INTUITIONISTIC FUZZY COMPLETELY REGULAR WEAKLY CONTINUOUS MAPPINGS

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(Received on: 20-11-13; Revised & Accepted on: 28-12-13)

ABSTRACT

In this paper we introduce intuitionistic fuzzy completely regular weakly continuous mappings and some of their properties are studied.

Key words and phrases: Intuitionistic fuzzy topology, intuitionistic fuzzy point, intuitionistic fuzzy regular weakly closed set intuitionistic fuzzy regular weakly open set and intuitionistic fuzzy completely regular weakly continuous mappings.

AMS Subject Classification Msc 2000: 54A40, 54D20.

1. INTRODUCTION

In 1965, Zadeh [16] introduced fuzzy sets and in 1968, Chang [2] introduced fuzzy topology. After the introduction of fuzzy sets and fuzzy topology, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In 1997 Coker [3] introduced the concept of intuitionistic fuzzy topological spaces. In this present paper, we introduce the concepts of intuitionistic fuzzy completely regular weakly continuous mappings in intuitionistic fuzzy topological space.

2. PRELIMINARIES

Throughout this paper, (X, τ) or X denotes the intuitionistic fuzzy topological spaces (briefly IFTS). For a subset A of X, the closure, the interior and the complement of A are denoted by cl(A), int(A) and A^c respectively. We recall some basic definitions that are used in the sequel.

Definition: 2.1 [1] Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form A = {〈x, µ_A(x), ν_A(x)〉/ x ∈ X} where the functions µ_A : X → [0,1] and ν_A : X → [0,1] denote the degree of membership (namely µ_A(x)) and the degree of non-membership (namely ν_A(x)) of each element x ∈ X to the set A, respectively, and 0 ≤ µ_A(x) + ν_A(x) ≤ 1 for each x ∈ X. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition: 2.2 [1] Let A and B be IFSs of the form A = {〈x, µ_A(x), ν_A(x)〉/ x ∈ X} and B = {〈x, µ_B(x), ν_B(x)〉/ x ∈ X}.

(i) A ⊆ B if and only if µ_A(x) ≤ µ_B(x) and ν_A(x) ≥ ν_B(x) for all x ∈ X,
(ii) A = B if and only if A ⊆ B and B ⊆ A,
(iii) A^c = {〈x, µ_A(x), ν_A(x)〉/ x ∈ X},
(iv) A ∩ B = {〈x, µ_A(x) ∧ µ_B(x), ν_A(x) ∨ ν_B(x)〉/ x ∈ X},
(v) A ∪ B = {〈x, µ_A(x) ∨ µ_B(x), ν_A(x) ∧ ν_B(x)〉/ x ∈ X}.

For the sake of simplicity, we shall use the notation A = {〈x, µ_A(x), ν_A(x)〉/ x ∈ X} instead of A = {〈x, µ_A(x), ν_A(x)〉/ x ∈ X}. Also for the sake of simplicity, we shall use the notation A = {〈x, (µ_A, µ_B), (ν_A, ν_B)〉/ x ∈ X} instead of A = {〈x, (µ_A, µ_B), (ν_A, ν_B)〉/ x ∈ X}. The intuitionistic fuzzy sets 0 = {〈x, 0, 1〉/ x ∈ X} and 1 = {〈x, 1, 0〉/ x ∈ X} are respectively the empty set and the whole set of X.

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Definition: 2.3 [3] An intuitionistic fuzzy topology (IFT in short) on X is a family $\tau$ of IFSs in X satisfying the following axioms:

(i) $0_\tau, 1_\tau \in \tau$, 
(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$, 
(iii) $\bigcup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair $(X, \tau)$ is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in $\tau$ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement $A^c$ of an IFOS A in an IFTS $(X, \tau)$ is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition: 2.4 [3] Let $(X, \tau)$ be an IFTS and $A = (x, \mu_A, \nu_A)$ be an IFS in X. Then

(i) $\text{int}(A) = \bigcup \{ G / G \text{ is an IFOS in X and } G \subseteq A \}$, 
(ii) $\text{cl}(A) = \bigcap \{ K / K \text{ is an IFCS in X and } A \subseteq K \}$, 
(iii) $\text{cl}(A^c) = (\text{int}(A))^c$, 
(iv) $\text{int}(A^c) = (\text{cl}(A))^c$.

