wgα-closed and wαg-closed in Ideal Topological Spaces

K. Indirani¹, V. Rajendran² and P. Sathishmohan³*

¹Department of Mathematics, Nirmala college for woman, Coimbatore, (T.N.), India.
², ³Department of Mathematics, KSG college, Coimbatore, (T.N.), India.

(Received on: 02-10-13; Revised & Accepted on: 16-11-13)

ABSTRACT

In this paper some properties of wgα-I-closed sets and wαg-I-closed sets are studied.

Keywords: gα-I-closed, αg-I-closed, wgα-I-closed, wαg-I-closed.

1. INTRODUCTION AND PRELIMINARIES

An ideal I on a topological space (X, τ) is a non-empty collection of subsets of X which satisfies the following properties. (1) A ∈ I and B ⊆ A implies B ∈ I, (2) A ∈ I and B ∈ I implies A ∪ B ∈ I. An ideal topological space is a topological space (X, τ) with an ideal I on X and is denoted by (X, τ, I). For a subset A ⊆ X, A*(I, τ) = {x ∈ X: A ∩ U /∈ I for every U ∈ τ (X, x)} is called the local function of A with respect to I and τ [7]. We simply write A* in case there is no chance for confusion. A kuratowski closure operator cl*(.) for a topology τ*(I, τ) called the *- topology, finer than τ is defined by cl*(A) = A ∪ A* [11]. If A ⊆ X, cl(A) and int(A) will respectively, denote the closure and interior of A in (X, τ).

Definition: 1.1 A subset A of a topological space (X, τ) is called
1. α-closed [10], if cl (int (cl(A))) ⊆ A
2. αg-closed [5], if α cl (A) ⊆ U whenever A ⊆ U and U is open in (X, τ)
3. gα-closed [5], if αcl(A) ⊆ U whenever A ⊆ U and U is α-open in (X, τ)
4. wgα-closed [6], if αcl (int(A)) ⊆ U whenever A ⊆ U and U is α-open in (X, τ).
5. wαg-closed [6], if αcl (int(A)) ⊆ U whenever A ⊆ U and U is open in (X, τ).
6. g-closed [8], if cl (A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
7. gs-closed [1], if scl (A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
8. sg-closed [3], if scl (int(A)) ⊆ U whenever A ⊆ U and U is semi open in (X, τ)
9. β-closed [10], if int (cl (int(A))) ⊆ A

The complements of the above mentioned closed sets are called their respective open sets.

Definition: 1.3 A subset A of an ideal topological spaces (X, τ, I) is said to be
1. α-I-closed [4], if cl (int(αcl(A))) ⊆ A
2. αg-I-closed [9], if αcl (A) ⊆ U whenever A ⊆ U and U is α-open in X.
3. gα-I-closed [9], if α cl (A) ⊆ U whenever A ⊆ U and U is open in X.

The complements of the above mentioned closed sets are called their respective open sets.

2. wgαI-closed and wαgI-closed sets

Definition: 2.1 A subset A of an Ideal topological space (X, τ, I) is said to be
1) wgαI-closed set, if αcl (Int (A)) ⊆ U whenever A ⊆ U and U is α-open in X.
2) wαgI-closed set, if αcl (Int (A)) ⊆ U whenever A ⊆ U and U is open in X.

Corresponding author: P. Sathishmohan³*

², ³Department of Mathematics, KSG college, Coimbatore, (T.N.), India.
Proposition 2.2 Every $\alpha$-I-closed set is w$\alpha$I-closed set but not conversely.

Proof: Assume that a subset $A$ of $(X, \tau, I)$ is $\alpha$-I-closed set. Let $U$ be an $\alpha$-open set containing $A$. Then $\alpha cl(A) \subseteq U$ as $A$ is $\alpha$-I-closed. So $\alpha cl(\text{Int}(A)) \subseteq \alpha cl(A) \subseteq U$. This implies that $\alpha cl(\text{Int}(A)) \subseteq U$. Hence $A$ is w$\alpha$I-closed.

Example 2.3 Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, X\}$ and $I = \{\phi, \{c\}\}$. Then $A = \{c\}$ is a w$\alpha$I-closed but not $\alpha$-I-closed.

Proposition 2.4 Every $g\alpha$-closed set is w$\alpha$I-closed set but not conversely.

