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HEAT AND MASS TRANSFER IN FREE CONVECTION FLOW PAST AN OSCILLATING PLATE IN POROUS MEDIA

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ABSTRACT

The influence of various flow entities that influence velocity and skin friction in case of mixed convection flow over an inclined plate has been examined at length in this paper. It is seen that as the Schmidt number increases the velocity also increases. Not much of significant variation is seen near the boundary layer, but more dispersion is observed as we move away from the lower plate. Further, in some cases, backward flow is noticed and this could be due to the increase in the frequency of oscillation. Also, it has been observed that, the nature of velocity profiles don't change qualitatively even when the Grashof number is changed marginally. This indicates that the Grashof number does not play any significant role for marginal increases. In a situation, when both Gr and Gm are identical, the velocity decreasing initially and thereafter increases in the core region subsequently decreases. However, a backward flow is not seen in this case. In general, it is seen that as the Schmidt number increases the boundary surface does not experience any friction. However as the Grashof number increases the skin friction appears to be increasing and up core minutely. It is concluded that while all other parameters remaining same, increase in Grashof number contributes increase in skin friction. Increase in Gr (modified Grashof number) is found to be relatively proportional to increase in skin friction.

Key words: *Velocity, Skin friction, modified Grashof number, Grashof number, Schmidt number.*

INTRODUCTION:

In several biological and in engineering systems flow through a channel plays an important role. Some examples in living organisms are fluid transport mechanisms among which one blood flow in human body , air flow in lungs, flow system for transporting lymph; using circulatory system and transpiration of cooling in internal combustion engines. The applications are many and are diversified in their nature. However, the transport phenomena remains unchanged. A classical example is in nuclear power station where the separation of Uranium U_{235} from U_{238} by gases diffusion. In many chemical processing industries generally slurry adheres to the reactor vessels and gets consolidated. As a result of this, the chemical compounds within the reactor vessel percolates through the boundaries causing loss of production and then consuming more reaction time. The slurry thus consolidated inside the reactor vessel often acts as a porous boundary for the next cycle of chemical processing.

Flow through porous media has been the subject of considerable research activity in recent years because of its several important applications notably in drug permeation through human skin, chemical reactor for economical separation or purification of mixtures, the evaluation of the capability of heat removal from particulate nuclear fuel debris that may result from a hypothetical accident in a nuclear reactor, the flow of oil through porous rock, the extraction of geothermal energy from the deep interior of the earth to the shallow layers, the filtration of solids from liquids, flow of liquids through ion-exchange beds and so on.

The viscous force imparted by a flowing fluid in a dense swarm of particles was first estimated and analysed by Brinkman [1]. Subsequently, the development of boundary layer growth with the introduction of suction and injection was examined by Hassmoto [2]. The two dimensional steady state flow of an incompressible viscous fluid with parallel rigid with porous wall with the flow being given by uniform suction or injection was studied by Berman [3].Later, the work of Berman was examined by Sellars [4] for high section Reynolds's number. The flow between two vertical plates which are electrically non-conducting and under the assumption that the wall temperature varies linearly in the direction of the flow and the existence of the heat source in the vertical channel was discussed by Mori[5]. Later, Macey [6] discussed the flow in the renal tubules as viscous flow through a circular tube of uniform cross section and permeable boundary by prescribing their radial velocity at the wall as exponentially decreasing function of axial distance. Subsequently, the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate, was studied by England and Emery [7] while Soundlegkar and Thakar [8] had examined the radiative free convective flow of an optically thin gray gas past a semi infinite vertical plate. Thereafter, the

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steady flow of a non-Newtonian fluid past a porous plate with a suction or injection has been studied by Mansutti *et.al [*9]. Sattar [10] had reported that, the free convection and mass transfer flow through a porous medium past an infinite vertical porous plate with time dependent temperature and concentration. Later, Das *et al* [11] had studied the radiation effects on flow past an impulsively started infinite isothermal vertical plate while, Chowdhary and Dass [12] investigated the magneto hydro dynamic boundary layer flow of a non Newtonian fluid past a flat plate. Later, Das *et al* [13] obtained numerically approximations for the mass transfer effects on unsteady flow past an accelerated vertical porous plate. Recently, Ramana Reddy and Ramana Murthy [14] discussed the mixed convective MHD flow and mass transfer past an accelerated infinite vertical porous plate wherein the influence of various flow entities and their effect on velocity field has been examined in detail. It has been pointed out that Grashof heat and mass transfer parameters significantly influence the flow entities.

The study reported herein is related to the unsteady mixed convective heat and mass transfer flow of a viscous incompressible electrically conducting fluid past an accelerating infinite vertical porous flat plate with suction in the presence of transverse magnetic field. These governing equations of motion are solved to a better converging solution. The flow phenomena has been characterized with the help of flow parameter and the effect of these parameters on the velocity field and skin friction have been analyzed and the results are presented graphically and discussed qualitatively.

