International Journal of Mathematical Archive-4(12), 2013, 223-229

B(1, 2) $+\pi g\alpha$ -REGULAR AND NORMAL SPACES IN SIMPLE EXPANSION

I. Arockiarani* and K. Mohana

Department of Mathematics, Nirmala College for Women, Coimbatore-641 018, Tamil Nadu, India

(Received on: 24-11-13; Revised & Accepted on: 17-12-13)

ABSTRACT

In this paper, we generalize the concept of simple expansion due to Levine to the bitopological setting. Also, we present some new classes of set and spaces namely, $B(1, 2)^{+}-\pi ga$ -closed set, $B(1, 2)^{+}-\pi ga$ -Regular and Normal Spaces. Here, we define the notion of $(1, 2)^{*}-\pi ga$ -closed sets in the extended topology as a subset A of a space X is called $B(1, 2)^{+}-\pi ga$ -closed set if $B(1, 2)^{+}-\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1, 2}$ - π -open in X. This class of sets and spaces not only depend on $(1, 2)^{*}$ -topology, but also on its simple expansion. Further, we discuss their properties and some of their characterizations are obtained in the light of $B(1, 2)^{+}-\pi ga$ -closed sets.

Keywords: $B(1, 2)^+$ - πga -closed set, $B(1, 2)^+$ - πga -irresolute, $B(1, 2)^+$ - πga -regular space, $B(1, 2)^+$ - πga -normal space.

1. INTRODUCTION

In 1963 Levine [5] started the study of generalized open sets with the introduction of semi-open sets. Since then many topologists have utilized the concepts to the various notions of subsets, weak axioms, weak regularity, weak normality and weaker and stronger forms of covering axioms in the literature. The concept of g-regular and g-normal spaces in topological spaces were introduced and studied by Munshi [6]. Further, Noiri and Popa [7] investigated the concepts introduced by Munshi [6]. M. E. Abd El. Monsef *et al* [1] have developed and studied Bg-closed sets, gB-continuity and gB-irresolute map. In 2010 Vadivel [10] have defined the notion of B-Generalized Regular and B-Generalized Normal spaces in topological spaces. Using τ_p^+ g-closed and $\tau_{\beta g}^+$ -closed sets, Nirmala Irudayam and I. Arockiarani [8, 9] introduced τ_p^+ g-regular, normal spaces and $\tau_{\beta g}^+$ -regular, normal spaces in simple extension. The notion of (1, 2)*- $\pi g\alpha$ -closed set was introduced by I. Arockiarani and K. Mohana [2]. In this paper we apply B(1, 2) + $\pi g\alpha$ -closed sets as a tool to extend the two classes of spaces called B(1, 2) + $\pi g\alpha$ -regular and B(1, 2) + $\pi g\alpha$ -Normal spaces in bitopological spaces. Also we characterize the basic properties along with weaker forms of regularity and normality.

2. PRELIMINARIES

We list some definitions which are useful in the following sections. Throughout, this paper (X, τ_1 , τ_2), (Y, σ_1 , σ_2), (Z, η_1 , η_2) [briefly X, Y, Z] represent non-empty bitopological spaces on which no separation axiom is defined, unless otherwise mentioned.

Definition: 2.1 [4] A subset S of a bitopological space X is said to be $\tau_{1, 2}$ -open if S=A \cup B where $A \in \tau_1$ and $B \in \tau_2$. A subset S of X is said to be (i) $\tau_{1, 2}$ -closed if the complement of S is $\tau_{1, 2}$ -open. (ii) $\tau_{1, 2}$ -clopen if S is both $\tau_{1, 2}$ -open and $\tau_{1, 2}$ -closed.

Definition: 2.2 [4] Let S be a subset of the bitopological space X. Then the $\tau_{1, 2}$ -interior of S denoted by $\tau_{1, 2}$ -int(S) is defined by \cup {G: G \subseteq S and G is $\tau_{1, 2}$ -open set} and the $\tau_{1, 2}$ -closure of S denoted by $\tau_{1, 2}$ -cl(S) is defined by \cap {F: S \subseteq F and F is $\tau_{1, 2}$ - closed set}.

The family of $all\tau_{1,2}$ -closed sets of X will be denoted by $\tau_{1,2}$ -C(X).

The set $\tau_{1, 2}$ -C(X, x) = {V $\in \tau_{1, 2}$ -C(X) / x $\in V$ }, for x $\in X$.

Remark: 2.3 [4] $\tau_{1, 2}$ -open sets need not form a topology.

Corresponding author: I. Arockiarani* and K. Mohana

Department of Mathematics, Nirmala College for Women, Coimbatore-641 018, Tamil Nadu, India

I. Arockiarani* and K. Mohana/B(1, 2)^{*}-πgα-Regular and Normal Spaces in Simple Expansion/IJMA- 4(12), Dec.-2013.

Definition: 2.4 [4] A subset A of a bitopological space X is called

(i) $(1, 2)^*$ -regular open if $A = \tau_{1, 2}$ -int $(\tau_{1, 2}$ -cl (A)).

(ii) $(1, 2)^{*}-\alpha$ -open if $A \subseteq \tau_{1, 2}$ -int $(\tau_{1, 2}$ -cl $(\tau_{1, 2}$ -int (A))).

The complement of the sets mentioned from (i) and (ii) are called their respective closed sets.

Definition: 2.5 [2] The finite union of $(1, 2)^*$ -regular open sets is said to be $\tau_{1, 2}$ - π -open. The complement of $\tau_{1, 2}$ - π -open is said to be $\tau_{1, 2}$ - π -closed.

Definition: 2.6 [2] A subset A of a bitopological space X is called $(1, 2)^*$ - $\pi g \alpha$ - closed if $(1, 2)^*$ - $\alpha cl (A) \subseteq U$ whenever A \subset U and U is $\tau_{1,2}$ - π -open. The complement of the above set is called $(1, 2)^*$ - $\pi g \alpha$ -open set.

Definition: 2.7[3] A function f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $(1, 2)^*$ - $\pi g \alpha$ -irresolute if f⁻¹(V) is $(1, 2)^*$ - $\pi g \alpha$ -closed in X, for every $(1, 2)^*$ - $\pi g \alpha$ -closed set V of Y.

