Fuzzy Soft ideals of K-algebras

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ABSTRACT

In this paper, we introduce the concept of fuzzy soft ideals of K-algebras and investigate some of their properties. We discuss about fuzzy soft inverse images of fuzzy soft ideals. Also we introduce the notion of \((\varepsilon, \varepsilon \sqcup q)\)-fuzzy soft ideals of \(K\) and obtain it's characterization.

Keywords: K-algebras, fuzzy soft ideals of K-algebras, \((\varepsilon, \varepsilon \sqcup q)\) - fuzzy soft ideals.

1. INTRODUCTION

The notion of a K-algebra \((G, \cdot, e)\) was first introduced by Dar & Akram[3] in 2003. A K-algebra is an algebra built on a group \((G, \cdot, e)\) by adjoining an induced binary operation \(\otimes\) on \(G\) which is attached to an abstract K-algebra \((G, \cdot, \otimes, e)\). This system is in general non commutative and non associative with a right identity \(e\) if \((G, \cdot, e)\) is non commutative. Fuzzy sets and soft sets are two different computing models for representing uncertainty & vagueness. In this paper we apply these models in combination to study uncertainty & vagueness in fuzzy soft ideals of K-algebras.

2. PRELIMINARIES

In this section, we recall some basic concepts that are necessary for subsequent discussion.

Definition: 2.1 Let \((G, \cdot, e)\) be a group in which each non identity element is not of order 2. Then a K-algebra is a structure \(K = (G, \cdot, \otimes, e)\) on a group \(G\) in which induced binary operation \(\otimes\) : \(G \times G \rightarrow G\) is defined by \(\otimes(x, y) = x \cdot y^{-1}\) and satisfies the following axioms

\[(K1) (x \otimes y) \otimes (x \otimes z) = (x \otimes (e \otimes y) \otimes (e \otimes y)) \otimes x\]

\[(K2) x \otimes (x \otimes y) = (x \otimes (e \otimes y)) \otimes x\]

\[(K3) x \otimes x = e\]

\[(K4) x \otimes e = x\]

\[(K5) e \otimes x = x^{-1}\] for all \(x, y, z \in G\)

Definition: 2.2 Let \(X\) be a non-empty set. A fuzzy subset of \(X\) is defined as a mapping from \(X\) into \([0, 1]\).

Definition: 2.3 A fuzzy set \(\mu\) in a set \(X\) of the form \(\mu(y) = \begin{cases} t \in (0, 1) & \text{if } y = x \\ 0 & \text{if } y \neq x \end{cases}\) is said to be a fuzzy point with support \(x\) & value \(t\) and is denoted by \(x, t\). For a fuzzy point \(x, t\) and a fuzzy set \(\mu\) in a set \(X\) the symbol \(x, t \in \mu\) if \(\mu(x) \geq t\). A fuzzy point \(x, t\) is quasi coincident with a fuzzy set \(\mu\) written as \(x, t \in \mu\) if \(\mu(x) + t > 1\). \(x, t \in \mu\) (respectively \(x, t \in \mu\) for \(x, t \in \mu\) means that \(x, t \in \mu\) or \(x, t \in \mu\) holds for all \(x, y \in G\).

Definition: 2.4 A non-empty subset \(H\) of a K-algebra \(K\) is called a sub-algebra [6] of the K-algebra \(K\) if \(a \otimes b \in H\) for all \(a, b \in H\). We note that every sub algebra of a K-algebra \(K\) contains the identity \(e\) of the group.

Definition: 2.5 A fuzzy set \(\mu\) on a K-algebra \(K\) is a fuzzy ideal if \(\mu: G \rightarrow [0, 1]\) is such that \(\mu(e) \geq \mu(x)\) for all \(x \in G\) and \(\mu(x) \geq \mu(y) \mu(y \otimes (y \otimes x))\) for all \(x, y \in G\).