MATHEMATICAL FORMULATION:

We consider unsteady, free convection flow of an incompressible and electrically conducting viscous fluid along an infinite non-conducting vertical flat plate through a porous medium. The x- axes taken along the plate in the vertically upward direction and y-axes is taken normal to the plate. A magnetic field of uniform strength B_0 is applied in the direction of flow and the induced magnetic field is neglected. Initially, the plate and the fluid are at same temperature T_∞ in a stationary condition with concentration level C_∞ at all points. At time $t>0$ the plate starts oscillating in its own plane with a velocity $U_0 \cos \omega t$ .Its temperature raise to C_w .Using the Bossiness approximation, the governing equations for the flow are given by:

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta(T-T_\infty) + g\beta^*(C-C_\infty) - \frac{\sigma B_0^2 u}{\rho} - \frac{\upsilon u}{K} \tag{1}$$

$$\frac{\partial T}{\partial t} = \frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \tag{2}$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2} \tag{3}$$

The boundary conditions are given by

$$u=0, T=T_\infty, C=C_\infty \text{ for all } y,t \le 0 \tag{4}$$

$$u=U_0\cos\omega t, T=T_w, C=C_w \text{ as } y=0, t>0$$

$$u=0, T=T_\infty, C=C_\infty, \text{ as } y\to\infty, t>0 \tag{5}$$

Introducing the non-dimensional variables

$$u^* = \frac{u}{U_0}, t^* = \frac{tU_0^2}{\upsilon}, y^* = \frac{yU_0}{\upsilon}, K^* = \frac{U_0^2 K}{\upsilon^2},$$

$$M = \frac{\sigma B_0^2 \upsilon}{\rho U_0^2}, \Pr = \frac{\mu C_p}{K}, \omega^* = \frac{\omega\upsilon}{U_0^2}$$

$$Gr = \frac{\upsilon g\beta(T_w - T_\infty)}{U_0^3}, Gm = \frac{\upsilon g\beta^*(C_w - C_\infty)}{U_0^3}$$

$$Sc = \frac{\upsilon}{D}, \theta^* = \frac{T-T_\infty}{T_w - T_\infty}, \phi^* = \frac{C-C_\infty}{C_w - C_\infty} \tag{6}$$

Where D is mass diffusivity, Gr is Grashof number, Gm is modified Grashof number, K is permeability parameter, M is magnetic parameter, Pr is Prandtl number, Sc is Schmidt number, β is thermal expansion coefficient and β^* is concentration expansion coefficient and ω is frequency of oscillation. Other physical variables have their usual meanings. With the help of (6), the governing equations with the boundary conditions reduce to

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gm\phi - \left(M + \frac{1}{K}\right)u \tag{7}$$

$$\Pr\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} \tag{8}$$

$$Sc\frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial y^2} \tag{9}$$

$$u=0, \theta=0, \phi=0 \text{ for all } y,t\le 0 \tag{10}$$

$$\left.\begin{aligned} &u=\cos\omega t, \theta=1, \phi=1 \text{ at } y=0, t>0 \\ &u=0, \theta=0, \phi=0 \text{ at } y\to\infty, t>0 \end{aligned}\right\} \tag{11}$$

SOLUTION FOR PROBLEM:

Let the solution of the equation (7),(8),and (9) be of the form

$$u(y,t) = u_0(y)e^{-i\omega t} \tag{12}$$

$$\theta(y,t) = \theta_0(y)e^{-i\omega t} \tag{13}$$

$$\phi(y,t) = \phi_0(y)e^{-i\omega t} \tag{14}$$

With the modified boundary conditions as:

$$u_0 = e^{i\omega t}\cos\omega t, \theta_0 = e^{i\omega t}, \phi_0 = e^{i\omega t} \text{ at } y=0$$

$$u_0 = 0, \theta_0 = 0, \phi_0 = 0 \text{ as } y\to\infty \tag{15}$$

Substituting equation (12), (13), (14) and (15) in equation (7), (8), and (9), we obtain

$$u(y,t)=\begin{bmatrix} \exp(-Ry)\cos\omega t \\ +\dfrac{Gr}{R^2+\omega^2\Pr^2}\begin{pmatrix}\exp(-Ry)- \\ \exp(-i\omega\Pr y)\end{pmatrix} \\ +\dfrac{Gm}{R^2+\omega^2 Sc^2}\begin{pmatrix}\exp(-Ry)- \\ \exp(-i\omega Scy)\end{pmatrix} \\ +\dfrac{G^*\sin\alpha}{R^2}(\exp(-Ry)-1) \end{bmatrix} \tag{16}$$

$$\theta(y,t)=\exp(-i\omega\Pr y) \tag{17}$$

$$\phi(y,t)=\exp(-i\omega Scy) \tag{18}$$

The relation for Skin - friction is:

$$\frac{\partial u}{\partial y}\Big|_{y=0}=\begin{bmatrix} -R\cos\omega t \\ +\dfrac{Gr}{R^2+\omega^2\Pr^2}(-R+i\omega\Pr) \\ +\dfrac{Gm}{R^2+\omega^2 Sc^2}(-R+i\omega Sc) \\ -\dfrac{G^*\sin\alpha}{R} \end{bmatrix} \tag{19}$$