Definition: 2.5 [5] An IFS A of an IFTS $(X, \tau)$ is an

(i) intuitionistic fuzzy semiclosed set (IFSCS for short) if $\text{int}(\text{cl}(A)) \subseteq A$, 
(ii) intuitionistic fuzzy semiopen set (IFSOS for short) if $A \subseteq \text{cl}(\text{int}(A))$.

Definition: 2.6 [5] An IFS A of an IFTS $(X, \tau)$ is an

(i) intuitionistic fuzzy regular closed (IFRCS for short) if $\text{cl}(\text{int}(A)) = A$, 
(ii) intuitionistic fuzzy regular open (IFROS for short) if $\text{int}(\text{cl}(A)) = A$, 
(iii) intuitionistic fuzzy preclosed set (IFPCS for short) if $\text{cl}(\text{int}(A)) \subseteq A$, 
(iv) intuitionistic fuzzy preopen set (IFPOS for short) if $A \subseteq \text{int}(\text{cl}(A))$.

Note that every IFOS in $(X, \tau)$ is an IFPOS in X.

Definition: 2.7 [15] An IFS A of an intuitionistic fuzzy topological space $(X, \tau)$ is called an intuitionistic fuzzy regular semi open (IFRSOS for short) if there is a regular open set U such that $U \subseteq A \subseteq \text{cl}(U)$.

Definition: 2.8 [5] If A is an IFS in intuitionistic fuzzy topological space $(X, \tau)$ then

(i) $\text{scl}(A) = \bigcap \{ F : A \subseteq F, F \text{ is intuitionistic fuzzy semi closed} \}$, 
(ii) $\text{pcl}(A) = \bigcap \{ F : A \subseteq F, F \text{ is intuitionistic fuzzy pre closed} \}$.

Definition: 2.9 An IFS A of an intuitionistic fuzzy topological space $(X, \tau)$ is called:

(i) Intuitionistic fuzzy $g$-closed (IFGCS for short) if $\text{cl}(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy open.[8]
(ii) Intuitionistic fuzzy $g$-open (IFGOS for short) if its complement $A^c$ is intuitionistic fuzzy $g$-closed.[8]
(iii) Intuitionistic fuzzy $rg$-closed (IFRGCS for short) if $\text{cl}(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy regular open.[10]
(iv) Intuitionistic fuzzy $rg$-open (IFRGOS for short) if its complement $A^c$ is intuitionistic fuzzy $rg$-closed.[10]
(v) Intuitionistic fuzzy $w$-closed (IFWCS for short) if $\text{cl}(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy semi open.[12]
(vi) Intuitionistic fuzzy $w$-open (IFWOS for short) if its complement $A^c$ is intuitionistic fuzzy $w$-closed.[12]
(vii) Intuitionistic fuzzy $gpr$-closed (IFGPROS for short) if $\text{pcl}(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy regular open.[14]
(viii) Intuitionistic fuzzy $gpr$-open (IFGPROS for short) if its complement $A^c$ is intuitionistic fuzzy $gpr$-closed. [14]

Remark: 2.10 Every intuitionistic fuzzy closed set is intuitionistic fuzzy $g$-closed but its converse may not be true. [8]

Remark: 2.11 Every intuitionistic fuzzy $g$-closed set is intuitionistic fuzzy $rg$-closed but its converse may not be true. [10]

Remark: 2.12 Every intuitionistic fuzzy $w$-closed (resp. Intuitionistic fuzzy $w$-open) set is intuitionistic fuzzy $g$-closed (intuitionistic fuzzy $g$-open) but its converse may not be true. [12]

Remark: 2.13 Every intuitionistic fuzzy $g$-closed (resp. Intuitionistic fuzzy $g$-open) set is intuitionistic fuzzy $gpr$-closed (intuitionistic fuzzy $gpr$-open) but its converse may not be true. [14]

Definition: 2.14 [13] An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X, \tau)$ is called an intuitionistic fuzzy $rw$-closed (IFRWCS for short) if $\text{cl}(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy regular semi open in X.
Every IFCS, IFGCS, IFWCS is an IFRWCS and every IFRWCS is an IFRGCS and IFGPRCS but the converses may not be true in general.