3. B (1, 2) ⁺-πgα-CLOSED SETS

Definition: 3.1 Let B be a subset of a bitopological space X then $\tau_{1,2}^+(B)$ is defined by $\tau_{1,2}^+(B) = \{O \cup (O^1 \cap B): O \in \tau_{1,2} \text{ and called its simple expansion of } \tau_{1,2}, O^1 \in \tau_{1,2}\}$ by B, where B $\notin \tau_{1,2}(O)$

Definition: 3.2 A subset of X belonging to $\tau_{1, 2}$ (B) is denoted by B $\tau_{1, 2}$ -open set, the complement of B $\tau_{1, 2}$ -open set is denoted by B $\tau_{1, 2}$ -closed set.

Definition: 3.3 The $B\tau_{1, 2}$ -interior of S, denoted by $B\tau_{1, 2}$ -int(S) is defined by $\cup \{G: G \subseteq S \text{ and } G \text{ is } \tau_{1, 2}$ -open in $\tau_{1, 2}(B)\}$. The $B\tau_{1, 2}$ -closure of S, denoted by $\cap \{F: S \subseteq F \text{ and } F \text{ is } \tau_{1, 2}$ -closed in $\tau_{1, 2}(B)\}$.

Definition: 3.4 A subset A of a bitoplogical space X is called B (1, 2) ⁺- α -open if A \subseteq B $\tau_{1, 2}$ -int (B $\tau_{1, 2}$ -cl (B $\tau_{1, 2}$ -int (A))). The complement of B (1, 2) ⁺- α -open set is called B (1, 2) ⁺- α -closed set.

Definition: 3.5

- (i) A subset A of a space X is B $(1, 2)^+ \alpha$ -regular if A is both B $(1, 2)^+ \alpha$ -open and B $(1, 2)^+ \alpha$ -closed.
- (ii) The B (1, 2) ⁺- α -closure of S, denoted by B (1, 2) ⁺- α cl(S) is defined by \cap {F: S \subseteq F and F is (1, 2) ⁺- α -closed in $\tau_{1, 2}$ (B)}.

The family of all B (1, 2) ⁺- α -open [B $\tau_{1, 2}$ -open] sets is denoted by B (1, 2) ⁺- α O(X) [B $\tau_{1, 2}$ -O(X)] and the family of all B (1, 2) ⁺- α -closed [B $\tau_{1, 2}$ -closed] sets is denoted by B (1, 2) ⁺- α C(X) [B $\tau_{1, 2}$ -C(X)].

Definition: 3.6 A subset A of a bitopological space X is said to be

- (i) B (1, 2) ⁺-g-closed set if B $\tau_{1,2}$ -cl (A) \subseteq U whenever A \subseteq U and U is $\tau_{1,2}$ -open in X.
- (ii) B (1, 2) ⁺- π g-closed set if B $\tau_{1, 2}$ -cl (A) \subseteq U whenever A \subseteq U and U is $\tau_{1, 2}$ - π -open in X.
- (iii) B (1, 2) +- π ga-closed set if B (1, 2) +- α cl (A) \subset U whenever A \subset U and U is $\tau_{1,2}$ - π -open in X.

Example: 3.7 Consider $X = \{a, b, c\}, \tau_1 = \{\phi, X, \{a\}\}, \tau_2 = \{\phi, X, \{b\}\}$. Let $B = \{c\}$. Here the B (1, 2) ⁺- $\pi g\alpha$ -closed sets are $\{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.

Theorem: 3.8 The union of two B $(1, 2)^+$ - π ga-closed set is B $(1, 2)^+$ - π ga-closed.

Proof: Let A and B be B (1, 2) ⁺- π g α -closed sets. Let U be $\tau_{1,2}$ - π -open set such that $A \cup B \subseteq U$. This implies $A \subseteq U$ and $B \subseteq U$. Since A is B (1, 2) ⁺- π g α -closed set, we have B (1, 2) ⁺- α cl (A) \subseteq U. Also B is B (1, 2) ⁺- π g α -closed set, we have B (1, 2) ⁺- α cl (B) \subseteq U. This implies B (1, 2) ⁺- π g α -closed set, we have B (1, 2) ⁺- α cl (B) \subseteq U. This implies B (1, 2) ⁺- α cl (A) \cup B (1, 2) ⁺- α cl (B) \subseteq U. This implies B (1, 2) ⁺- α cl (A) \cup B (1, 2) ⁺- α cl (B) \subseteq U. This implies B (1, 2) ⁺- α cl (A) \cup B (1, 2) ⁺- α cl (B) \subseteq U. This implies B (1, 2) ⁺- α cl (A) \cup B (1, 2) ⁺- α cl (B) \subseteq U. This implies B (1, 2) ⁺- α cl (A) \cup B (1, 2) ⁺- α cl (B) \subseteq U. This implies B (1, 2) ⁺- α cl (A) \cup B (1, 2) ⁺- α cl (B) \subseteq U. This implies B (1, 2) ⁺- α cl (A) \cup B (1, 2) ⁺- α cl (B) \subseteq U. This implies B (1, 2) ⁺- α cl (A) \cup B (1, 2) ⁺- α cl (B) \subseteq U. This implies B (1, 2) ⁺- α cl (A) \cup B (1, 2) ⁺- α cl (B) \subseteq U. This implies B (1, 2) ⁺- α cl (A) \cup B (1, 2) ⁺- α cl (B) \subseteq U. This implies B (1, 2) ⁺- α cl (A) \cup B (1, 2) ⁺- α cl (B) \subseteq U. This implies B (1, 2) ⁺- α cl (B) \subseteq U whenever A \cup B \subseteq U where U is $\tau_{1,2}$ - π -open.

Theorem: 3.9

(i) Every closed set in $\tau_{1,2}$ (B) is B (1, 2) ⁺- π g α -closed set. (ii)Every B (1, 2) ⁺-g-closed set in $\tau_{1,2}$ (B) is B (1, 2) ⁺- π g α -closed set. (iii) Every B (1, 2) ⁺- π g-closed set in $\tau_{1,2}$ (B) is B (1, 2) ⁺- π g α -closed set.