RESULTS AND CONCLUSIONS:

The Influence of Schmidt number on velocity profiles has been illustrated in figure -1, figure -2 and figure-3. In figure -1, it is seen that as the Schmidt number increases the velocity also increases. However, not much of significant variation is seen near the boundary layer, but more dispersion is observed as we move away from the lower plate.

In figure-2, similar such effect is observed but the velocity profiles appear to be converged as we move faraway from the bounding surface. Such convergence can be attributed due to the increase in the frequency of oscillation (ω)

The influence of Schmidt number with respective to velocity is illustrated in figure-3 and figure-4. It is observed that as the Schmidt number increases the velocity increases. In this case, a backward flow is noticed. Such a backward flow is due to increase in time as well as and increase in the frequency of oscillation. Similar such effect can be seen in figure – 5.

A consolidated effect of Schmidt number with respective to the Grashof number is illustrated in figure-6. It is noted that from figure-5 and figure-6, the Grashof number play an important role in the fluid velocity. It is observed that, as the Schmidt number increases the velocity decreases, which is contrast to observation in figure-5. Even in this case a backward flow is observed. But it is found to be less significant when compared to figure-5.

From figure-7 it can be seen that the nature of velocity profiles do not change qualitatively even when the Grashof number is changed marginally. In this situation, this indicates that the Grashof number does not play any significant role for marginal increases in the Grashof number

The influence of Schmidt number with respective to Gm (modified Grashof number) is illustrated in figure-8. In this case as the Schmidt number increases the velocity increases. In this situation also, near the boundary layer region, a back word flow is observed. But as we move faraway from the boundary surface, more of farward flow is seen. From the figure-5, figure-6, figure-7and figure-8 it can be concluded that the driving forces required to move the fluid, in the forward direction - the contribution of Grashof number and Gm (modified Grashof number) is essential.

From the figure-8 and figure-9. It is seen that Grashof number and Gm (modified Grashof number) contributes to more of backward flow. In case of figure-9, as the Schmidt number increases the velocity decreases.

The combined influence of Grashof number and Gm (modified Grashof number) with respective to the Schmidt number has been illustrated in figure – 10. In this situation, when both Gr and Gm are identical, the velocity decreases initially and then increases in the core region and there after it decreases. However, a backward flow is not seen in this case.

The influence of the Schmidt number on the velocity profiles is shown in figure-11. In this situation the phenomena is as stated in figure-10 remains same. However, a back word flow is observed. Such a backward flow is particularly due to the change in the frequency of oscillation (ω) and also due to a time (t).

Figure – 12 depicts the effect of the Schmidt number on the velocity profiles. It is seen that, as the Schmidt number increases the velocity decreases. When this situation is compared with figure-11, it is seen that no backward flow is observed. Such a result can be attributed to increases in the time parameter.

In figure-13. It is observed that, as the Schmidt number increases, as usual the velocity decreases. In this case when the values of Grashof number, Gm (modified Grashof number) are held at same level, backward flow is observed. When compared, figure-13 with figure-14, it can be seen that primarily the backward flow is due to the combined influence of decrease in the frequency of oscillation as well as time.

Figre-14 shows the influence the Schmidt number on skin friction. In general, it is seen that as the Schmidt number increases the boundary surface does not experience any friction. Further, as the Schmidt number increases the skin friction is found to be very less. However, as the Grashof number increases the skin friction appears to be increasing minutely.

Figure-16 shows the influences of Schmidt number on skin friction. When the bounding surface is inclined at $\pi/6$. In this situation, as the Schmidt number increases the skin friction on the boundary increases in general. As seen in the earlier cases, increases in Grashof number contribute to increasing in the skin friction.

Figure-17 shows the nature of skin friction when Grashof number is 9.0. From figure-16 and figure-17, it is concluded that increase in Grashof number, while all other parameters remains the same, the skin friction increases. As seen in the earlier cases increase Gr (modified Grashof number) is proportional to increase in the skin friction.

Figure-18 illustrates the behavior of skin friction with respect to Schmidt number and Gr (modified Grashof number) when the bounding surface indiclined at $\pi/4$. When this situation is compared with figure-16 in general, it is observed that the Skin Friction decreases more rapidly. In figure-18, figure-16 when Grashof number is held constant the skin friction decreases at fast rate. Similar such observation is noted in figure-17 and figure-19.

