1 = \frac{7n+2}{2}, \lambda_2 = \frac{13n}{2}\) and \(\lambda_3 = \frac{13n-2}{2}\).

Hence the Ladder \(L_n\) is an edge trimagic total when \(n\) is even.

Therefore, by case 1 and case 2 the ladder \(L_n\) admits an edge trimagic total labeling.

**Theorem 2.2** The Ladder \(L_n = P_n \times P_2\) is a super edge trimagic total for all \(n \geq 2\).

**Proof:** We proved that the Ladder \(L_n = P_n \times P_2\) is an edge trimagic total graph for all \(n\) with \(2n\) vertices. The labeling given in Theorem 2.1 is as follows:

When \(n\) is odd,

\[ f(u_i) = \begin{cases} 
\frac{i+1}{2}, & \text{i is odd} \\
\frac{n+i+1}{2}, & \text{i is even}
\end{cases} \]

\[ f(v_i) = \begin{cases} 
\frac{3n+i}{2}, & \text{i is odd} \\
\frac{2n+i}{2}, & \text{i is even}
\end{cases} \]

When \(n\) is even,

\[ f(u_i) = \begin{cases} 
\frac{i+1}{2}, & \text{i is odd} \\
\frac{n+i}{2}, & \text{i is even}
\end{cases} \]

\[ f(v_i) = \begin{cases} 
\frac{3n+i+1}{2}, & \text{i is odd} \\
\frac{2n+i}{2}, & \text{i is even}
\end{cases} \]
Hence the 2n vertices get labels 1, 2, ..., 2n. Therefore, the ladder \( L_n \) is a super edge trimagic total for all \( n \).

**Example: 2.3** An edge trimagic total labeling of the Ladders \( L_7 \) and \( L_6 \) are given in figure 1 and figure 2, respectively.

**Theorem: 2.4** The triangular Ladder \( TL_n \) admits an edge trimagic total labeling for all \( n \geq 2 \).

**Proof:** Let \( V= \{v_i, u_i / 1 \leq i \leq n\} \) be the vertex set and \( E = \{v_i v_{i+1}, u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{u_i v_i / 1 \leq i \leq n-1\} \) be the edge set of the triangular Ladder \( TL_n \). Then \( TL_n \) has 2n vertices and 4n–3 edges.

**Case: 1** \( n \) is odd.

Define a bijection \( f: V \cup E \rightarrow \{1, 2, \ldots, 6n-3\} \) such that:

- \( f(u_i) = 2i, 1 \leq i \leq n \);
- \( f(v_i) = 2i-1, 1 \leq i \leq n \);
- \( f(u_iu_{i+1}) = 6n-4i-2, 1 \leq i \leq n-1 \);
- \( f(v_iv_{i+1}) = 6n-4i, 1 \leq i \leq n-1 \);
- \( f(u_iv_i) = 6n+1, 1 \leq i \leq n \) and \( f(u_iv_{i+1}) = 6n-4i-1, 1 \leq i \leq n-1 \).

Now, we prove this labeling is an edge trimagic total.

For the edge \( u_iu_{i+1}, 1 \leq i \leq n-1 \),

\[ f(u_i)+f(u_iu_{i+1})+f(u_{i+1}) = 2i+6n-4i+2(i+1) = 6n+2 = \lambda_1 \text{ (say)} . \]

For the edges \( v_iv_{i+1}, 1 \leq i \leq n-1 \),

\[ f(v_i)+f(v_iv_{i+1})+f(v_{i+1}) = 2i-1+6n-4i+2(i+1)-1 = 6n-2 = \lambda_2 \text{ (say)} . \]

For the edges \( u_iv_i, 1 \leq i \leq n \),

\[ f(u_i)+f(u_iv_i)+f(v_i) = 2i+6n-4i+1+2i-1 = 6n = \lambda_3 \text{ (say)} . \]

Also, for the edges \( u_iv_{i+1}, 1 \leq i \leq n-1 \),

\[ f(u_i)+f(u_iv_{i+1})+f(v_{i+1}) = 2i+6n-4i+1+2(i+1)-1 = 6n = \lambda_3 . \]

Hence for each edge \( uv \in E \), \( f(u)+f(uv)+f(v) \) yields any one of the magic constant \( \lambda_1 = 6n+2 \), \( \lambda_2 = 6n-2 \) and \( \lambda_3 = 6n \).