Proof: The proof is obvious.

The converse of the above theorem is not true in general. The following examples shows this result.

Example: 3.10 Consider $X = \{a, b, c\}, \tau_1 = \{\phi, X, \{a\}\}, \tau_2 = \{\phi, X, \{a, b\}\}$. Let $B = \{c\}$. Here the B (1, 2) ⁺- π ga-closed sets are $\{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Then $\{a, c\}$ is B (1, 2) ⁺- π ga-closed set, but not B $\tau_{1, 2}$ -closed set.

Example: 3.11 In example 3. 10, the B (1, 2) ⁺-g-closed sets are { ϕ , X, {b}, {c}, {a, b}, {a, c}, {b, c}}. Then {a} is B (1, 2) ⁺- π g α -closed set but not B (1, 2) ⁺-g-closed set.

Example: 3.12 Consider $X = \{a, b, c, d\}, \tau_1 = \{\phi, X, \{d\}, \{c, d\}, \{a, d\}, \{a, c, d\}\}, \tau_2 = \{\phi, X, \{a, c\}\}$. Let $B = \{a\}$. Here the B(1, 2) ⁺- π g α -closed sets are $\{\phi, X, \{b\}, \{c\}, \{b, c\}, \{a, b\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}\}$ and B (1, 2) ⁺- π g-closed sets are $\{\phi, X, \{b\}, \{b, c\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$. Then $\{c\}$ is B(1, 2) ⁺- π g α -closed set but not B (1, 2) ⁺- π g-closed set.

4. B (1, 2) ⁺-πgα-REGULAR SPACES

In this section, we introduce $B(1, 2)^+$ - $\pi g\alpha$ -regular space in bitopological spaces and also we obtain some characterizations of $B(1, 2)^+$ - $\pi g\alpha$ -regular spaces.

Definition: 4.1 A subset A of a space X is said to be B $(1, 2)^+$ - π g α -regular if for every B $(1, 2)^+$ - π g α -closed set F and a point x \notin F, there exist disjoint B $(1, 2)^+$ - α -open sets U and V such that F \subset U and x \in V.

Theorem: 4.2 For a bitopological spaces X, the following are equivalent:

- 1. X is B(1, 2) $+\pi g\alpha$ -regular.
- 2. Every B(1, 2) $+\pi g\alpha$ -open set U is a union of B (1, 2) $+\alpha$ -regular sets.
- 3. Every $B(1, 2)^+$ - $\pi g\alpha$ -closed set A is an intersection of B $(1, 2)^+$ - $\pi g\alpha$ -regular sets.

Proof:

[1] => [2]: Let U be a B (1, 2) ⁺- π g α -open set and let $x \in U$. If A = X-U, then A is B(1, 2) ⁺- π g α -closed. By assumption there exist disjoint B (1, 2) ⁺- α -open subsets W₁ and W₂ of X such that $x \in W_1$ and A $\subseteq W_2$. If V = B (1, 2) ⁺- α cl (W₁) then V is B (1, 2) ⁺- α -closed and V \cap A \subseteq V \cap W₂ = ϕ . It follows that $x \in V \subseteq U$. Thus U is a union of B (1, 2) ⁺- α -regular sets.

[2] => **[3]:** This is obvious.

[3] => [1]: Let A be B (1, 2) ⁺- π g α -closed and let $x \notin A$. By assumption there exists a B (1, 2) ⁺- α -regular set V such that A \subseteq V and $x \notin V$. If U = X/V, then U is B (1, 2) ⁺- α -open set containing x and U \cap V = φ . Thus, X is B(1, 2) ⁺- π g α -regular.

Definition: 4.3 A bitopological space X is called a

- (1) B (1, 2) $+\pi g\alpha$ -open-T₀ if for each pair of distinct points, there exists an B (1, 2) $+\pi g\alpha$ -open set containing one point but not the other.
- (2) $B(1, 2)^+ -\pi g\alpha$ -open- T_1 if for any two distinct points x, y in X, there exists a pair of $B(1, 2)^+ -\pi g\alpha$ -open sets, one containing x but not y and the other containing y but not x.
- (3) B (1, 2) ⁺- α -open-T₂ if for each pair of distinct points x and y in X, there exist disjoint B (1, 2) ⁺- α -open sets U and V in X such that $x \in U$ and $y \in V$.
- (4) (B, B (1, 2) ⁺- π ga-open-R₀)-space if B (1, 2) ⁺- π gacl ({x}) \subseteq U whenever U is B (1, 2) ⁺- π ga-open and x \in U.

Theorem: 4.4 Every B $(1, 2)^+$ - $\pi g\alpha$ -regular space X is both B $(1, 2)^+$ - α -T₂ and (B, B $(1, 2)^+$ - $\pi g\alpha$ -open-R₀)-space.

Proof: Let X be B $(1, 2)^{+}$ - $\pi g \alpha$ -regular space and let x, $y \in X$ such that $x \neq y$. By theorem 4. 2. {x} is either B $(1, 2)^{+}$ - α -open (or) B $(1, 2)^{+}$ - α -closed. Since every space is B $(1, 2)^{+}$ - α -T₂. If {x} is B $(1, 2)^{+}$ - α -open, hence B $(1, 2)^{+}$ - $\pi g \alpha$ -open. Thus {x} and X - {x} are separating B $(1, 2)^{+}$ - α -open sets. If {x} is B $(1, 2)^{+}$ - α -closed, then X - {x} are B $(1, 2)^{+}$ - α -open and hence by theorem 4. 2 X is the union of B $(1, 2)^{+}$ - α -regular sets. Hence there is a B $(1, 2)^{+}$ - α -regular set V $\subseteq X - {x}$ containing y. This proves that X is B $(1, 2)^{+}$ - α -T₂. By Theorem 4. 2. it follows immediately that X is also (B, B $(1, 2)^{+}$ - $\pi g \alpha$ -open-R₀)-space.

Definition: 4.5 The intersection of all B $(1, 2)^+$ - $\pi g\alpha$ -closed sets containing A is called B $(1, 2)^+$ - $\pi g\alpha$ -closure of A and is denoted by B $(1, 2)^+$ - $\pi g\alpha$ -cl (A).