Figure-20 and figure-21 illustrates the influence of other parameters on skin friction in both these situations when Grashof number increases, the skin friction decreases more. However, the pattern of the profiles remains unchanged.

Figure-22 and figure-23 shows the behavior of skin friction when the bounding surface is held purely vertical. In both these situations the influence of Grashof number is not felt much. However, the nature of the Schmidt number and the Gr (modified Grashof number) doesn't qualitatively alter the nature of profiles of skin-friction

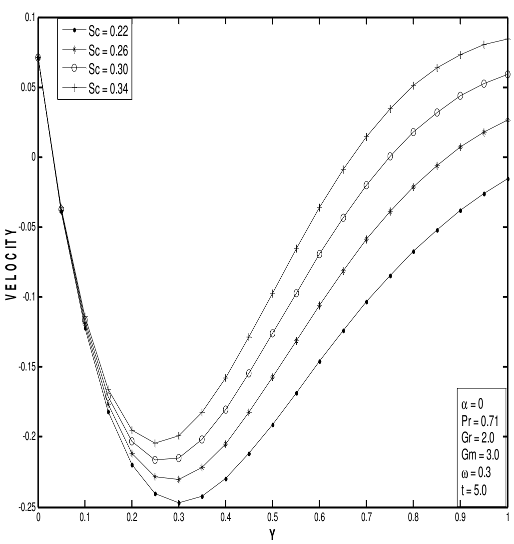

Figure -1: Effect of Schmidt No. on Velocity Profiles

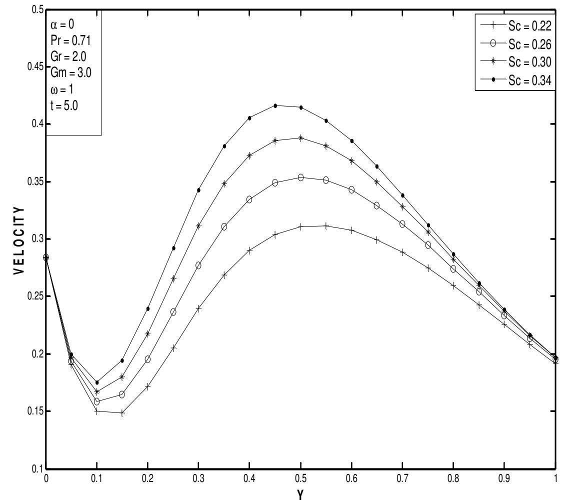

Figure – 2: Influence of Schmidt No. on Velocity Profiles

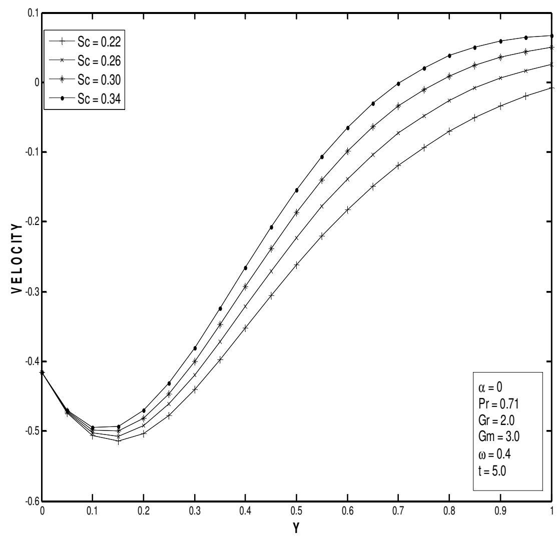

Figure – 3: Influence of Schmidt No. on Velocity Profiles

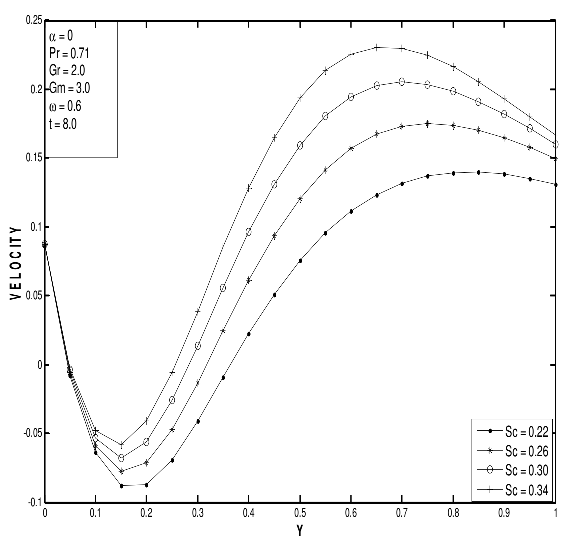

Figure – 4: Effect of Schmidt No. on Velocity Profiles

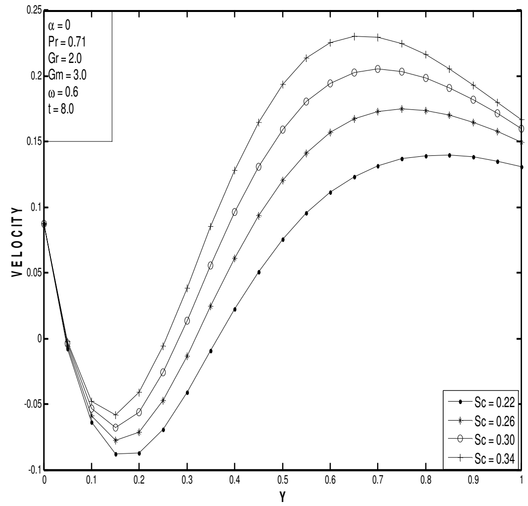

Figure – 5: Influence of Schmidt No. on Velocity Profiles

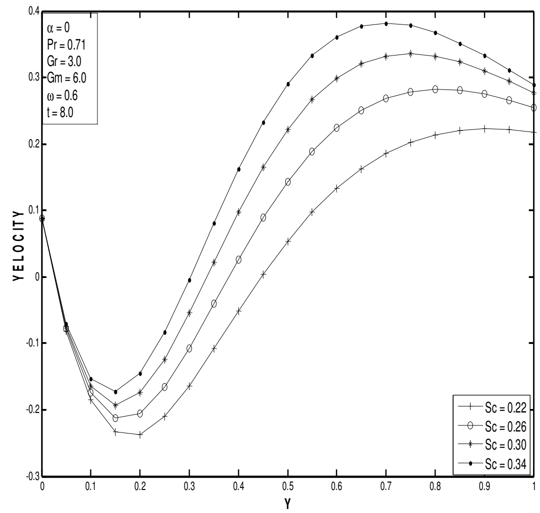

Figure – 8: Contribution of of Schmidt No. on Velocity Profiles