Proof: Let $A$ be a subset of $(X, \tau)$ which is $g\alpha$-closed and let $U$ be an $\alpha$-open set containing $A$. Since $A$ is $g\alpha$-closed, $\alpha cl(A) \subseteq U$, $\alpha cl(A) \subseteq \alpha cl(A) \subseteq U$. This implies that $\alpha cl(\text{Int}(A)) \subseteq \alpha cl(A) \subseteq U$. Hence $A$ is w$\alpha$I-closed.

Example 2.5 Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a, b\}, X\}$ and $I = \{\phi, \{a\}\}$. Then $A = \{b\}$ is a w$\alpha$I-closed but not $g\alpha$-closed.

Proposition 2.6 Every $g\alpha$-I-closed set is w$\alpha$I-closed set but not conversely.

Proof: Assume that a subset $A$ of $(X, \tau, I)$ is $g\alpha$-I-closed set. Let $U$ be an $\alpha$-open set containing $A$. Therefore $\alpha cl(I\text{Int}(A)) \subseteq \alpha cl(A) \subseteq U$, therefore $A$ is w$\alpha$I-closed.

Example 2.7 Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a, b\}, X\}$ and $I = \{\phi, \{a\}\}$. Then $A = \{b\}$ is a W$\alpha$I-closed but not $g\alpha$-closed.

Remark 2.8 Suppose $I = \{\phi\}$, then the notion of w$\alpha$I-closed and w$\alpha$I-closed sets coincide with w$\alpha$-closed and w$\alpha$-closed set.

Remark 2.9 The following examples show that the concepts of g-closed and w$\alpha$I-closed sets are independent.

Example 2.10 Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a, b\}, X\}$ and $I = \{\phi, \{a\}\}$. Then $A = \{b\}$ is a w$\alpha$I-closed but not $g$-closed set.

Example 2.11 Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, X\}$ and $I = \{\phi, \{c\}\}$. Then $A = \{a, b\}$ is a $g$-closed but not w$\alpha$I-closed.

Remark 2.12 The following examples show that the concepts of sg-closed and w$\alpha$I-closed sets are independent.

Example 2.13 Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a, b\}, X\}$ and $I = \{\phi, \{a\}\}$. Then $A = \{b\}$ is a w$\alpha$I-closed but not sg-closed.

Example 2.14 Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $I = \{\phi, \{a\}\}$. Then $A = \{b\}$ is a sg-closed but not w$\alpha$I-closed.

Remark 2.15 The following examples show that the concept of gs-closed and w$\alpha$I-closed sets is independent.

Example 2.16 Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $I = \{\phi, \{a\}\}$. Then $A = \{c\}$ is a gs-closed but not w$\alpha$I-closed.

Example 2.17 Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, X\}$ and $I = \{\phi, \{a\}\}$. Then $A = \{a\}$ is a w$\alpha$I-closed but not gs-closed.

Proposition 2.18 Every $\alpha$-I-closed set is w$\alpha$I-closed set but not conversely.

Proof: Assume that a subset $A$ of $(X, \tau, I)$ is $\alpha$-I-closed set. Let $U$ be an $\alpha$-open set containing $A$. Then $\alpha cl(A) \subseteq U$, as $A$ is $\alpha$-I-closed. So $\alpha cl(\text{Int}(A)) \subseteq \alpha cl(A) \subseteq U$. This implies that $\alpha cl(\text{Int}(A)) \subseteq U$. Hence $A$ is w$\alpha$I-closed.

Example 2.19 Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, X\}$ and $I = \{\phi, \{a\}, \{b\}, \{a, b\}\}$. Then $A = \{a, b\}$ is a w$\alpha$I-closed but not $\alpha$-I-closed.

Proposition 2.20 Every $g\alpha$-closed set is w$\alpha$I-closed set but not conversely.

Proof: Assume that a subset $A$ of $(X, \tau, I)$ is $g\alpha$-closed set. Let $U$ be an open set containing $A$. Then $\alpha cl(A) \subseteq U$, as $A$ is $g\alpha$-closed. Since every $\alpha$-I-closed set is $\alpha$-closed, $\alpha cl(\text{Int}(A)) \subseteq \alpha cl(A) \subseteq U$. Hence $A$ is w$\alpha$I-closed.
Example: 2.21 Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a, b\}, X\}$ and $I = \{\emptyset, \{a\}\}$. Then $A = \{a\}$ is a wαg-I-closed but not αg-closed.

Proposition: 2.22 Every pαgI-closed set is wαgI-closed set but not conversely.

Proof: Assume that a subset $A$ of $(X, \tau, I)$ is pαgI-closed set. Let $U$ be an open set containing $A$. Then $\alpha \text{cl}(A) \subseteq U$ and $\alpha \text{cl}(\text{Int}(A)) \subseteq \alpha \text{cl}(A) \subseteq U$. Hence $A$ is wαgI-closed.