**Definition: 2.15** [13] An intuitionistic fuzzy set \( A \) of an intuitionistic fuzzy topological space \( (X, \tau) \) is called intuitionistic fuzzy rw-open (IFRWOS for short) if and only if its complement \( A^c \) is intuitionistic fuzzy rw-closed.

**Definition: 2.16** [5] Let \( (X, \tau) \) and \( (Y, \sigma) \) be two intuitionistic fuzzy topological spaces and let \( f: X \to Y \) be a function. Then \( f \) is said to be an intuitionistic fuzzy continuous if the pre image of each intuitionistic fuzzy closed set in \( Y \) is an intuitionistic fuzzy closed set in \( X \).

**Definition: 2.17** Let \( (X, \tau) \) and \( (Y, \sigma) \) be two intuitionistic fuzzy topological spaces and let \( f: X \to Y \) be a function. Then \( f \) is said to be an
(i) intuitionistic fuzzy g-continuous if the pre image of every intuitionistic fuzzy closed set in \( Y \) is intuitionistic fuzzy g-closed in \( X \). [9]
(ii) intuitionistic fuzzy w-continuous if the pre image of every intuitionistic fuzzy closed set in \( Y \) is intuitionistic fuzzy w-closed in \( X \). [12]
(iii) intuitionistic fuzzy rg-continuous if the pre image of every intuitionistic fuzzy closed set in \( Y \) is intuitionistic fuzzy rg-closed in \( X \). [11]
(iv) intuitionistic fuzzy gpr-continuous if the pre image of every intuitionistic fuzzy closed set in \( Y \) is intuitionistic fuzzy gpr-closed in \( X \). [14]

**Remark: 2.18** Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy gcontinuous, but the converse may not be true [9].

**Remark: 2.19** Every intuitionistic fuzzy w-continuous mapping is intuitionistic fuzzy gcontinuous, but the converse may not be true [12].

**Remark: 2.20** Every intuitionistic fuzzy g-continuous mapping is intuitionistic fuzzy rgcontinuous, but the converse may not be true [8].

**Remark: 2.21** Every intuitionistic fuzzy g-continuous mapping is intuitionistic fuzzy gpr-continuous, but the converse may not be true [14].

**Definition: 2.22** [4] Let \( \alpha, \beta \in [0, 1] \) and \( \alpha + \beta \leq 1 \). An intuitionistic fuzzy point (IFP for short) \( p_{(\alpha, \beta)} \) of \( X \) is an IFS of \( X \) defined by
\[
p_{(\alpha, \beta)}(y) = \begin{cases} (\alpha, \beta) & \text{if } y = x \\ (0,1) & \text{if } y \neq x \end{cases}
\]

**Definition: 2.23** [7] Let \( p_{(\alpha, \beta)} \) be an IFP of an IFTS \( (X, \tau) \). An IFS \( A \) of \( X \) is called an intuitionistic fuzzy neighborhood (IFN for short) of \( p_{(\alpha, \beta)} \) if there exists an IFOS \( B \) in \( X \) such that \( p_{(\alpha, \beta)} \in B \subseteq A \).

**Definition: 2.24** [6] An IFS \( A \) is said to be intuitionistic fuzzy dense (IFD for short) in another IFS \( B \) in an IFT \( (X, \tau) \) if \( \text{cl}(A) = B \).

**Definition: 2.25** [5] Two IFSs \( A \) and \( B \) are said to be q-coincident (\( A \approx B \) in short) if and only if there exists and element \( x \in X \) such that \( \mu_A(x) > \nu_B(x) \) or \( \nu_A(x) < \mu_B(x) \).