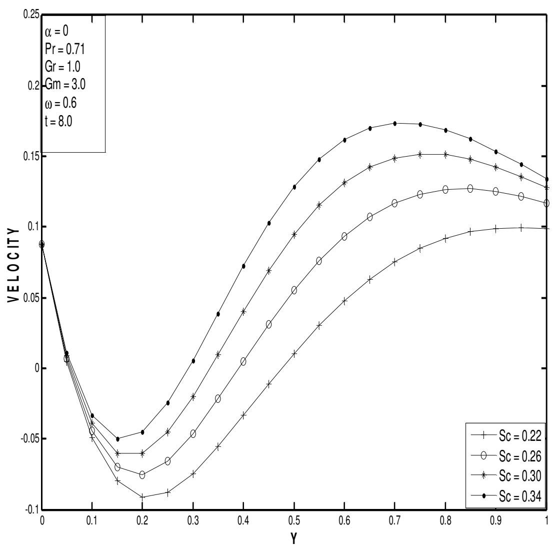

Figure – 6: Effect of Schmidt No. on Velocity Profiles

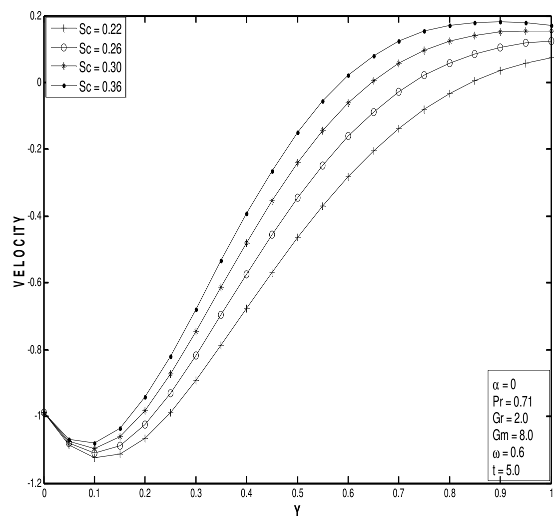

Figure -9: Effect of Schmidt No. on Velocity Profiles

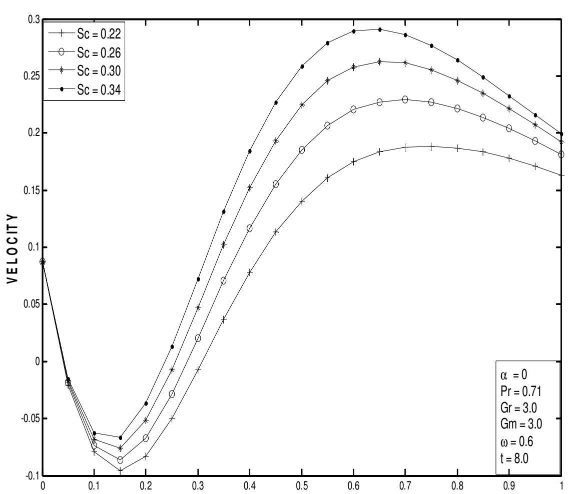

Figure – 7: Influence of Schmidt No. on Velocity Profiles

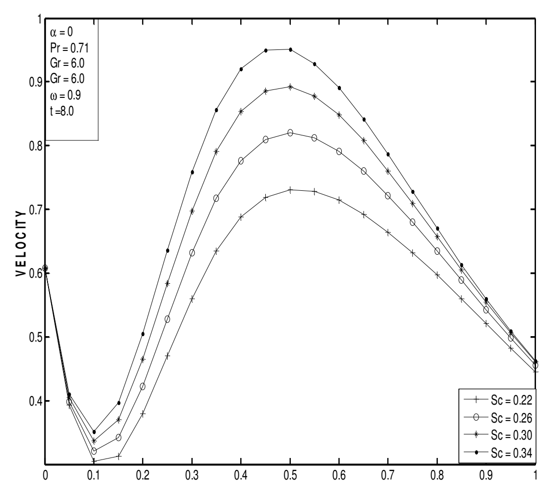

Figure –10: Influence of Schmidt No. on Velocity Profiles

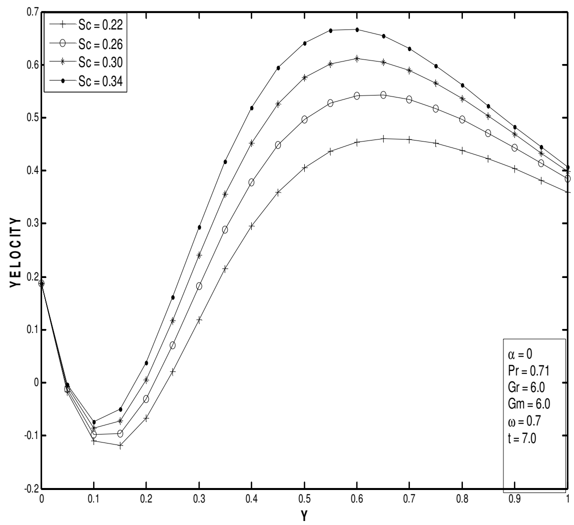

Figure –11: Contribution of Schmidt No. on Velocity Profiles

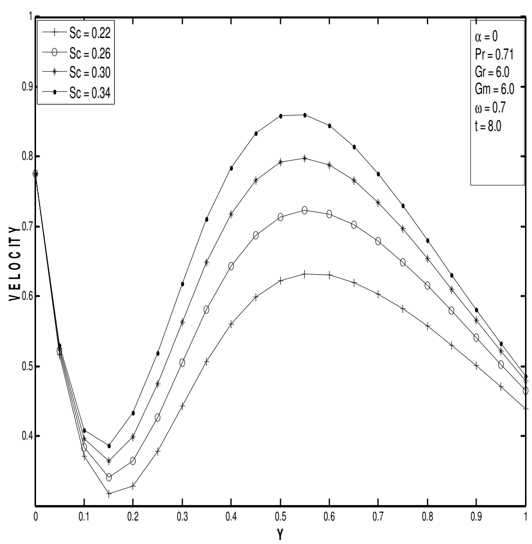