Definition: 2.6 A fuzzy set \(\mu\) in \(K\) is called a \((\varepsilon, \varepsilon \sqcup q)\) - fuzzy ideal of \(K\) if it satisfies the following conditions:

\(x, e \in \mu \Rightarrow e, v \sqcup q \in \mu\) for all \(x \in G\) and for all \(t \in (0, 1]\), \(x \otimes y, e \in \mu, (y \otimes (y \otimes x)), e \in \mu \Rightarrow x, t \in \mu\) for all \(x \in G\).
Molodtsov [5] defined the notion of soft set in the following way: Let \( U \) be an initial Universe and \( E \) be the set of parameters. Let \( P(U) = [0, 1] \) denote the power set of \( U \) and let \( A \) be a non-empty subset of \( E \). A pair \((f, A)\) is called a soft set over \( U \) where \( f \) is a mapping given by \( f: A \rightarrow P(U) \). In other words a soft set over \( U \) is parameterized family of subsets of \( U \). For \( \xi \in A \), \((f(\xi), C)\) may be considered as the set of \( \xi \)- approximate elements of the soft set \((f, A)\). Let \( f: A \rightarrow [0, 1], 1 = [0, 1] \) then \((f, A)\) is called a fuzzy soft set over \( U \). In general for all \( \xi \in A \), \( f(\xi) \) is a fuzzy set of \( U \) and it is called fuzzy value set of parameter \( x \).

1. LEVEL SOFT SETS OF FUZZY SOFT SET

Definition: 2.7 Let \((f, A)\) be a fuzzy soft set over \( U \). For each \( t \in [0, 1] \) the set \((f, A)^t = (f^t, A)\) is called a \( t \)-level soft set of \((f, A)\) where \( f^t = \{ x \in U / f(\xi)(x) \geq t \} \) for all \( \xi \in A \). Clearly \((f, A)^t\) is a soft set over \( U \).

Example: 3.2 Consider the K-algebra \( \mathcal{K} = (S_3, \cdot, \varnothing, e) \) on the symmetric group \( S_3 = \{e, a, b, x, y, z\} \) where \( e = (1) \), \( a = (1\ 2\ 3) \), \( b = (1\ 3\ 2) \), \( x = (1\ 2) \), \( y = (1\ 3) \) and \( z = (2\ 3) \). Then \( (S_3, \cdot) \) is given by the Cayley table:

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<thead>
<tr>
<th>( \cdot )</th>
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In this section we introduce fuzzy soft ideals of K-algebras and study about their properties.

Definition: 3.1 Let \((f, A)\) be a soft set over a K-algebra \( \mathcal{K} \). Then \((f, A)\) is called a soft ideal over \( \mathcal{K} \) if \( f(t) \) is an ideal of \( K \) for all \( t \in A \). (i.e.,)

i. \( e \in f(t) \)

ii. \( x \varnothing y \in f(t) \) & \( (y \varnothing (y \varnothing x)) \in f(t) \) \( \Rightarrow x \in f(t) \)

Example: 3.2 Consider the K-algebra \( \mathcal{K} = (S_3, \cdot, \varnothing, e) \) on the symmetric group \( S_3 = \{e, a, b, x, y, z\} \) where \( e = (1) \), \( a = (1\ 2\ 3) \), \( b = (1\ 3\ 2) \), \( x = (1\ 2) \), \( y = (1\ 3) \) and \( z = (2\ 3) \). Then \((f, A)\) is called a soft ideal over \( \mathcal{K} \) if \( f(t) \) is an ideal of \( K \) for all \( t \in A \). (i.e.,)

i. \( e \in f(t) \)

ii. \( x \varnothing y \in f(t) \) & \( (y \varnothing (y \varnothing x)) \in f(t) \) \( \Rightarrow x \in f(t) \)
Let \((f, A)\) be a soft set over \(\mathcal{K}\) where \(A = \mathcal{K}\) and \(f: A \rightarrow P(\mathcal{K})\) is a set valued function.

Then \(f(e) = \{e\}, f(a) = f(b) = \{e, a, b\}, f(x) = \{e, x\}, f(y) = \{e, y\}, f(z) = \{e, z\}\) are ideals of \(\mathcal{K}\). Then \((f, A)\) is a soft ideal of \(\mathcal{K}\).

**Definition: 3.3** Let \((f, A)\) be a soft set over a K-algebra \(\mathcal{K}\). Therefore \((f, A)\) is said to be a fuzzy soft ideal over \(\mathcal{K}\) if \(f_e\) is a fuzzy ideal of \(\mathcal{K}\) for all \(e \in A\). (i.e.,)

1. \(f_e(x) \geq f_e(x)\) for all \(x \in G\)
2. \(f_e(x) \geq \min \{f_{\mathcal{E}}(x \odot y), f_{\mathcal{E}}(y \odot (y \odot x))\}\) for all \(x, y \in G\).