### 3. INTUITIONISTIC FUZZY COMPLETELY REGULAR WEAKLY CONTINUOUS MAPPINGS

In this section, we introduced intuitionistic fuzzy completely regular weakly continuous mappings and studied some of its properties.

**Definition: 3.1** A mapping \( f: (X, \tau) \to (Y, \sigma) \) is called an intuitionistic fuzzy completely regular weakly continuous (IFcRW continuous in short) mapping if \( f^{-1}(B) \) is an IFRC in \( (X, \tau) \) for every IFRWCS \( B \) of \( (Y, \sigma) \).

For the sake of simplicity, we shall use the notation \( A = (x, (\mu, \nu), (v, v)) \) instead of \( A = (x, (\mu_{\alpha}, b / \mu_{\beta}), (a / v_{\alpha}, b / v_{\beta})) \) in all the examples used in this paper. Similarly we shall use the notation \( B = (x, (\mu, \nu), (v, v)) \) instead of \( B = (x, (u / \mu_{\alpha}, v / \mu_{\beta}), (u / v_{\alpha}, v / v_{\beta})) \) in the following examples.

**Example: 3.2** Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( G_1 = (x, (0.3, 0.4), (0.7, 0.6)) \), \( G_2 = (y, (0.4, 0.4), (0.6, 0.6)) \). Then \( \tau = \{0, G_1, 1\} \) and \( \sigma = \{0, G_2, 1\} \) are IFT on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \to (Y, \sigma) \) by \( f(a) = u \) \( f(b) = v \).
and \( f(b) = v \). Then \( f \) is an IFcRW continuous mapping, since \( B = \{ (y, (0.7, 0.6), (0.3, 0.4)) \} \) is an IFRWCS in \( Y \) then \( f^{-1}(B) = \{ (x, (0.7, 0.6), (0.3, 0.4)) \} \) is IFRCS in \( X \).

**Theorem: 3.3** Every IFcRW continuous mapping is an IF continuous mapping but not conversely.

**Proof:** Assume that \( f: (X, \sigma) \rightarrow (Y, \sigma) \) is an IFcRW continuous mapping. Let \( B \) be an IFCS in \( Y \). Then \( B \) is an IFRWCS in \( Y \). Since \( f \) is an IFcRW continuous mapping, \( f^{-1}(B) \) is an IFRCS in \( X \). This implies \( f^{-1}(B) \) is an IFGRCS in \( X \). Hence the mapping \( f \) is an IF continuous mapping.

**Example: 3.4** Let \( X = \{ a, b \} \), \( Y = \{ u, v \} \), and \( G_1 = (x, (0.2, 0.3), (0.7, 0.5)) \). Then \( \tau = \{ 0, G_1, 1 \} \) and \( \sigma = \{ 0, G_2, 1 \} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IF continuous mapping. But \( f \) is not an IFcRW continuous mapping, since \( B = \{ y, (0.8, 0.5), (0.2, 0.5) \} \) is an IFRWCS in \( Y \) but \( f^{-1}(B) = \{ (x, (0.8, 0.5), (0.2, 0.5)) \} \) is not an IFRCS in \( X \).

**Theorem: 3.5** Every IFcRW continuous mapping is an IFG continuous mapping but not conversely.

**Proof:** Assume that \( f: (X, \tau) \rightarrow (Y, \sigma) \) is an IFcRW continuous mapping. Let \( B \) be an IFCS in \( Y \). Since \( f \) is an IFcRW continuous mapping, \( f^{-1}(B) \) is an IFRCS in \( X \). This implies \( f^{-1}(B) \) is an IFGCS in \( X \). Hence \( f \) is an IFG continuous mapping.

**Example: 3.6** Let \( X = \{ a, b \} \), \( Y = \{ u, v \} \), and \( G_1 = (x, (0.3, 0.4), (0.7, 0.6)) \). Then \( \tau = \{ 0, G_1, 1 \} \) and \( \sigma = \{ 0, G_2, 1 \} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IFG continuous mapping. But \( f \) is not an IFcRW continuous mapping, since \( B = \{ y, (0.9, 0.3), (0.1, 0.7) \} \) is an IFRWCS in \( Y \) but \( f^{-1}(B) = \{ (x, (0.9, 0.3), (0.1, 0.7)) \} \) is not an IFRCS in \( X \).