Theorem: 4.6 The following are equivalent for a space X:

- 1. X is B $(1, 2)^+$ - π g α -regular.
- 2. B (1, 2) $+\pi g\alpha$ -cl (A) = \bigcap {F: A \subseteq F and F is B (1, 2) $+\pi g\alpha$ -closed in X} for each subset A of X.
- 3. B (1, 2) $+\pi g\alpha$ -cl (A) = A for each B (1, 2) $+\pi g\alpha$ -closed A.

Proof:

(1) => (2): For any subset A of X, we have $A \subseteq \bigcap \{F: A \subseteq F \text{ and } F \text{ is } B(1, 2)^+ -\pi g\alpha \text{-closed in } X\} \subseteq B(1, 2)^+ -\pi g\alpha \text{-cl}$ (A). Suppose $x \in X \setminus \bigcap \{F: A \subseteq F \text{ and } F \text{ is } B(1, 2)^+ -\pi g\alpha \text{-closed in } X\}$, then there exists a B $(1, 2)^+ -\pi g\alpha \text{-closed set } F$ such that $x \in X \setminus F$, $A \subseteq F$. Since X is B $(1, 2)^+ -\pi g\alpha \text{-regular}$, there exists disjoint B $(1, 2)^+ -\alpha \text{-open sets } U$ and V such that $x \in U$ and $F \subseteq A$. Therefore, we have $x \in U \subseteq B(1, 2)^+ -\alpha \operatorname{cl}(U) \subseteq X \setminus V \subseteq X \setminus F \subseteq X \setminus A$.

Hence, B (1, 2) $+\alpha cl(U) \cap A = \phi$. Therefore, $x \in X \setminus B$ (1, 2) $+\pi g\alpha - cl$ (A). Hence $\bigcap \{F: A \subseteq F \text{ and } F \text{ is } B$ (1, 2) $+\pi g\alpha - cl$ (A). Hence $\bigcap \{F: A \subseteq F \text{ and } F \text{ is } B$ (1, 2) $+\pi g\alpha - cl$ (A).

(2) => (3): We always have $A \subset B(1, 2)^+ -\pi g\alpha$ -cl (A) to be true.

Now to prove B (1, 2) $+\pi g\alpha$ -cl (A) \subset A. B (1, 2) $+\pi g\alpha$ -cl (A) is the smallest B (1, 2) $+\pi g\alpha$ -closed set and A is a B (1, 2) $+\pi g\alpha$ -closed set contained in F.

Therefore, B (1, 2) $^{+}-\pi g\alpha$ -cl (A) \subset A => A = B (1, 2) $^{+}-\pi g\alpha$ -cl (A).

(3) => (1): Let F be any B (1, 2) ⁺- π ga-closed set and $x \in X \setminus F$, then $x \in B$ (1, 2) ⁺- π ga-cl (F). Then there exists an B (1, 2) ⁺- α -open set U such that B (1, 2) ⁺- α cl(U) \cap F = ϕ . Therefore, we have F $\subset X \setminus B$ (1, 2) ⁺- α cl(U). Also U and X \B (1, 2) ⁺- α cl(U) are disjoint hence X is B (1, 2) ⁺- π ga-regular. Hence the proof.

Theorem: 4.7 Let X be a bitopological space. Then the following statements are equivalent.

- (1) X is B (1, 2) $+\pi g\alpha$ -regular.
- (2) For each point $x \in X$ and for each B $(1, 2)^+$ - $\pi g \alpha$ -open nbhd W of x, there exists a B $(1, 2)^+$ - α -open set U of X such that B $(1, 2)^+$ - $\alpha cl(U) \subseteq W$.
- (3) For each point $x \in X$ and for each B(1, 2) ⁺- π g α -closed set F not containing x, there exists a B (1, 2) ⁺- α -open set V of X such that B (1, 2) ⁺- α cl(V) \cap F = ϕ .

Proof:

(1) => (2): Let W be any B (1, 2) $^+$ - π g α -open nbhd of x. Then there exists an B (1, 2) $^+$ - π g α -open set G such that $x \in G \subseteq W$. Since G^c is B(1, 2) $^+$ - π g α -closed and $x \notin G^c$, by hypothesis, there exist B (1, 2) $^+$ - α -open sets U and V such that $G^c \subseteq U$, $x \in V$ and $U \cap V = \phi$ and so $V \subseteq U^c$. Now B (1, 2) $^+$ - α cl (V) \subseteq B (1, 2) $^+$ - α cl (U^c) = U^c and $G^c \subseteq U$ implies $U^c \subseteq G \subseteq W$. Thus B (1, 2) $^+$ - α cl (U) $\subseteq W$.

(2) => (1): Let F be any B (1, 2) ⁺- π g α -closed set and $x \notin F$. Then $x \in F^c$ and F^c is a B (1, 2) ⁺- π g α -open and so F^c is a B (1, 2) ⁺- π g α -nphd of x. By hypothesis, there exists a B (1, 2) ⁺- α -open set V of x such that $x \in V$ and B (1, 2) ⁺- α cl(V) $\subseteq F^c$, which implies $F \subseteq (B(1, 2)^+ - \alpha cl(V))^c$. Then $(B(1, 2)^+ - \alpha cl(V))^c$ is B (1, 2) ⁺- α -open set containing F and $V \cap (B(1, 2)^+ - \alpha cl(V))^c = \phi$. Therefore X is B(1, 2) ⁺- π g α -regular.

(2)=> (3): Let $x \in X$ and F be a B(1, 2) ⁺- π g α -closed set such that $x \notin F$. Then F^c is a B (1, 2) ⁺- π g α -nbhd of x and by hypothesis, there exists a B (1,2) ⁺- α -open set V of X, such that B (1,2) ⁺- α cl(V) \subseteq F^c and hence B (1,2) ⁺- α - cl(V) \cap F = ϕ .