**Example: 3.4** Consider the K-algebra \(\mathcal{K} = (G, \cdot, e, \odot, e)\) where \(G = \{e, a, a^2, a^3\}\) is the cyclic group of order 4 and \(\odot\) is given by the following Cayley table:

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</table>

Let \(A = \{e_1, e_2, e_3\}\) & \(f: A \rightarrow P(G)\) be a set valued function defined by

\(f(e_1) = \{(e, .7), (a, .3), (a^2, .6), (a^3, .3)\}\);

\(f(e_2) = \{(e, .6), (a, .2), (a^2, .5), (a^3, .2)\}\) and \(f(e_3) = \{(e, .7), (a, .1), (a^2, .3), (a^3, .1)\}\)

Let \(B = \{e_2, e_3\}\) & \(g : B \rightarrow P(G)\) be a set valued function defined by

\(g(e_2) = \{(e, .5), (a, .2), (a^2, .4), (a^3, .2)\}\) and \(g(e_3) = \{(e, .6), (a, .1), (a^2, .3), (a^3, .1)\}\).

Clearly \((f, A)\) and \((g, B)\) be two fuzzy soft sets over \(\mathcal{K}\). By routine calculation we see that they are fuzzy soft ideals of \(\mathcal{K}\).

**Definition: 3.5** Let \((f, A)\) and \((g, B)\) be two fuzzy soft ideals of \(\mathcal{K}\). Then \((f, A)\) is a fuzzy soft subideal of \((g, B)\) if \(\forall \xi \in A\) \((i)\) \(f(\xi) \geq g(\xi)\) for all \(x \in G\).

**Theorem: 3.8** Let \((f, A)\) be a soft set over a K-algebra \(\mathcal{K}\). Then \((f, A)\) is a fuzzy soft ideal of \(\mathcal{K}\) if and only if \((f, A)^t\) is a soft ideal of \(\mathcal{K}\) for all \(t \in [0, 1]\).

**Proof:** Suppose that \((f, A)\) is a fuzzy soft ideal of \(\mathcal{K}\). Then \(f_e(x) \geq f_e(x)\) for all \(x \in G\). Therefore \(f_{\mathcal{E}}(e) \geq f_{\mathcal{E}}(x)\) for all \(x \in G\). Now let \(x \in (x \odot y) \in (f, A)^t\) & \((y \odot (y \odot x)) \in (f, A)^t\). This implies \(f_{\mathcal{E}}(x \odot y) \geq t \& f_{\mathcal{E}}(y \odot (y \odot x)) \geq t\). Since \((f, A)\) is a fuzzy soft ideal \(f_{\mathcal{E}}(x) \geq \min \{f_{\mathcal{E}}(x \odot y), f_{\mathcal{E}}(y \odot (y \odot x))\} \geq t \Rightarrow x \in (f, A)^t\).

Conversely let \(t = \min \{f_{\mathcal{E}}(x \odot y), f_{\mathcal{E}}(y \odot (y \odot x))\}\). Then \(x \odot y \in (f, A)^t\) & \((y \odot (y \odot x)) \in (f, A)^t\). Since \((f, A)\) is a soft ideal of \(\mathcal{K}\) this implies that \(x \odot y \odot (y \odot x) = f_{\mathcal{E}}(x) \geq t\). Thus \(f_{\mathcal{E}}(x) \geq t \Rightarrow \min \{f_{\mathcal{E}}(x \odot y), f_{\mathcal{E}}(y \odot (y \odot x))\}\).

Since \((f, A)^t\) is a soft ideal of \(\mathcal{K}\) for all \(t \in [0, 1]\), \(e \in (f, A)^t\) for all \(t \in [0, 1]\) \(\Rightarrow f_{\mathcal{E}}(e) \geq t \Rightarrow f_{\mathcal{E}}(e) \geq t \Rightarrow f_{\mathcal{E}}(e) \geq t \forall x \in G\). Thus \((f, A)\) is a fuzzy soft ideal of \(\mathcal{K}\).

**Theorem: 3.9** Let \((g, B)\) be a fuzzy soft ideal of \(\mathcal{K}\). Let \((\varphi, \psi)\) be a fuzzy soft homomorphism from \(\mathcal{K}, 1\) to \(\mathcal{K}, 2\). Then \((\varphi, \psi)^{-1}\) is a fuzzy soft ideal of \(\mathcal{K}\).