Example: 2.23 Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, c\}, X\}$ and $I = \{\emptyset, \{a\}\}$. Then $A = \{a\}$ is a wαgI-closed but not αg-I-closed.

Remark: 2.24 The following examples show that the concept of β-closed and wαg-I-closed sets are independent.

Example: 2.25 Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $I = \{\emptyset, \{b\}\}$. Then $A = \{a, b\}$ is a wαgI-closed but not β-closed.

Example: 2.26 Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $I = \{\emptyset, \{a\}\}$. Then $A = \{a\}$ is β-closed but not wαg-I-closed.

Remark: 2.27 The following examples show that the concepts of sg-closed and wαgI-closed sets are independent.

Example: 2.28 Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, c\}, X\}$ and $I = \{\emptyset, \{a\}\}$. Then $A = \{a\}$ is wαgI-closed but not sg-closed.

Example: 2.29 Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $I = \{\emptyset, \{a\}\}$. Then $A = \{b\}$ is sg-closed but not wαg-I-closed.

Remark: 2.27 The following examples show that the concepts of gs-closed and wαgI-closed sets are independent.

Example: 2.30 Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}$ and $I = \{\emptyset, \{c\}\}$. Then $A = \{c\}$ is gs-closed but not wαgI-closed.

Example: 2.31 Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, c\}, X\}$ and $I = \{\emptyset, \{a\}\}$. Then $A = \{a\}$ is wαgI-closed but not gs-closed.

Proposition: 2.32 Every g-closed set is wαgI-closed set but not conversely.

Proof: Assume that a subset $A$ of $(X, \tau, I)$ is g-closed set. Let $U$ be an open set containing $A$. Then $\text{cl}(A) \subseteq U$, as $A$ is g-closed. Then $\alpha \text{cl}(A) \subseteq \text{cl}(A) \subseteq U$. Since every $\alpha$-I-closed set is $\alpha$-closed. $\alpha \text{cl}(\text{Int}(A)) \subseteq \alpha \text{cl}(A) \subseteq U$. Hence $A$ is wαgI-closed.

Example: 2.32 Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, c\}, X\}$ and $I = \{\emptyset, \{a\}\}$. Then $A = \{a\}$ is wαgI-closed but not in gαI-closed.

Proposition: 2.33 Every wgαI-closed set is wαgI-closed set but not conversely.

Proof: Assume that a subset $A$ of $(X, \tau, I)$ is wgαI-closed set. Let $U$ be an open set containing A. From the above theorems, $\alpha \text{cl}(\text{Int}(A)) \subseteq U$. Since every open set is $\alpha$-open. Hence A is wαgI-closed.

Example: 2.34 Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, c\}, X\}$ and $I = \{\emptyset, \{a\}\}$. Then $A = \{c\}$ is wαgI-closed but not in g-closed.

Proposition: 2.35 Every gαI-closed set is wαgI-closed set but not conversely.

Proof: Assume that a subset $A$ of $(X, \tau, I)$ is gαI-closed set. Let $U$ be an open set containing A. From the above theorems, $\alpha \text{cl}(\text{Int}(A)) \subseteq U$. Since every open set is $\alpha$-open. Hence A is wαgI-closed.

Example: 2.36 Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ and $I = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. Then $A = \{a, c\}$ is wαgI-closed but not in gαI-closed.

Proposition: 2.37 Every wgαI-closed set is wαgI-closed set but not conversely.

Proof: Let $A$ be a wgαI-closed set in $(X, \tau, I)$ and Let $U$ be an open set containing A. Since A is wαgI-closed. So $\alpha \text{cl}(\text{Int}(A)) \subseteq U$. Hence A is wαgI-closed in $(X, \tau, I)$.

Example: 2.38 Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$ and $I = \{\emptyset, \{c\}\}$. Then $A = \{a, b\}$ is wαgI-closed but not in wgαI-closed.
REFERENCES

5. H.Maki, R.devi and K.Balachandran, Associate topologies if generalized $\alpha$-closed sets and $\alpha$-generalized closedsets, Mem Fac,Kochi,Univ,Ser., (1994), 51-63.
6. K.Ramasamy, A.Viswanathan and A.Parvathi., On Weakly $\alpha$-closed sets and weakly $\alpha g$-closed sets into topological spaces, AJM(to appear).

Source of support: Nil, Conflict of interest: None Declared