(3)=> (2): Let $x \in X$ and W be a B(1, 2) ⁺- π g α -nbhd of x. Then there exists a B (1, 2) ⁺- π g α -open set G such that $x \in G \subseteq W$. Since G^c is B (1, 2) ⁺- π g α -closed and $x \notin G^c$, by hypothesis there exists a B (1, 2) ⁺- α -open set U of x such that B (1, 2) ⁺- α cl(U) $\cap G^c = \phi$. Therefore, B(1, 2) ⁺- α cl(U) $\subseteq G \subseteq W$.

Theorem: 4.8 A bitopological space X is B (1, 2) $^+$ - π g α -regular if and only if for each B (1, 2) $^+$ - π g α -closed set F of X and each $x \in F^c$, there exist B (1, 2) $^+$ - α -open sets U and V of X such that $x \in U$ and F \subseteq V and B (1, 2) $^+$ - α cl(U) \cap B (1, 2) $^+$ - α cl(V) = ϕ .

Proof: Let F be a B (1, 2) ⁺- π g α -closed set of X and x \notin F. Then there exist B (1, 2) ⁺- α -open sets U_x and V suchthat x \in U_x, F \subseteq V and U_x \cap V = ϕ . Which implies that U_x \cap B (1, 2) ⁺- α cl(V) = ϕ . Since X is B (1, 2) ⁺- π g α -regular, there exist B (1, 2) ⁺- α -open sets G and H of X such that x \in G, B (1, 2) ⁺- α cl(V) \subseteq H and G \cap H = ϕ . This implies B (1, 2) ⁺- α cl(G) \cap H = ϕ . Now put U = U_x \cap G, then U and V are B (1, 2) ⁺- α -open sets of X such that x \in U and F \subseteq V and B (1, 2) ⁺- α cl(U) \cap B (1, 2) ⁺- α cl(V) = ϕ .

Converse is obvious.

© 2013, IJMA. All Rights Reserved

I. Arockiarani* and K. Mohana/ B(1, 2)^{*}-πgα-Regular and Normal Spaces in Simple Expansion/IJMA- 4(12), Dec.-2013.

Definition: 4.9 A space X is said to be B $(1, 2)^+$ - α -symmetric if for any distinct points x and y of X, $x \in B(1, 2)^+$ - $\alpha cl\{y\}$ implies that $y \in B(1, 2)^+$ - $\alpha cl\{x\}$.

Definition: 4.10 A space X is called B $(1, 2)^+ - \alpha$ -Urysohn if for every pair of points x, $y \in X$, $x \neq y$ there exist $U \in B(1, 2)^+ - \alpha O(x)$, $V \in B(1, 2)^+ - \alpha O(y)$ such that $B(1, 2)^+ - \alpha cl(U) \cap B(1, 2)^+ - \alpha cl(U) = \phi$.

Theorem: 4.11 A space X is $B(1, 2)^+ -\alpha$ -symmetric if and only if $\{x\}$ is $B(1, 2)^+ -\pi g\alpha$ -closed in X for each $x \in X$.

Proof: Sufficiency part: Suppose $x \in B(1, 2)^+ - \alpha \operatorname{cl}\{y\}$. But $y \notin B(1, 2)^+ - \alpha \operatorname{cl}\{x\}$, then $y \in B(1, 2)^+ - \alpha \operatorname{cl}\{x\}^c$. Thus B $(1, 2)^+ - \alpha \operatorname{cl}\{y\} \in B(1, 2)^+ - \alpha \operatorname{cl}\{x\}$, then $x \in B(1, 2)^+ - \alpha \operatorname{cl}\{x\}$, which is a contradiction.

Necessity Part: Suppose $\{x\} \subseteq O \in \tau_{1, 2}$. But $\{x\} \not\subset O$ then $B(1, 2)^+ -\alpha \operatorname{cl}\{x\} \cap O^c = \phi$. Let $y \in B(1, 2)^+ -\alpha \operatorname{cl}\{x\} \cap O^C$ therefore $x \in B(1, 2)^+ -\alpha \operatorname{cl}\{y\} \subset O^c$ and $x \notin O$, which is a contradiction. Hence the proof.

Theorem: 4.12 If a space X is B $(1, 2)^+$ - π g α -regular and B $(1, 2)^+$ - α -symmetric, then it is B $(1, 2)^+$ - α -Urysohn.

Proof: Let x and y be any distinct points of X. Since X is B $(1, 2)^+$ - α -symmetric, {x} and {y} are B $(1, 2)^+$ - π g α -closed in X. By theorem 4. 8, there exists B $(1, 2)^+$ - α -open sets U and V such that $x \in U$ and $y \in V$ and B $(1, 2)^+$ - α cl $(U) \cap B$ $(1, 2)^+$ - α cl $(V) = \phi$. This implies X is B $(1, 2)^+$ - α -Urysohn.

Theorem: 4.13 If a space X is B $(1, 2)^+$ - π g α -regular and B $(1, 2)^+$ - α -symmetric, then it is B $(1, 2)^+$ - α -open-T₂.

Proof: Let x and y be any distinct points of X. Since X is B $(1, 2)^+$ - α -symmetric, $\{x\}$ and $\{y\}$ are B $(1, 2)^+$ - $\pi g\alpha$ -closed in X. Since X is B $(1, 2)^+$ - $\pi g\alpha$ -regular, there exists disjoint B $(1, 2)^+$ - α -open sets U and V such that $\{x\} \subseteq U$ and $\{y\} \subset V$ and $U \cap V = \phi$. This implies X is B $(1, 2)^+$ - α -open-T₂.

Lemma: 4.14 If Y is a B $(1, 2)^+$ - $\pi g\alpha$ -closed set of a space X and A is a B $(1, 2)^+$ - $\pi g\alpha$ -closed set of the subspace Y, then A is B $(1, 2)^+$ - $\pi g\alpha$ -closed set of X.

Proof: Let $A \subseteq O$. Suppose O is B (1, 2) ⁺- α -open in X, then $A \subseteq Y \cap O$. Hence B (1, 2) ⁺- α cl(A) $\subseteq Y \cap O$. It follows that $Y \cap B$ (1, 2) ⁺- α cl (A) $\subseteq Y \cap O$ and $Y \subseteq O \cup [B (1, 2)$ ⁺- α cl (A)]^c. Therefore, B (1, 2) ⁺- α cl (A) $\subseteq B$ (1, 2) ⁺- α cl(Y) $\subseteq O \cup [B (1, 2)$ ⁺- α cl (A)]^c and B (1, 2) ⁺- α cl (A) $\subseteq O$. This shows that, A is B (1, 2) ⁺- π ga-closed set in X.