**Theorem: 3.7** Every IFcRW continuous mapping is an IFRG continuous mapping but not conversely.

**Proof:** Assume that \( f: (X, \tau) \rightarrow (Y, \sigma) \) is an IFcRW continuous mapping. Let \( B \) be an IFCS in \( Y \). Since \( f \) is an IFcRW continuous mapping, \( f^{-1}(B) \) is an IFRCS in \( X \). This implies \( f^{-1}(B) \) is an IFWCS in \( X \). Hence \( f \) is an IFR continuous mapping.

**Example: 3.8** Let \( X = \{ a, b \} \), \( Y = \{ u, v \} \), and \( G_1 = (x, (0.5, 0.3), (0.2, 0.6)) \). Then \( \tau = \{ 0, G_1, 1 \} \) and \( \sigma = \{ 0, G_2, 1 \} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IFR continuous mapping. But \( f \) is not an IFcRW continuous mapping, since \( B = \{ y, (0.2, 0.4), (0.8, 0.5) \} \) is an IFRWCS in \( Y \) but \( f^{-1}(B) = \{ (x, (0.2, 0.4), (0.8, 0.5)) \} \) is not an IFRCS in \( X \).

**Theorem: 3.9** Every IFcRW continuous mapping is an IFGPR continuous mapping but not conversely.

**Proof:** Assume that \( f: (X, \tau) \rightarrow (Y, \sigma) \) is an IFcRW continuous mapping. Let \( B \) be an IFCS in \( Y \). Since \( f \) is an IFcRW continuous mapping, \( f^{-1}(B) \) is an IFRCS in \( X \). This implies \( f^{-1}(B) \) is an IFGRCS in \( X \). Hence \( f \) is an IFGPR continuous mapping.

**Example: 3.10** Let \( X = \{ a, b \} \), \( Y = \{ u, v \} \), and \( G_1 = (x, (0.5, 0.7), (0.5, 0.3)) \). Then we define \( \tau = \{ 0, G_1, 1 \} \) and \( \sigma = \{ 0, G_2, 1 \} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IFGPR continuous mapping. But \( f \) is not an IFcRW continuous mapping since \( B = \{ y, (0.8, 0.1), (0.2, 0.8) \} \) is an IFRWCS in \( Y \) but \( f^{-1}(B) = \{ (x, (0.8, 0.1), (0.2, 0.8)) \} \) is not an IFRCS in \( X \).

**Theorem: 3.11** Every IFcRW continuous mapping is an IFGPR continuous mapping but not conversely.

**Proof:** Assume that \( f: (X, \tau) \rightarrow (Y, \sigma) \) is an IFcRW continuous mapping. Let \( B \) be an IFCS in \( Y \). Since \( f \) is an IFcRW continuous mapping, \( f^{-1}(B) \) is an IFRCS in \( X \). This implies \( f^{-1}(B) \) is an IFGRPRCS in \( X \). Hence \( f \) is an IFGPR continuous mapping.

**Example: 3.12** Let \( X = \{ a, b \} \), \( Y = \{ u, v \} \), and \( G_1 = (x, (0.5, 0.3), (0.5, 0.7)) \). Then we define \( \tau = \{ 0, G_1, 1 \} \) and \( \sigma = \{ 0, G_2, 1 \} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IFGPR continuous mapping. But \( f \) is not an IFcRW continuous mapping since \( B = \{ y, (0.8, 0.3), (0.2, 0.2) \} \) is an IFRWCS in \( Y \) but \( f^{-1}(B) = \{ (x, (0.8, 0.3), (0.2, 0.2)) \} \) is not an IFRCS in \( X \).
The relation between various types of intuitionistic fuzzy continuity is given in the following diagram. In this diagram ‘cts’ means continuous.

\[
\text{IFG cts} \quad \text{IF cts} \quad \text{IFW cts} \quad \text{IFGPR cts} \\
\text{IF cts} \quad \text{IFcRW cts} \quad \text{IFG cts} \\
\text{IFG cts} \quad \text{IFcRW cts} \quad \text{IFR cts}
\]

The reverse implications are not true in general in the above diagram.