Therefore, the triangular Ladder \( TL_n \) admits an edge trimagic total labeling when \( n \) is odd.

**Case: 2** \( n \) is even.

Define a bijection \( f: V \cup E \rightarrow \{1, 2, \ldots, 6n-3\} \) such that:

- \( f(u_i) = 2i-1, 1 \leq i \leq n \);
- \( f(v_i) = 2i, 1 \leq i \leq n \);
- \( f(u_iu_{i+1}) = 6n-4i-1, 1 \leq i \leq n-1 \);
- \( f(v_iv_{i+1}) = 6n-4i, 1 \leq i \leq n-1 \);
- \( f(u_iv_i) = 6n-4i+1, 1 \leq i \leq n \) and \( f(u_iv_{i+1}) = 6n-4i-2, 1 \leq i \leq n-1 \).
Now we prove this labeling is an edge trimagic total.

For the edges $u_iu_{i+1}$, $1 \leq i \leq n-1$,

$$f(u_i)+f(u_iu_{i+1})+f(u_{i+1}) = 2i-1+6n-4i+1+2(i+1) -1 = 6n-1 = \lambda_3 \text{(say)}.$$

For the edges $v_iv_{i+1}$, $1 \leq i \leq n-1$,

$$f(v_i)+f(v_iv_{i+1})+f(v_{i+1}) = 2i+6n-4i-2(i+1) = 6n+2 = \lambda_2 \text{(say)}.$$

For the edges $u_iv_i$, $1 \leq i \leq n$,

$$f(u_i)+f(u_iv_i)+f(v_i) = 2i-1+6n-4i+1+2i = 6n = \lambda_3 \text{ (say)}.$$

Also, for the edges $u_iv_{i+1}$, $1 \leq i \leq n-1$,

$$f(u_i)+f(u_iv_{i+1})+f(v_{i+1}) = 2i-1+6n-4i-2+2(i+1) = 6n-1 = \lambda_1.$$

Hence for each edge $uv \in E$, $f(u)+f(uv)+f(v)$ yields any one of the magic constant $\lambda_1 = 6n-1$, $\lambda_2 = 6n+2$, and $\lambda_3 = 6n$.

Therefore, the triangular Ladder $TL_n$ admits an edge trimagic total labeling for even $n$.

Hence by case 1 and case 2, the triangular Ladder $TL_n$ admits an edge trimagic total labeling.

**Theorem: 2.5** The triangular ladder $TL_n$ admits a super edge trimagic total labeling.

**Proof:** We have proved that the triangular ladder $TL_n$ has an edge trimagic total labeling with $2n$ vertices. The labeling given in the proof of Theorem 2.4, is as follows:

For odd $n$, $f(u_i) = 2i$, $1 \leq i \leq n$ and $f(v_i) = 2i-1$, $1 \leq i \leq n$.

For even $n$, $f(u_i) = 2i-1$, $1 \leq i \leq n$ and $f(v_i) = 2i$, $1 \leq i \leq n$.

Hence the $2n$ vertices get labels 1, 2, ..., $2n$. Therefore, the triangular ladder $TL_n$ admits a super edge trimagic total labeling for all $n \geq 2$.

**Example: 2.6** An super edge trimagic total labeling of the triangular ladders $TL_7$ and $TL_6$ are given in figure 3 and figure 4, respectively.

**Figure 3:** $TL_7$ with magic constants $\lambda_1 = 40$, $\lambda_2 = 42$ and $\lambda_3 = 44$.

**Figure 4:** $TL_6$ with magic constants $\lambda_1 = 35$, $\lambda_2 = 36$ and $\lambda_3 = 38$. 

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Theorem: 2.7 The generalized Petersen graph \( P(n, \frac{n-1}{2}) \) admits an edge trimagic total labeling (n is odd).

Proof: Consider a generalized Petersen graph \( P(n, \frac{n-1}{2}) \) with the vertex set
\[
V = \{x_i, y_i / 0 \leq i \leq n-1\}
\]
and the edge set
\[
E = \{xy_i / 0 \leq i \leq n-1\} \cup \{yy_{i+1} / 0 \leq i \leq n-2\} \cup \{xx_{i+n-1}/ 0 \leq i \leq n-1\} \cup \{y_0y_{n-1}\},
\]
where the subscripts taken modulo n.