Figure –12: Influence of Schmidt No. on Velocity Profiles

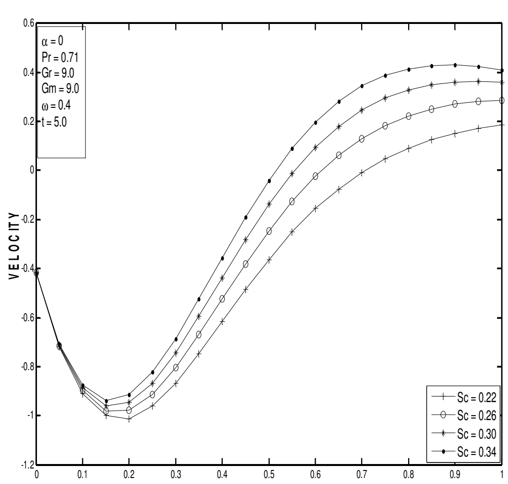

Figure –13: Influence of Schmidt No. on Velocity Profiles

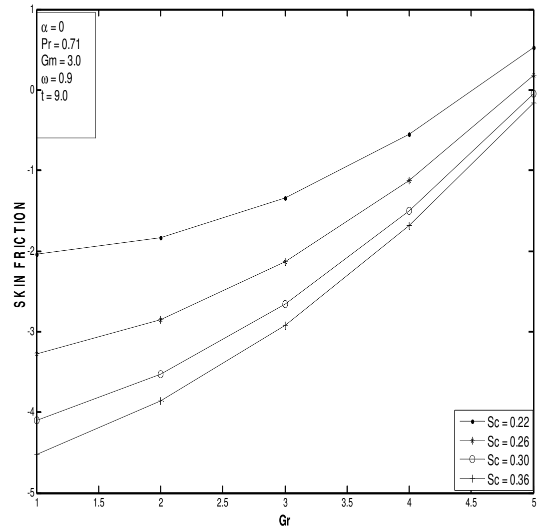

Figure –14: Influence of Schmidt No. on Skin Friction

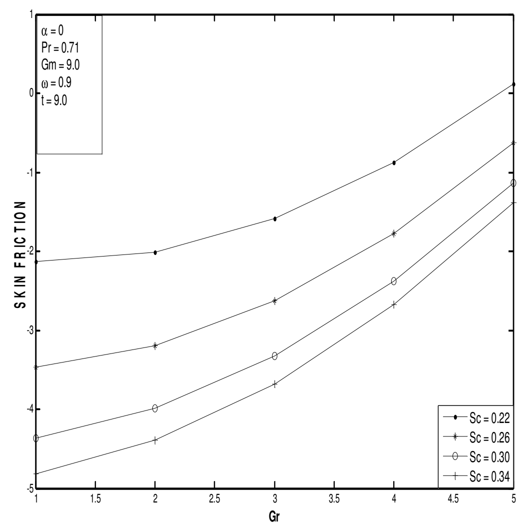

Figure 15: Contribution of Schmidt No. on Skin Friction

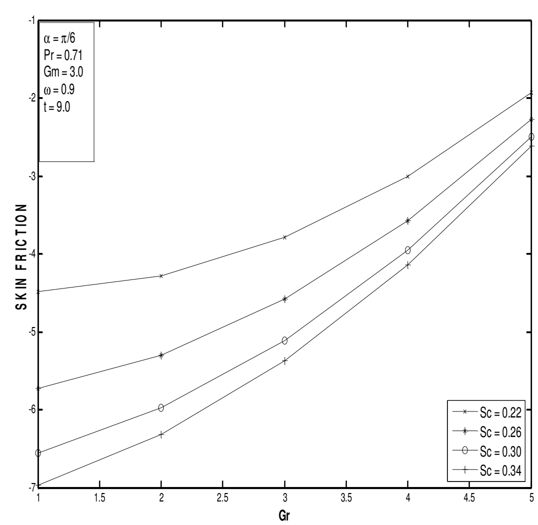

Figure 16: Effect of Schmidt No. on Skin Friction

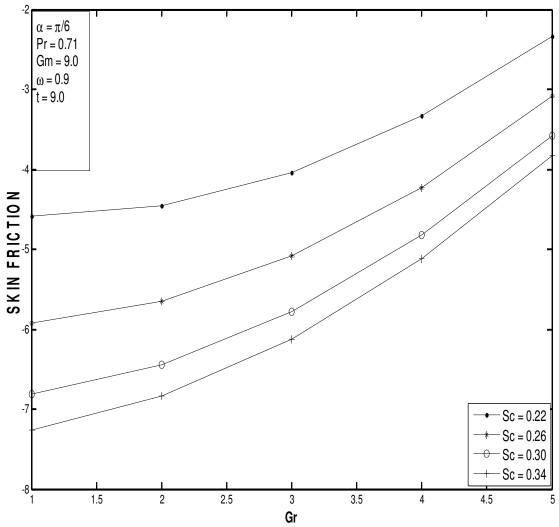

Figure 17: Influence of Schmidt No. on Skin Friction

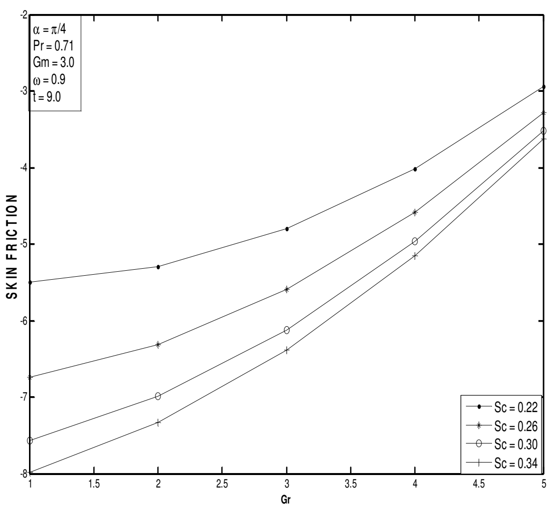