**Definition:** 3.13 A mapping \( f: (X, \tau) \to (Y, \sigma) \) is called an intuitionistic fuzzy regular weakly irresolute (IFRW irresolute in short) mapping if \( f^{-1}(B) \) is an IFRWCS in \((X, \tau)\) for every IFRWCS \( B \) of \((Y, \sigma)\).

**Example:** 3.14 Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( G_1 = \langle x, (0.3, 0.4), (0.7, 0.6) \rangle \), and \( G_2 = \langle y, (0.4, 0.4), (0.6, 0.6) \rangle \). Then \( \tau = \{0\text{~}, G_1, 1\text{~}\} \) and \( \sigma = \{0\text{~}, G_2, 1\text{~}\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \to (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IFR RW irresolute mapping, since \( B = \langle y, (0.9, 0.3), (0.1, 0.7) \rangle \) is an IFRWCS in \( Y \) then \( f^{-1}(B) = \langle x, (0.9, 0.3), (0.1, 0.7) \rangle \) is an IFRWCS in \( X \).

**Theorem:** 3.15 Every IFRW continuous mapping is an IFRW irresolute mapping.

**Proof:** Assume that \( f: X \to Y \) is an IFRW continuous mapping. Let \( B \) be an IFRWCS in \( Y \). Since \( f \) is an IFRW continuous, \( f^{-1}(B) \) is an IFRCS in \( X \). This implies \( f^{-1}(B) \) is an IFRWCS in \( X \). Hence \( f \) is an IFRW irresolute mapping.

**Theorem:** 3.16 A mapping \( f: X \to Y \) is an IFRW continuous mapping if and only if the inverse image of each IFRWOS in \( Y \) is an IFROS in \( X \).

**Proof:** If \( A \) be an IFRWOS in \( Y \). This implies \( A^c \) is an IFRWCS in \( Y \). Since \( f \) is an IFRW continuous, \( f^{-1}(A^c) \) is an IFRCS in \( X \). Since \( f^{-1}(A^c) = (f^{-1}(A))^c \), \( f^{-1}(A) \) is an IFROS in \( X \).

**Sufficiency:** Let \( A \) be an IFRWCS in \( Y \). This implies \( A^c \) is an IFRWOS in \( Y \). By hypothesis \( f^{-1}(A^c) \) is an IFROS in \( X \). Since \( f^{-1}(A^c) = (f^{-1}(A))^c \), \( f^{-1}(A) \) is an IFRCS in \( X \). Hence \( f \) is an IFRW continuous mapping.

**Theorem:** 3.17 Let \( p(\alpha, \beta) \) be an IFP in an IFTS \((X, \tau)\). A mapping \( f: X \to Y \) is an IFRW continuous mapping if for every IFRWOS \( A \) in \( Y \) with \( f(p(\alpha, \beta)) \in A \), there exists an IFROS \( B \) in \( X \) with \( p(\alpha, \beta) \in B \subseteq f^{-1}(A) \).

**Proof:** Let \( A \) be an IFRWOS in \( Y \) and let \( p(\alpha, \beta) \in A \). Then there exists an IFROS \( B \) in \( X \) with \( p(\alpha, \beta) \in B \subseteq \text{cl}(f^{-1}(A)) \). Since \( B \) is an IFROS, \( \text{int}\left(\text{cl}(f^{-1}(A))\right) = \text{cl}(f^{-1}(A)) \). That is \( \text{int}(f^{-1}(A)) = \text{cl}(f^{-1}(A)) \). This implies \( f^{-1}(A) \) is also an IFROS in \( X \).

**Theorem:** 3.18 If a mapping \( f: X \to Y \) is an IFRW continuous mapping, then for every IFP \( p(\alpha, \beta) \in X \) and for every IFN \( A \) of \( f(p(\alpha, \beta)) \), there exists an IFROS \( B \subseteq X \) such that \( p(\alpha, \beta) \in B \subseteq f^{-1}(A) \).