**Proof:** Since \((g, B)\) is a fuzzy soft ideal of \(\mathcal{K}\),

(i) \(\varphi(\psi(e)) \geq \varphi(\psi(y)) \forall y \in G\) for all \(y \in G\)
\[ g \cdot \varphi (e_t) \geq g \cdot \varphi (x) \text{ where } x \in G_1 \]

\[ \varphi^{-1} g \cdot \varphi (e_t) \geq \varphi^{-1} g \cdot \varphi (x) \text{ for all } x \in G_1 \]

\[ ii) g \cdot \varphi^{-1} (y) \geq \min \{ g \cdot \varphi^{-1} (y \circ_2 z), g \cdot \varphi^{-1} ((z \circ_3 z) \circ_4 y) \} \]

\[ = \min \{ g \cdot \varphi^{-1} (\varphi (x) \circ_2 \varphi (x')), g \cdot \varphi^{-1} ((\varphi (x') \circ_3 (\varphi (x') \circ_4 \varphi (x))) \} \]

\[ \text{where } y = \varphi (x), \]

\[ z = \varphi (x') = \min \{ g \cdot \varphi^{-1} (x \circ \varphi (x)), \]

\[ g \cdot \varphi^{-1} (x' \circ_3 \varphi (x)) \} = \min \{ \varphi^{-1} g \cdot \varphi (x) \circ_2 \varphi (x'), \varphi^{-1} g \cdot \varphi ((x') \circ_3 (x' \circ_4 x)) \} \]

\[ \text{Then } \varphi^{-1} g \cdot \varphi^{-1} (x) \geq \min \{ \varphi^{-1} g \cdot \varphi (x \circ \varphi (x')), \varphi^{-1} g \cdot \varphi ((x') \circ_3 (x' \circ_4 x)) \} \text{ which implies } (\varphi, \psi)^{-1} (g, B) \text{ is a fuzzy soft ideal of } \mathcal{K}. \]

4. **(ε, ε ∨ q)-FUZZY SOFT IDEALS OF THE K-ALGEBRA**

**Definition:** 4.1 Given a fuzzy set \( \mu \) in \( \mathcal{K} \) and \( A \subseteq [0, 1] \), we define two set valued functions \( f: A \rightarrow \mathbb{P}(\mathcal{K}) \) and \( f_q: A \rightarrow \mathbb{P}(\mathcal{K}) \) by \( f(t) = \{ x \in G / x \in \mu \} \) and \( f_q(t) = \{ x \in G / x \in \mu \} \) for all \( t \in A \). Then \( f(t) \) is called an \( \varepsilon \)-soft set and \( f_q(t) \) is called \( q \)-soft set over \( \mathcal{K} \).

**Example:** 4.2 Consider the K-algebra \( \mathcal{K} = \{ G, \cdot, \circ, e \} \) where \( G = \{ e, a, a^2, a^3, a^4 \} \) is the cyclic group of order 5 & \( \circ \) is given by the following Cayley table:

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<thead>
<tr>
<th></th>
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<td>a^2</td>
<td>a^1</td>
<td>e</td>
<td>a</td>
</tr>
</tbody>
</table>

Let \( \mu \) be a fuzzy set in \( G \) defined by \( \mu(e) = .7, \mu(a) = .8, \mu(a^2) = .8, \mu(a^3) = .4 \). Then \( \mu \) is a fuzzy ideal of \( \mathcal{K} \).

**Proposition:** 4.3 Let \( \mu \) be a fuzzy set in a K-algebra \( \mathcal{K} \). Let \( (f, A) \) be an \( \varepsilon \)-soft set on \( \mathcal{K} \) with \( A = (0, 1] \). Then \( (f, A) \) is a soft ideal of \( \mathcal{K} \) if and only if \( \mu \) is a fuzzy ideal of \( \mathcal{K} \).

**Proof:** Assume that \( (f, A) \) is a soft ideal of \( \mathcal{K} \). Then \( f(t) \) is an ideal of \( \mathcal{K} \) for all \( t \in A \). If \( \mu \) is not a fuzzy ideal of \( \mathcal{K} \) then there exists \( x, y \in G \) such that \( \mu(x) < \mu(x \circ_3 y) \). Take \( t \in A \) such that \( \mu(x) < t \leq \min \{ \mu(x \circ_3 y), \mu(y \circ_3 (y \circ_3 x)) \} \). Since \( \mu(x) < t \leq \min \{ \mu(x \circ_3 y), \mu(y \circ_3 (y \circ_3 x)) \} \) implies \( x \in \mathcal{K} \) (i.e.,) \( x \in \mathcal{K} \) is a contradiction.