Then \( P(n, \frac{n-1}{2}) \) has 2n vertices and 3n edges.

Define a bijection \( f: V \cup E \rightarrow \{1, 2, \ldots , 5n\} \) such that
\[
f(x_i) = 2n-i, 0 \leq i \leq n-1;
f(y_i) = n-i, 0 \leq i \leq n-1;
f(y_{i+1}) = 3n+2i+2, 0 \leq i \leq n-2;
f(y_0y_{n-1}) = 5n;
f(x_iy_i) = 3n+2i+1, 0 \leq i \leq n-1
\]
and 
\[
f(x_{i+n-1}) = 2n+2i+1, 0 \leq i \leq n-1.
\]

Now we have to prove that the generalized Petersen graph \( P(n, \frac{n-1}{2}) \) admits an edge trimagic total labeling.

For the edges \( y_iy_{i+1}, 0 \leq i \leq n-2; \)
\[
f(y_i)+f(y_{i+1})+f(y_{i+1}) = n-i+3n+2i+2+n-(i+1) = 5n+1 = \lambda_1 \text{(say)}.
\]

For the edge \( y_0y_{n-1}; \)
\[
f(y_0)+f(y_0y_{n-1})+f(y_{n-1}) = n-0+5n+n-(n-1) = 6n+1 = \lambda_2 \text{(say)}.
\]

For the edges \( x_iy_i, 0 \leq i \leq n-1; \)
\[
f(x_i)+f(x_iy_i)+f(y_i) = 2n-i+3n+2i+1+n-i = 6n+1 = \lambda_2.
\]

For the edges \( x_i x_{i+n-1}, 0 \leq i \leq n-1 \) with i taken modulo n.
\[
f(x_i)+f(x_{i+n-1})+f(x_{i+n-1}) = 2n-i+2n+2i+1+2n-(i+n-1) = 11n+3 = \frac{11n+3}{2} = \lambda_3 \text{(say)}.
\]

Hence for each edge \( uv \in E \), \( f(u)+f(uv)+f(u) \) yields any one of the magic constants \( \lambda_1 = 5n+1, \lambda_2 = 6n+1 \) and \( \lambda_3 = \frac{11n+3}{2} \).

Therefore, the generalized Petersen graph \( P(n, \frac{n-1}{2}) \) admits an edge trimagic total labeling.

Theorem: 2.8 The generalized Petersen graph \( P(n, \frac{n-1}{2}) \) admits a super edge trimagic total labeling.

Proof: We have proved that the generalized Petersen graph \( P(n, \frac{n-1}{2}) \) admits edge trimagic total labeling. The labeling given in the Theorem 2.7 for the vertices is, \( f(x_i) = 2n-i, 0 \leq i \leq n-1 \) and \( f(y_i) = n-i, 0 \leq i \leq n-1 \). Since the vertices get labels 1, 2, \ldots , 2n the generalized Petersen graph \( P(n, \frac{n-1}{2}) \) is a super edge trimagic total.

Example: 2.9 Generalized Petersen graph \( P(9, 4) \) is super edge trimagic total.

Figure 5: Petersen graph \( P(9, 4) \) with \( \lambda_1 = 46, \lambda_2 = 55 \) and \( \lambda_3 = 51 \).
Theorem: 2.10 The Helm graph $H_n$ has an edge trimagic total labeling for every positive even integer $n$.

Proof: Let $V = \{u\} \cup \{v_i : 1 \leq i \leq n\} \cup \{w_i : 1 \leq i \leq n\} \cup \{v_{i+1} : 1 \leq i \leq n-1\} \cup \{v_{1n}\}$ be the vertex set and $E = \{uv, vw_{i}, v_{i+1} : 1 \leq i \leq n\}$ be the edge set of the helm graph $H_n$. Then $H_n$ has $2n+1$ vertices and $3n$ edges.