Figure 18: Effect of Schmidt No. on Skin Friction

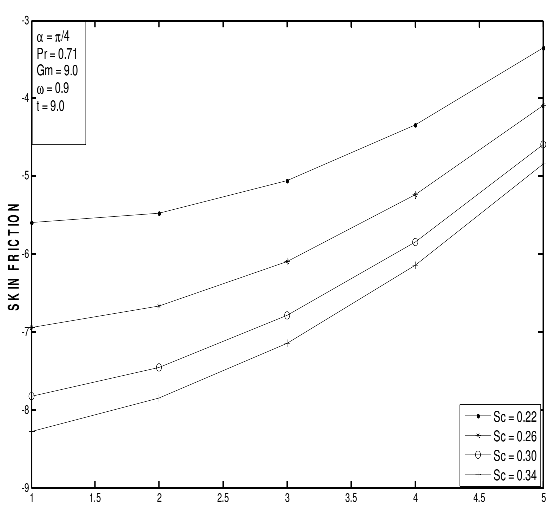

Figure 19: Influence of Schmidt No. on Skin Friction

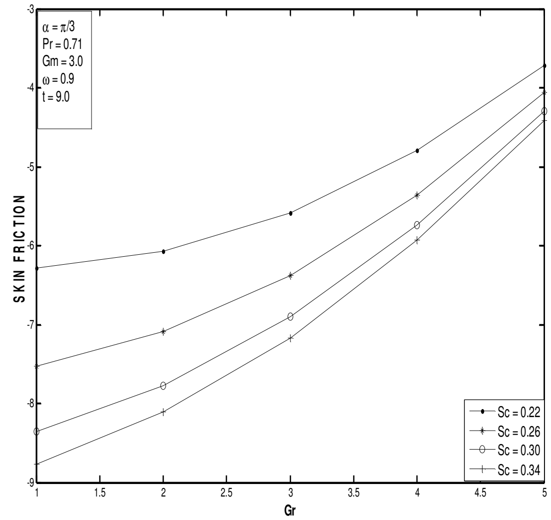

Figure 20: Effect of Schmidt No. on Skin Friction

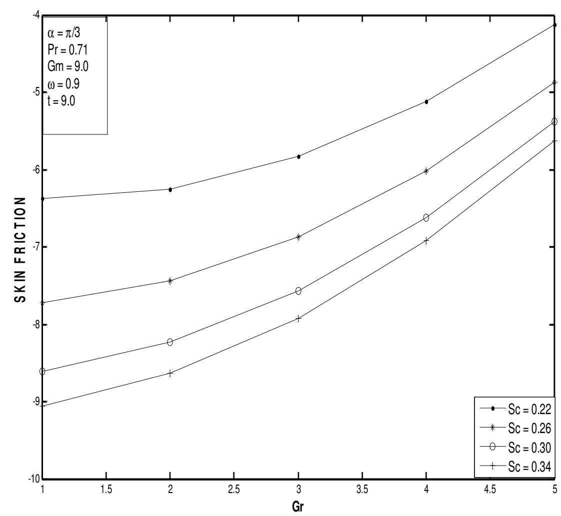

Figure 21: Influence of Schmidt No. on Skin Friction

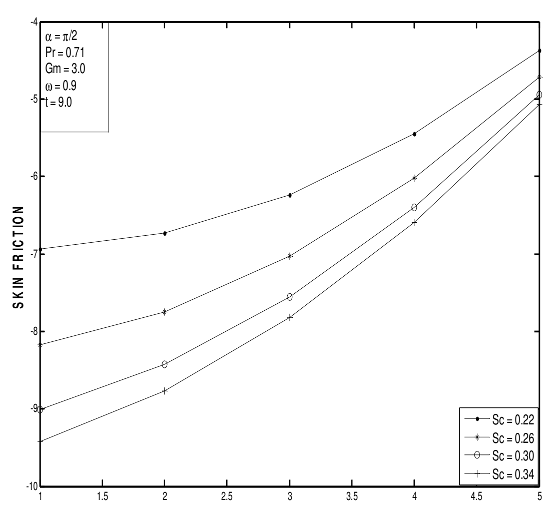

Figure 22: Effect of Schmidt No on Skin Friction

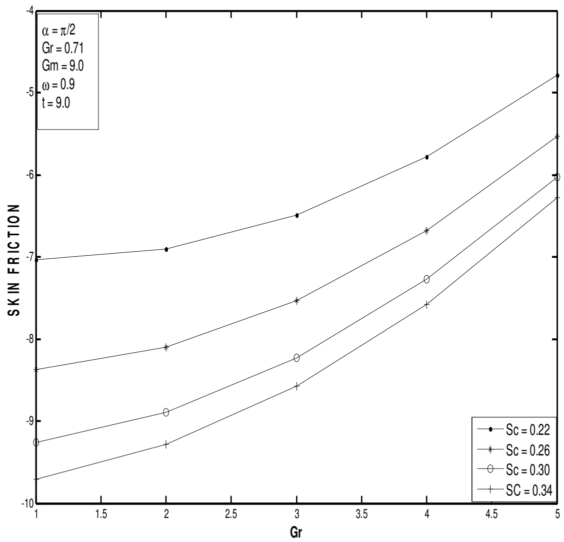

Figure 23: Contribution of Schmidt No. on Skin Friction

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