Conversely suppose \( \mu \) is a fuzzy ideal of \( \mathcal{K} \). Take \( t \in A \), \( x \circ_3 y \in f(t) \) and \( (y \circ_3 (y \circ_3 x)) \in f(t) \) implies \( x \in \mathcal{K} \) (i.e.,) \( x \in \mathcal{K} \) is an ideal of \( \mathcal{K} \). Hence \( f(t) \) is an ideal for all \( t \in (0, 1] \). Thus \( (f, A) \) is a soft ideal of \( \mathcal{K} \).

**Proposition:** 4.4 Let \( \mu \) be a fuzzy set in a K-algebra \( \mathcal{K} \) and let \( (f_q, A) \) be a q-soft set over \( \mathcal{K} \) with \( A = (0, 1] \). Then \( (f_q, A) \) is a soft ideal of \( \mathcal{K} \) if and only if \( \mu \) is a fuzzy ideal of \( \mathcal{K} \).

**Proof:** Suppose that \( \mu \) is a fuzzy ideal of \( \mathcal{K} \). Let \( t \in A \) & \( x, y \in G \) be such that \( \mu(x \circ_3 y) + t > 1 \) & \( \mu(y \circ_3 (y \circ_3 x)) + t > 1 \). We have

\[ \mu(x) \geq \min \{ \mu(x \circ_3 y), \mu(y \circ_3 (y \circ_3 x)) \} \]

\[ \Rightarrow \mu(x) + t \geq \min \{ \mu(x \circ_3 y), \mu(y \circ_3 (y \circ_3 x)) \} + t \]

\[ = \min \{ \mu(x \circ_3 y) + t, \mu(y \circ_3 (y \circ_3 x)) + t \} \]

\[ > 1 \]

implies \( x \in f_q(t) \).

Since \( \mu(e) \geq \mu(x) \) for all \( x \in G \), in particular \( \mu(e) \geq \mu(x) \) for all \( x \in f_q(t) \). Therefore \( \mu(e) \geq \mu(x) > 1 - t \Rightarrow \mu(e) + t > 1 \Rightarrow e \in f_q(t) \).
Conversely if $\mu$ is not a fuzzy ideal of $\mathcal{K}$ then there exists $t \in A$ such that $\mu(x) < 1 - t \leq \min \{ \mu(y \odot y), \mu(y \odot (y \odot x)) \} \Rightarrow \mu(x \odot y) \geq 1 - t \& \mu(y \odot (y \odot x)) \geq 1 - t \Rightarrow \mu(x \odot y) + t > 1 \& \mu(y \odot (y \odot x)) + t > 1 \Rightarrow x \in f(t)$ (i.e.,) $\mu(x) + t > 1 \Rightarrow \mu(x) > 1 - t$ a contradiction.

**Theorem:** 4.5 Let $\mu$ be a fuzzy set in a K-algebra $\mathcal{K}$. Let $(f, A)$ be an $e\epsilon$- soft set on $K$ with $A = (0.5, 1]$. Then the following assertions are equivalent:

i. $(f, A)$ is a soft ideal of $\mathcal{K}$

ii. $\max (\mu(x), .5) \geq \min \{ \mu(x \odot y), \mu(y \odot (y \odot x)) \}$

**Proof:**

(i) $\Rightarrow$ (ii): Let $(f, A)$ be a soft ideal of $\mathcal{K}$. If one of $\mu(x \odot y)$ or $\mu(y \odot (y \odot x))$ or both $\leq .5$. Then $\mu(x \odot y) \geq \min \{ \mu(x \odot y), \mu(y \odot (y \odot x)) \} \leq .5$. If $\mu(x) \leq .5$ then $\max \{ \mu(x), .5 \} \geq \min \{ \mu(x \odot y), \mu(y \odot (y \odot x)) \}$.

If $\mu(x) > .5$ then $\max \{ \mu(x), .5 \} = \mu(x)$ then $\max \{ \mu(x), .5 \} \geq \min \{ \mu(x \odot y), \mu(y \odot (y \odot x)) \}$.