Define a bijection $f: V \cup E \rightarrow \{1, 2, \ldots, 5n+1\}$ such that

- $f(u) = 1$,
- $f(v_i) = \begin{cases} \frac{i+1}{2} + 1, & \text{if } i \text{ is odd} \\ \frac{i+n}{2} + 1, & \text{if } i \text{ is even} \end{cases}$
- $f(w_i) = \begin{cases} n + \frac{i+1}{2} + 1, & \text{if } i \text{ is odd} \\ n + \frac{i+n}{2} + 1, & \text{if } i \text{ is even} \end{cases}$
- $f(uv_i) = \begin{cases} 5n - \frac{i+1}{2} + 2, & \text{if } i \text{ is odd} \\ 5n - \frac{n+i}{2} + 2, & \text{if } i \text{ is even} \end{cases}$
- $f(v_{1n}) = 4n+1$, $f(v_{i+1}) = 4n-i+1$, $1 \leq i \leq n-1$ and $f(v_{1i}) = 3n-i+2$, $1 \leq i \leq n$.

Now we prove this labeling is an edge trimagic total.

Consider the edges $uv_i$, $1 \leq i \leq n$.

For odd $i$, $f(u)+f(uv_i)+f(v_i) = 1 + \frac{i+1}{2} + 1 + 5n - \frac{i+1}{2} + 2 = 5n+4 = \lambda_1$ (say).

For even $i$, $f(u)+f(uv_i)+f(v_i) = 1 + 5n - \frac{i+n}{2} + 2 + \frac{i+n}{2} + 1 = 5n+4 = \lambda_1$.

Consider the edges $v_{i+1}$, $1 \leq i \leq n-1$.

For odd $i$, $f(v_i)+f(v_{i+1})+f(v_{i+1}) = \frac{i+1}{2} + 4n - i + 1 + \frac{i+1}{2} + 1 = 4n + \frac{n}{2} + 4 = \lambda_2$ (say).

For even $i$, $f(v_i)+f(v_{i+1})+f(v_{i+1}) = \frac{i+n}{2} + 4n - i + 1 + \frac{i+n}{2} + 1 = 4n + \frac{n}{2} + 4 = \lambda_2$.

Consider the edges $w_i$, $1 \leq i \leq n$.

For odd $i$, $f(v_i)+f(v_{i+1})+f(w_i) = \frac{i+1}{2} + 3n - i + 2 + n + \frac{i+1}{2} + 1 = 4n + 5 = \lambda_3$ (say).

For even $i$, $f(v_i)+f(v_{i+1})+f(w_i) = \frac{i+n}{2} + 3n - i + 2 + n + \frac{i+n}{2} + 1 = 5n+4 = \lambda_3$.

For the edge $v_{1n}$, $f(v_i)+f(v_{1n})+f(v_{1n}) = \frac{i+1}{2} + 4n + 1 + \frac{2n}{2} + 1 = 5n+4 = \lambda_1$.

Hence for each edge $uv \in E$, $f(u)+f(uv)+f(v)$ yields any one of the constants $\lambda_1 = 5n+4$, $\lambda_2 = 4n + \frac{n}{2} + 4$ and $\lambda_3 = 4n + 5$.

Therefore, the helm graph $H_n$ admits an edge trimagic total labeling for every positive even integer $n$.

Theorem: 2.11 The helm graph $H_n$ is a super edge trimagic total for even $n$.

Proof: We have proved that the helm graph $H_n$ is an edge trimagic total for even $n$. The labeling given in the proof of Theorem 2.10, the labeling for the vertices are $f(u) = 1$,

$$f(v_i) = \begin{cases} \frac{i+1}{2} + 1, & \text{if } i \text{ is odd} \\ \frac{i+n}{2} + 1, & \text{if } i \text{ is even} \end{cases}$$

and $f(w_i) = \begin{cases} n + \frac{i+1}{2} + 1, & \text{if } i \text{ is odd} \\ n + \frac{i+n}{2} + 1, & \text{if } i \text{ is even} \end{cases}$
Since the helm graph $H_n$ has $2n+1$ vertices and gets labels $1, 2, ..., 2n+1$, the helm graph $H_n$ is a super edge trimagic total labeling.