**Proof:** Let \( p(\alpha, \beta) \in X \) and let \( A \) be an IFN of \( f(p(\alpha, \beta)) \). Then there exists an IFOS \( C \) in \( Y \) such that \( f(p(\alpha, \beta)) \in C \subseteq A \). Since every IFOS is an IFRWOS, \( C \) is an IFRWOS in \( Y \). Hence by hypothesis, \( f^{-1}(C) \) is an IFROS in \( X \) and \( p(\alpha, \beta) \in f^{-1}(C) \). Now, let \( f^{-1}(C) = B \). Therefore \( p(\alpha, \beta) \in B \subseteq f^{-1}(A) \).

**Theorem:** 3.19 If a mapping \( f: X \to Y \) is an IFRW continuous mapping, then for every IFP \( p(\alpha, \beta) \in X \) and for every IFN \( A \) of \( f(p(\alpha, \beta)) \), there exists an IFROS \( B \subseteq X \) such that \( p(\alpha, \beta) \in B \subseteq f^{-1}(A) \).

**Proof:** Let \( p(\alpha, \beta) \in X \) and let \( A \) be an IFN of \( f(p(\alpha, \beta)) \). Then there exists an IFOS \( C \) in \( Y \) such that \( f(p(\alpha, \beta)) \in C \subseteq A \). Since every IFOS is an IFRWOS, \( C \) is an IFRWOS in \( Y \). Hence by hypothesis, \( f^{-1}(C) \) is an IFROS in \( X \) and \( p(\alpha, \beta) \in f^{-1}(C) \). Now let \( f^{-1}(C) = B \). Therefore \( p(\alpha, \beta) \in B \subseteq f^{-1}(A) \). Thus \( f(B) \subseteq f(f^{-1}(A)) \subseteq A \), which implies \( f(B) \subseteq A \).

**Theorem:** 3.20 A mapping \( f: X \to Y \) is an IFRW continuous mapping then \( \text{int}(\text{cl}(f^{-1}(B))) \subseteq f^{-1}(B) \) for every IFS \( B \) in \( Y \).
Proof: Let \( B \subseteq Y \) be an IFS. Then, \( \text{int}(B) \) is an IFOS in \( Y \) and hence an IFRWOS in \( Y \). By hypothesis, \( f^{-1}(\text{int}(B)) \) is an IFROS in \( X \). Hence \( \text{int}(c(f^{-1}(\text{int}(B)))) = f^{-1}(\text{int}(B)) \subseteq f^{-1}(B) \).

Theorem 3.21 A mapping \( f: X \rightarrow Y \) is an IFcRW continuous mapping then the following are equivalent:
(i) for any IFRWOS \( A \) in \( X \) and for any IFP \( p_{(\alpha, \beta)} \subseteq X \) if \( f(p_{(\alpha, \beta)}) \subseteq A \), then there exists an IFP \( p_{(\alpha, \beta)} \subseteq X \) such that \( f^{-1}(A) \) is an IFROS in \( X \). Let \( A = f^{-1}(B) \). Then \( p_{(\alpha, \beta)} \cup B \) and \( f(B) \subseteq A \).

Proof:
(i) \rightarrow (ii): Let \( A \subseteq Y \) be an IFRWOS and let \( p_{(\alpha, \beta)} \not\subseteq X \). Then \( f(p_{(\alpha, \beta)}) \subseteq f^{-1}(A) \) and \( f(A) \subseteq \text{int}(B) \). Therefore, \( p_{(\alpha, \beta)} \cup B \) is an IFcRW continuous mapping.

(ii) \rightarrow (i): Let \( A \subseteq Y \) be an IFRWOS and let \( p_{(\alpha, \beta)} \subseteq X \). Suppose \( f(p_{(\alpha, \beta)}) \subseteq A \), then there exists an IFP \( B \) in \( X \) such that \( B \subseteq f^{-1}(B) \) and \( f(B) \subseteq A \). Now \( B \subseteq f^{-1}(f(B)) \subseteq f^{-1}(A) \). That is \( B = \text{int}(B) \subseteq f^{-1}(A) \). Therefore, \( p_{(\alpha, \beta)} \subseteq f^{-1}(A) \).