Theorem: 4.15 If X is a B $(1, 2)^+$ - $\pi g\alpha$ -regular space and Y is a B $(1, 2)^+$ - $\pi g\alpha$ -closed subset of X, then the subspace Y is B $(1, 2)^+$ - $\pi g\alpha$ -regular.

Proof: Let A be any B $(1, 2)^{+}$ - $\pi g\alpha$ -closed set of Y, $y \in Y \setminus A$, by lemma 4. 14, A is B $(1, 2)^{+}$ - $\pi g\alpha$ -closed set in X. Since X is B $(1, 2)^{+}$ - $\pi g\alpha$ -regular, there exists disjoint B $(1, 2)^{+}$ - α -open sets U and V of X such that $y \in U$ and $A \subseteq U$. Therefore U \cap Y and V \cap Y are disjoint B $(1, 2)^{+}$ - α -open sets of the subspace Y, such that $y \in U \cap Y$ and $A \subseteq V \cap Y$. This implies that the subspace Y is B $(1, 2)^{+}$ - $\pi g\alpha$ -regular.

Definition: 4.16 A map $f : X \rightarrow Y$ is called M - B (1, 2) ⁺- α -open (resp. M-B (1, 2) ⁺- α -closed) if f(V) is B (1, 2) ⁺- α -open(resp. B (1, 2) ⁺- α -closed) set in Y for every B (1, 2) ⁺- α -open(resp. B (1, 2) ⁺- α -closed) set V of X.

Definition: 4.17 A map f: $X \rightarrow Y$ is called B (1, 2) ⁺- π ga-continuous (resp. B (1, 2) ⁺- π ga-irresolute) if f¹(V) is B (1, 2) ⁺- π ga-openset in X, for every B $\tau_{1, 2}$ - (resp. B (1, 2) ⁺- π ga-) open set V of Y.

Theorem: 4.18 If $f: X \to Y$ is bijective $\tau_{1,2}$ -open and B $(1, 2)^+$ - $\pi g\alpha$ -continuous, then it is B $(1, 2)^+$ - $\pi g\alpha$ -irresolute.

Proof: Let A be a B (1, 2) ⁺- π ga-closed set in Y. Let f ⁻¹(A) \subseteq O, where O is B $\tau_{1, 2}$ -open in X. Therefore, A \subset f (O), holds. Since f (O) is B $\tau_{1, 2}$ -open and A is B (1, 2) ⁺- π ga-closed in Y. B (1, 2) ⁺- α cl (A) \subseteq f (O) holds. Hence f ⁻¹(B (1, 2) ⁺- α cl (A)) \subseteq O. Since f is B (1, 2) ⁺- π ga-continuous, B (1, 2) ⁺- α cl (A) is B (1, 2) ⁺- α -closed in Y, f ⁻¹(B (1, 2) ⁺- α cl (A)) \subseteq O. So B (1, 2) ⁺- α cl (f ⁻¹(A)) \subseteq O. Therefore, f ⁻¹(A) is B (1, 2) ⁺- π ga-closed in X. Hence, f is B (1, 2) ⁺- π ga-irresolute.

Theorem: 4.19 If f: X \rightarrow Y is an B $\tau_{1, 2}$ -open, B (1, 2) ⁺- π g α -continuous bijection and X is B (1, 2) ⁺- π g α -regular, then Y is B (1, 2) ⁺- π g α -regular.

Proof: Let F be any B (1, 2) $+\pi g\alpha$ -closed set in Y and $y \in Y \setminus F$. Since f is B $\tau_{1, 2}$ -open, B (1, 2) $+\pi g\alpha$ -continuous bijective then f is B (1, 2) $+\pi g\alpha$ -irresolute. Hence f $^{-1}(F)$ is B (1, 2) $+\pi g\alpha$ -closed in X, for every B (1, 2) $+\pi g\alpha$ -closed set F in Y. Let f(x) = y, we have $x \in X \setminus f^{-1}(F)$. Since X is B (1, 2) $+\pi g\alpha$ -regular, there exist disjoint B $\tau_{1, 2}$ -open sets U

I. Arockiarani* and K. Mohana/ B(1, 2)⁺-πgα-Regular and Normal Spaces in Simple Expansion/IJMA- 4(12), Dec.-2013.

and V such that $x \in U$, $f^{-1}(F) \subset V$. Since f is B $\tau_{1, 2}$ -open and bijective, $y \in f(U)$, $F \subset f(V)$. Hence $f(U) \cap f(V) = \phi$. Therefore, Y is B $(1, 2)^{+}$ - $\pi g \alpha$ -regular.

Theorem: 4.20 If X is B (1, 2) ⁺- π g α -regular space and f: X \rightarrow Y is bijective, B (1, 2) ⁺- π g α -irresolute and M- B (1, 2) ⁺- α -open, then Y is B (1, 2) ⁺- π g α -regular space.

Proof: Let $y \in Y$ and F be any B $(1, 2)^+$ - $\pi g \alpha$ -closed subset of Y with $y \notin F$. Since f is B $(1, 2)^+$ - $\pi g \alpha$ -irresolute, $f^1(F)$ is B $(1, 2)^+$ - $\pi g \alpha$ -closed set in X. Since f is bijective. Let f(x) = y, then $x \neq f^1(y)$. By hypothesis, there exist B $(1, 2)^+$ - α -open sets U and V such that $x \in U$ and $f^1(F) \subseteq V$. Since f is M- B $(1, 2)^+$ - α -open and bijective. We have $y \in f(U)$ and $F \subseteq f(V)$ and $f(U) \cap f(V) = f(U \cap V) = \phi$. Hence Y is B $(1, 2)^+$ - $\pi g \alpha$ -regular space.

Theorem: 4.21 If f: $X \rightarrow Y$ is B (1, 2) ⁺- π g α -irresolute and M- B (1, 2) ⁺- α -closed and A is a B (1, 2) ⁺- π g α -closed subset of X, then f (A) is B (1, 2) ⁺- π g α -closed.