If $\mu(x \odot y) > .5$ & $\mu(y \odot (y \odot x)) > .5$ then $\min \{ \mu(x \odot y), \mu(y \odot (y \odot x)) \} > .5$. Suppose $\mu(x) < t \leq \min \{ \mu(x \odot y), \mu(y \odot (y \odot x)) \}$ for all $t > .5$. Hence $(x \odot y) \in \mu, (y \odot (y \odot x)) \in \mu \Rightarrow x \in \mu, (y \odot (y \odot x)) \in \mu$. (i.e.,) $\mu(x) t > \mu(y \odot (y \odot x))$.

(ii) $\Rightarrow$ (i): To prove $(f, A)$ is a soft ideal where $A = (.5, 1]$, it is enough to prove that $\mu$ is a fuzzy ideal of $\mathcal{K}$. If not there exists $t \in A$ such that $\mu(x) < t \leq \min \{ \mu(x \odot y), \mu(y \odot (y \odot x)) \}$.

Now $\max(\mu(x), .5) \geq \min \{ \mu(x \odot y), \mu(y \odot (y \odot x)) \} \geq t$ where $t \in (.5, 1] \Rightarrow \mu(x) \geq t$ a contradiction. Thus $(f, A)$ is a soft ideal of $\mathcal{K}$.

**Definition:** 4.6 Let $(f, A)$ be a fuzzy soft set over $\mathcal{K}$. Then $(f, A)$ is said to be an $(\epsilon, \epsilon \lor q)$-fuzzy soft ideal of $\mathcal{K}$ if $f_\xi$ is an $(\epsilon, \epsilon \lor q)$-fuzzy ideal of $\mathcal{K}$ for all $\xi \in A$.

**Lemma:** 4.7 A fuzzy soft set $\mu$ in a K-algebra is an $(\epsilon, \epsilon \lor q)$-fuzzy soft ideal of $\mathcal{K}$ if and only if

i. $\mu(\epsilon) \geq \min \{ \mu(x), .5 \}$

ii. $\mu(x) \geq \min \{ \mu(x \odot y), \mu(y \odot (y \odot x)), .5 \}$ for all $x, y \in G$.

**Proposition:** 4.8 Let $\mu$ be a fuzzy set in a K-algebra $K$. Let $(f, A)$ be an $e\epsilon$- soft set on $\mathcal{K}$ with $A = (0.5, 1]$. Then the following assertions are equivalent:

(i) $\mu$ is an $(\epsilon, \epsilon \lor q)$-fuzzy soft ideal of $\mathcal{K}$

(ii) $(f, A)$ is a soft ideal of $\mathcal{K}$.

**Proof:**

(i) $\Rightarrow$ (ii) Let $t \in A, x \odot y \in f(t) \& y \odot (y \odot x) \in f(t)$. Therefore $\mu(x \odot y) \geq t \& \mu(y \odot (y \odot x)) \geq t$ where $t \in (0, .5)$. Now $x \in \mu \Rightarrow (x \odot y) \geq t$ or $\mu(x) + t > 1 \Rightarrow \mu(x) \geq \min \{ \mu(x \odot y), \mu(y \odot (y \odot x)), .5 \} \geq \min \{t, .5\} = t \Rightarrow x \in \mu$. Similarly $e \in f(t)$. Hence $(f, A)$ is a soft ideal of $\mathcal{K}$ for all $t \in A$.

(ii) $\Rightarrow$ (i): If there exists $x \odot y \in G \& y \odot (y \odot x) \in G$ such that $\mu(x) < \min \{ \mu(x \odot y), \mu(y \odot (y \odot x)), .5 \}$, Take $t \in (0, .5)$ such that $\mu(x) < t \leq \min \{ \mu(x \odot y), \mu(y \odot (y \odot x)), .5 \}$. Thus $\mu(x) \leq \mu(y \odot (y \odot x)) \in \mu \Rightarrow x \in \mu \leq \mu(\mu(x \odot y), \mu(y \odot (y \odot x)), .5) \}$ for all $x, y \in G$. Likewise $\mu(\epsilon) \geq \min \{ \mu(x), .5 \}$ for all $x \in G$. Hence $\mu$ is an $(\epsilon, \epsilon \lor q)$-fuzzy soft ideal of $\mathcal{K}$.

**REFERENCES**


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