Theorem 3.22 For any two IFcRW continuous mappings \( f_1 \) and \( f_2 \): \( (X, \tau) \rightarrow (Y, \sigma) \), the function \( (f_1, f_2): (X, \tau) \rightarrow (Y \times Y, \sigma \times \sigma) \) is also an IFcRW continuous mapping where \( (f_1, f_2)(x) = (f_1(x), f_2(x)) \) for every \( x \in X \).

Proof: Let \( A \subseteq X \) be an IFRWOS in \( Y \times Y \). Then \( (f_1, f_2)^{-1}(A) = (f_1(x), f_2(x)) = (x, \min(\mu_1(f_1(x)), \mu_2(f_2(x))), \max(\nu_1(f_1(x)), \nu_2(f_2(x)))) \) for every \( x \in X \).

Theorem 3.23 Let \( f: X \rightarrow Y \) be a mapping. Then the following are equivalent.
(i) \( f \) is an IFcRW continuous mapping.
(ii) \( f^1(B) \) is an IFcRW in \( X \) for every IFRWOS \( B \) in \( Y \).
(iii) for every IFP \( p_{(\alpha, \beta)} \subseteq X \) and for every IFRWOS \( B \) in \( Y \) such that \( f(p_{(\alpha, \beta)}) \subseteq B \) there exists an IFROS \( A \) in \( X \) such that \( p_{(\alpha, \beta)} \subseteq A \) and \( f(A) \subseteq B \).

Proof:
(i) \rightarrow (ii): is obviously.

(ii) \rightarrow (iii): Let \( p_{(\alpha, \beta)} \subseteq X \) and \( B \subseteq Y \) such that \( f(p_{(\alpha, \beta)}) \subseteq B \). This implies \( \text{int}(f^{-1}(B)) \subseteq f^{-1}(B) \). Since \( B \) is an IFRWOS in \( Y \), by hypothesis \( f^{-1}(B) \) is an IFROS in \( X \). Let \( A = f^{-1}(B) \). Then \( p_{(\alpha, \beta)} \subseteq f^{-1}(f(p_{(\alpha, \beta)})) \subseteq f^{-1}(B) = A \). Therefore \( p_{(\alpha, \beta)} \subseteq A \) and \( f(A) \subseteq B \).

(iii) \rightarrow (i): Let \( B \subseteq Y \) be an IFRWOS. Let \( p_{(\alpha, \beta)} \subseteq X \) and \( f(p_{(\alpha, \beta)}) \subseteq B \). By hypothesis, there exists an IFROS \( C \) in \( X \) such that \( p_{(\alpha, \beta)} \subseteq C \) and \( f(C) \subseteq B \). This implies \( C \subseteq f^{-1}(f(C)) \subseteq f^{-1}(B) \). Therefore, \( p_{(\alpha, \beta)} \subseteq C \subseteq f^{-1}(B) \). That is \( f^{-1}(B) = U_{p_{(\alpha, \beta)}} \subseteq f^{-1}(B) \subseteq f^{-1}(C) \subseteq f^{-1}(B) \). This implies \( f^{-1}(B) = U_{p_{(\alpha, \beta)}} \subseteq f^{-1}(B) \subseteq f^{-1}(C) \subseteq f^{-1}(B) \). Since union of IFROSs is IFROS, \( f^{-1}(B) \) is an IFROS in \( X \). Hence \( f \) is an IFcRW continuous mapping.

4. CONCLUSION
In this paper we have introduced intuitionistic fuzzy completely regular weakly continuous mapping and studied some of its basic properties. Also we have studied the relationship between intuitionistic fuzzy completely regular weakly continuous mappings and some of the intuitionistic fuzzy continuous mappings that already exist.

REFERENCES


Source of support: Nil, Conflict of interest: None Declared