Theorem: 4.22 Iff: $X \rightarrow Y$ is B (1, 2) ⁺- π g α -irresolute, M- B (1, 2) ⁺- α -closed and injective and Y is B (1, 2) ⁺- π g α -regular, then X is B (1, 2) ⁺- π g α -regular.

Proof: Let F be any B $(1, 2)^+$ - π g α -closed set of X and x \notin F. Since f is $(1, 2)^+$ - π g α -irresolute, M-B $(1, 2)^+$ - α -closed, by Theorem 4. 21., f (F) is B $(1, 2)^+$ - π g α -closed in Y and f(x) \notin f (F). Since Y is B $(1, 2)^+$ - π g α -regular and so there exist disjoint B $(1, 2)^+$ - α -open sets U and V in Y such that $f(x) \in U$ and f (F) \subseteq V. By hypothesis, $f^1(U)$ and $f^1(V) \in$ B $(1, 2)^+$ - α O(X) such that $x \in f^1(U)$ and $F \subseteq f^1(V)$ and $f^1(U) \cap f^1(V) = \phi$. Therefore X is B $(1, 2)^+$ - π g α -regular.

5. B(1, 2) ⁺-πgα-NORMAL SPACES

Definition: 5.1 A bitopological space X is called a B $(1, 2)^+$ - $\pi g\alpha$ -normal, if for every pair of disjoint B $(1, 2)^+$ - $\pi g\alpha$ -closed subsets A and B, there exist disjoint B $(1, 2)^+$ - α -open sets U and V of X such that A \subseteq U and B \subseteq V.

Theorem: 5.2 Let X be a bitopological space. Then the following statements are equivalent:

- (1) X is B (1, 2) $^{+}$ - π g α -normal.
- (2) For each B (1, 2) ⁺-πgα-closed set F and for each B (1, 2) ⁺-πgα-open set U containing F, there exists a B (1, 2) ⁺-α-open set V containing F such that B (1, 2) ⁺-αcl(V) (V).
- (3) For each pair of disjoint B (1, 2) $+\pi g\alpha$ -closed set A and B in X, there exists a B (1, 2) $+\alpha$ -open set U containing A such that B (1, 2) $+\alpha cl$ (U) \cap B = ϕ .
- (4) For each pair of disjoint B (1, 2) $+\pi g\alpha$ -closed set A and B in X, there exists a B (1, 2) $+\alpha$ open sets U and V such that A \subseteq U, B \subseteq V and B (1, 2) $+\alpha$ -cl(A) \cap B (1, 2) $+\alpha$ -cl(B) = ϕ .

Proof:

(1) \rightarrow (2): Let F be a B (1, 2) ⁺- π g α -closed set and U be a B (1, 2) ⁺- π g α -open set such that F \subseteq U. Then F \cap U^c = ϕ . By assumption, there exist B (1, 2) ⁺- α -open sets V and W such that F \subseteq U, U^c \subseteq W and V \cap W = ϕ , which implies B (1, 2) ⁺- α -cl(V) \cap W = ϕ . Now B (1,2) ⁺- α -cl (V) \cap U^c \subseteq B (1, 2) ⁺- α -cl (V) \cap W = ϕ and so B (1, 2) ⁺- α -cl(V) \subseteq U.

(2) \rightarrow (3): Let A and B be disjoint B (1, 2) ⁺- π ga-closed sets of X. Since $A \cap B = \phi$, $A \subseteq B^{c}$ and B^{c} is B (1, 2) ⁺- π ga-open. By assumption, there exists B (1, 2) ⁺- α -open set U containing A such that B (1, 2) ⁺- α cl(U) $\subseteq B^{c}$ and so B (1, 2) ⁺- α cl(U) $\cap B = \phi$.

(3) \rightarrow (4): Let A and B be B (1, 2) ⁺- π g α -closed sets of X. Then by assumption, there exists B (1, 2) ⁺- α -open set U containing A such that B (1, 2) ⁺- α cl(U) \cap B = ϕ . Since B (1, 2) ⁺- α cl(A) is B (1, 2) ⁺- α -closed, it is B (1, 2) ⁺- π g α -closed and so B and B (1, 2) ⁺- α cl(A) are disjoint B (1, 2) ⁺- π g α -closed sets in X. Therefore again by assumption, there exists a B (1, 2) ⁺- α -open set V containing B such that B (1, 2) ⁺- α cl(A) \cap B (1, 2) ⁺- α cl(B) = ϕ .

(4) \rightarrow (1): Let A and B be any disjoint B (1, 2) +- π ga-closed sets of X. By assumption, there exist B (1, 2) +- α -open sets U and V such that A \subseteq U, B \subseteq V and B (1, 2) +- α cl(A) \cap B (1, 2) +- α cl(B) = ϕ , we have U \cap V = ϕ and thus X is B (1, 2) +- π ga-normal.

Theorem: 5.3 If f: X \rightarrow Y is bijective, B (1, 2) ⁺- π g α -irresolute and M- B (1, 2) ⁺- α -open mapping and X is B (1, 2) ⁺- π g α -normal space, then Y is B (1, 2) ⁺- π g α -normal space.

Proof: Let A and B be any disjoint B(1, 2) $+\pi g\alpha$ -closed subset of Y. Since f is B (1, 2) $+\pi g\alpha$ -irresolute, $f^{1}(A)$ and $f^{1}(B)$ are disjoint B (1, 2) $+\pi g\alpha$ -closed set of X. As X is B (1, 2) $+\pi g\alpha$ -normal space, there exist disjoint B (1, 2) $+\alpha$ -open sets U and V such that $f^{1}(A) \subseteq U$ and $f^{1}(B) \subseteq V$. Since f is M- B (1, 2) $+\alpha$ -open and bijective. We have f(U) and \emptyset 2013, IJMA. All Rights Reserved 228

I. Arockiarani* and K. Mohana/ B(1, 2) $^{+}$ - π ga-Regular and Normal Spaces in Simple Expansion/IJMA- 4(12), Dec.-2013.

f(V) are B (1, 2) + α -open sets in Y such that $A \subseteq f(U)$ and $B \subseteq f(V)$ and $f(U) \cap f(V) = \phi$. Hence Y is B (1, 2) + $\pi g \alpha$ -normal space.