**Example: 2.12** The helm graph $H_n$ given in figure 6 admits a super edge trimagic total labeling with magic constants 29, 31, and 34.

\[ \text{Figure 6: Helm graph } H_6 \text{ with } \lambda_1 = 29, \lambda_2 = 31 \text{ and } \lambda_3 = 34. \]

**Theorem 2.13** The flower graph $F_n$ has an edge trimagic total labeling for all $n$.

**Proof:** Let $V = \{v_i, w_i / 1 \leq i \leq n\} \cup \{u\}$ be the vertex set and $E = \{u v_i, v_i w_i, u w_i / 1 \leq i \leq n\} \cup \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{v_1 v_n\}$ be the edge set of the flower graph $F_n$. Then the flower graph $F_n$ has $2n+1$ vertices and $4n$ edges.

Define a bijection $f: V \cup E \rightarrow \{1, 2, ..., 6n+1\}$ such that $f(u) = 1$, $f(v_i) = i+1$, $1 \leq i \leq n$; $f(w_i) = n+i+1$, $1 \leq i \leq n$; $f(u v_i) = 5n-i+2$, $1 \leq i \leq n$; $f(v_i v_{i+1}) = 4n-2i+1$, $1 \leq i \leq n-1$; $f(v_i w_i) = 4n-2i+2$, $1 \leq i \leq n$; $f(u w_i) = 6n-i+2$, $1 \leq i \leq n$ and $f(v_1 v_n) = 4n+1$.

Now, we prove the above labeling is an edge trimagic total.

For the edges $u v_i$, $1 \leq i \leq n$,

\[ f(u)+f(u v_i)+f(v_i) = 1+5n-i+2+i+1 = 5n+4 = \lambda_1 \text{ (say)}. \]

For all the edges $v_i v_{i+1}$, $1 \leq i \leq n-1$,

\[ f(v_i)+f(v_i v_{i+1})+f(v_{i+1}) = i+1+4n-2i+1+i+1+1 = 4n+4 = \lambda_2 \text{ (say)}. \]

For the edges $v_i w_i$, $1 \leq i \leq n$,

\[ f(v_i)+f(v_i w_i)+f(w_i) = i+1+4n-2i+2+n+i+1 = 5n+4 = \lambda_1. \]

For the edge $u w_i$, $1 \leq i \leq n$,

\[ f(u)+f(u w_i)+f(w_i) = 1+6n-i+2+i+1 = 7n+4 = \lambda_2 \text{ (say)}. \]

And for the edge $v_1 v_n$, $f(v_1)+f(v_1 v_n)+f(v_n) = 1+1+4n+1+n+1 = 5n+4 = \lambda_1$.

Hence for each edge $uv$, $f(u)+f(uv)+f(v)$ yields any one of the magic constant $\lambda_1 = 5n+4$, $\lambda_2 = 4n+4$ and $\lambda_3 = 7n+4$.

Therefore, the flower graph $F_n$ admits an edge trimagic total labeling for all $n$.

**Theorem 2.14** The flower graph $F_n$ is a super edge trimagic total for all $n \geq 3$.

**Proof:** We have proved that the flower graph $F_n$ admits an edge trimagic total labeling for $n \geq 3$. The labeling given in the proof of the Theorem 2.13, the labeling for the vertices are, $f(u) = 1$, $f(v_i) = i+1$, $1 \leq i \leq n$; $f(w_i) = n+i+1$, $1 \leq i \leq n$.

Since the flower graph $F_n$ has $2n+1$ vertices and gets labels $1, 2, ..., 2n+1$, the flower graph $F_n$ is a super edge trimagic total labeling.
Example: 2.15 The flower graph Fl₆ given in figure 7 is a super edge trimagic total graph with magic constants 34, 28 and 46.

![Flower graph Fl₆ with λ₁ = 34, λ₂ = 28 and λ₃ = 46.](image)

**Figure 7:** Flower graph Fl₆ with λ₁ = 34, λ₂ = 28 and λ₃ = 46.

**CONCLUSION**

In this paper, we have proved some classes of graphs namely, the ladder Lₙ, triangular ladder TLₙ, generalized Petersen graph P(n, n-1, 2), the helm graph Hₙ and the flower graph Flₙ are edge trimagic total and super edge trimagic total graphs. There will be many trimagic graphs can be constructed in future.

**REFERENCES**


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