Theorem: 5.4 If f: X \rightarrow Y is B (1, 2) ⁺- π g α -irresolute, M- B (1, 2) ⁺- α -closed and B (1, 2) ⁺- α -irresolute injective and Y is B (1, 2) ⁺- π g α -normal, then X is B (1, 2) ⁺- π g α -normal.

Proof: Let A and B be any two disjoint B $(1, 2)^{+}$ - $\pi g\alpha$ -closed sets of X. Since the map f is B $(1, 2)^{+}$ - $\pi g\alpha$ -irresolute, M- B $(1, 2)^{+}$ - α -closed, by Theorem 4. 12., f (A) and f (B) are disjoint B $(1, 2)^{+}$ - $\pi g\alpha$ -closed sets of Y. Since Y is B $(1, 2)^{+}$ - $\pi g\alpha$ -normal, there exist disjoint B $(1, 2)^{+}$ - α -open sets U and V in Y such that f (A) \subseteq U and f (B) \subseteq V. That is, A \subseteq f⁻¹(U) and B \subseteq f⁻¹(V) and f⁻¹(U) \cap f⁻¹(V) = ϕ . Since f is B $(1, 2)^{+}$ - α -irresolute, f⁻¹(U) and f⁻¹(V) are B $(1, 2)^{+}$ - α -open sets in X, we have X is B $(1, 2)^{+}$ - $\pi g\alpha$ -normal space.

Theorem: 5.5 Suppose that $B \subseteq A \subseteq X$, B is B (1, 2) ⁺- π g α -closed set relative to A and that A is B (1, 2) ⁺- α -open and B (1, 2) ⁺- π g α -closed in X. Then B is B (1, 2) ⁺- π g α -closed in X.

Theorem: 5.6 If X is B (1, 2) $+\pi g\alpha$ -normal space and Y is an B (1, 2) $+\alpha$ -open and B (1, 2) $+\pi g\alpha$ -closed subset of X, then the subspace Y is B (1, 2) $+\pi g\alpha$ -normal.

Proof: Let A and B be any two disjoint B $(1, 2)^{+}$ - π g α -closed sets of Y. By theorem 5.5 and A and B are B $(1, 2)^{+}$ - π g α -closed in X. Since X is B $(1, 2)^{+}$ - π g α -normal, there exists disjoint B $(1, 2)^{+}$ - α -open sets U and V of X such that A \subseteq U and B \subseteq V. Since Y is B $(1, 2)^{+}$ - α -open, U \cap Y and V \cap Y are disjoint B $(1, 2)^{+}$ - α -open sets of the subspace Y. Hence the subspace Y is B $(1, 2)^{+}$ - π g α -normal.

Theorem: 5.7 If f: X \rightarrow Y is bijective, B (1, 2) ⁺- π g α -continuous and M- B (1, 2) ⁺- α -open mapping and X is B (1, 2) ⁺- π g α -normal space, then Y is B (1, 2) ⁺- π g α -normal space.

Proof: Let A and B be any disjoint B $(1, 2)^+$ - $\pi g \alpha$ -closed subset of Y. The map f is B $(1, 2)^+$ - $\pi g \alpha$ -irresolute by theorem 4. 18 and so f¹(A) and f¹(B) are disjoint B $(1, 2)^+$ - $\pi g \alpha$ -closed set of X. Since X is B $(1, 2)^+$ - $\pi g \alpha$ -normal space, there exist disjoint B $(1, 2)^+$ - α -open sets U and V such that f¹(A) \subseteq U and f¹(B) \subseteq V. Since f is M- B $(1, 2)^+$ - α -open and bijective. We have f(U) and f(V) are B $(1, 2)^+$ - α -open sets in Y such that A \subseteq f(U) and B \subseteq f(V) and f(U) \cap f(V) = ϕ . Hence Y is B $(1, 2)^+$ - $\pi g \alpha$ -normal space.

ACKNOWLEDGEMENT

The authors gratefully acknowledge the University Grants Commission, India for the financial support.

REFERENCES

- 1. M. E. Abd El. Monsef, A. M. Kozae and R. A. Abu-Gdairi, New Approaches for generalised continuous function, Int. Journal of Math. Analysis, Vol. 4, 2010, No. 27, 1329-1339.
- 2. I. Arockiarani and K. Mohana, (1, 2)*-πgα -closed sets and (1, 2)*-quasi-α- normal spaces in Bitopological settings, Andartica J. Math., 7(3) (2010), 345-355.
- 3. I. Arockiarani and K. Mohana, (1, 2)*-πgα -continuous functions in Bitopological Spaces, Acta ciencia Indica, XXXVII (4) (2011), 819-829.
- 4. M. Lellis Thivagar and O. Ravi, On stronger forms of (1, 2)*-quotient mappings, Int. J. of Mathematics, Game theory and Algebra. 14(6)(2004), 481-492.
- 5. N. Levine, Simple Extension of Topologies, Amer. Math. Japan. Monthly, 71(1963), 22-105.
- 6. B. M. Munshi, Separation Axioms, Acta ciencia Indica, 12(1986), 140-144.
- 7. T. Noiri and V. Popa, On g-regular spaces and some functions, Mem. Fac. Sci. Kochi Univ, Ser. A. Math., 20(1999), 67-74.
- 8. F. Nirmala Irudayam and I. Arockiarani, On τ_p^+ Generalized closed sets, τ_p^+g regular and τ_p^+g normal spaces, IJMA, 2(8), 2011, 1405-1410.
- 9. F. Nirmala Irudayam and I. Arockiarani, On τ^+_{β} Generalized closed sets and τ^+_{β} g regular spaces, Inter. Journal of Advanced scientific and technical research, 2(1), 2012, 41-50.
- 10. A. Vadivel, R. Vijayalakshmi and D. Krishnaswamy, B-Generalized Regular and B-Generalized Normal Spaces, International Mathematical Forum, 5, 2010, No. 54, 2699- 2706.

Source of support: University Grants Commission, India, Conflict of interest: None Declared