International Journal of Mathematical Archive-4(12), 2013, 230-234
Available online through www.ijma.info ISSN 2229 – 5046

Fuzzy Soft ideals of K-algebras

***C. Parameswari and *N. Meenakumari**

***Department Of Mathematics, A. P. C. Mahalaxmi College for Women, Thoothukudi-628 002, India.**

(Received on: 24-11-13; Revised & Accepted on: 29-12-13)

ABSTRACT

In this paper, we introduce the concept of fuzzy soft ideals of K-algebras and investigate some of their properties. We discuss about fuzzy soft inverse images of fuzzy soft ideals. Also we introduce the notion of (ε, ε ∨q)-fuzzy soft ideals of $K - algebra$ *and obtain it's characterization.*

Keywords: *K-algebras, fuzzy soft ideals of K-algebras, (ε, ε ∨q) - fuzzy soft ideals.*

1. INTRODUCTION

The notion of a K-algebra (G, ·,⊙, e) was first introduced by Dar & Akram[3] in 2003. A K-algebra is an algebra built on a group (G, ., e) by adjoining an induced binary operation ⊙ on G which is attached to an abstract K-algebra (G, ·, ⊙, e). This system is in general non commutative and non associative with a right identity e if (G,·, e) is non commutative. Fuzzy sets and soft sets are two different computing models for representing uncertainty & vagueness. In this paper we apply these models in combination to study uncertainty & vagueness in fuzzy soft ideals of K-algebras.

2. PRELIMINARIES

In this section, we recall some basic concepts that are necessary for subsequent discussion.

Definition: 2.1 Let (G, ·, e) be a group in which each non identity element is not of order 2. Then a K -algebra is a structure $\mathcal{K}$= (G, ·, ⊙, e) on a group G in which induced binary operation ⊙: G×G → G is defined by ⊙ (x, y) = x y^{-1} and satisfies the following axioms

(K1) (x ⊙ y) ⊙ (x ⊙ z) = (x ⊙ (e ⊙z) ⊙ (e ⊙ y)) ⊙ x
(K2) x ⊙ (x ⊙ y) = (x ⊙ (e ⊙ y)) ⊙ x
(K3) x ⊙ x = e
(K4) x ⊙ e = x
(K5) e ⊙ x = x^{-1} for all x, y, z ∈ G

Definition: 2.2 Let X be a non-empty set. A fuzzy subset of X is defined as a mapping from X into [0, 1].

Definition: 2.3 A fuzzy set μ in a set X of the form $\mu(y) = \begin{cases} t \in (0,1] & \text{if } y = x \\ 0 & \text{if } y \neq x \end{cases}$ is said to be a fuzzy point with support x & value t and is denoted by x_t. For a fuzzy point x_t and a fuzzy set μ in a set X the symbol x_t α μ where α∈{ε, q, ε ∨ q, ε ∧ q}. A fuzzy point x_t is called belong to a fuzzy set μ written as x_t ε μ if μ(x) ≥ t. A fuzzy point x_t is called quasi coincident with a fuzzy set μ written as x_t q μ if μ(x) + t > 1. x_t ε ∨ q μ (respectively x_t ε ∧ q μ) means that x_t ε μ or x_t q μ (respectively x_t ε μ and x_t q μ). x_t $\overline{\alpha}$ μ means that x_t α μ does not hold.

Definition: 2.4 A non-empty subset H of a K-algebra $\mathcal{K}$ is called a sub-algebra [6] of the K-algebra $\mathcal{K}$ if a ⊙ b ∈ H for all a, b ∈ H. We note that every sub algebra of a K-algebra $\mathcal{K}$ contains the identity e of the group.

Definition: 2.5 A fuzzy set μ on a K- algebra $\mathcal{K}$ is a fuzzy ideal if μ: G → [0, 1] is such that μ(e) ≥ μ(x) for all x ∈ G and μ(x) ≥ min { μ(x ⊙ y), μ(y ⊙ (y ⊙ x)} for all x, y ∈ G.

Definition: 2.6 A fuzzy set μ in K is called a (ε, ε ν q) – fuzzy ideal of $\mathcal{K}$ if it satisfies the following conditions:
x_t ε μ ⇒ e_t ε ∨q μ for all x ∈ G and for all t ∈ (0, 1], $(x ⊙ y)_t$ ε μ, $(y ⊙ (y ⊙ x))_s$ ε μ ⇒ $x_{\min(s,t)}$ ε ν q μ

Corresponding author: *N. Meenakumari
****Department Of Mathematics, A. P. C. Mahalaxmi College for Women, Thoothukudi-628 002, India***

Molodtsov [5] defined the notion of soft set in the following way: Let U be an initial Universe and E be the set of parameters. Let $P(U) = I^U$ denote the power set of U and let A be a non-empty subset of E. A pair (f, A) is called a soft set over U where f is a mapping given by f: $A \to P(U)$. In other words a soft set over U is parameterized family of subsets of U. For $\xi \in A$, $f(\xi)$ may be considered as the set of ξ– approximate elements of the soft set (f, A). Let f: $A \to I^U$, I = [0, 1] then (f, A) is called a fuzzy soft set over U. In general for all $\xi \in A$, $f(\xi) = f_\xi$ is a fuzzy set of U and it is called fuzzy value set of parameter x.

t- LEVEL SOFT SETS OF FUZZY SOFT SET

Definition: 2.7 Let (f, A) be a fuzzy soft set over U. For each $t \in [0, 1]$ the set $(f, A)^t = (f^t, A)$ is called a t-level soft set of (f, A) where $f_\xi{}^t = \{x \in U / f_\xi(x) \geq t$ for all $\xi \in A\}$. Clearly $(f, A)^t$ is a soft set over U.

Definition: 2.8 Let (f, A) and (g, B) be two fuzzy soft sets over U. We say that (f, A) is a fuzzy soft subset of (g, B) and write $(f, A) \subset (g, B)$ if $A \subseteq B$ and for all $\xi \in A$, $f(\xi) \subseteq g(\xi)$.

Definition: 2.9 Let (f, A) and (g, B) be two fuzzy soft sets over U. Then their extended intersection is a fuzzy soft set denoted by (h, C) where $C = A \cup B$ and

$$h(\xi) = \begin{cases} f_\xi \ if \ \xi \in A - B \\ g_\xi \ if \ \xi \in B - A \\ f_\xi \cap g_\xi \ if \ \xi \in A \cap B \end{cases} \quad \text{for all } \xi \in C.$$

Definition: 2.10 Extended Union of two fuzzy soft sets (f, A) and (g, B) is denoted by (h, C) where $C = A \cup B$ and

$$h(\xi) = \begin{cases} f_\xi \ if \ \xi \in A - B \\ g_\xi \ if \ \xi \in B - A \\ f_\xi \cup g_\xi \ if \ \xi \in A \cap B \end{cases} \quad \text{for all } \xi \in C$$

Definition: 2.11 Let $\varphi : X \to Y$ and $\psi: A \to B$ be two functions, A and B are parametric sets from the crisp sets X and Y. Then the pair (φ, ψ) is called a fuzzy soft function from X to Y.

Definition: 2.12 Let (f, A) and (g, B) be two fuzzy soft sets over G_1 and G_2 respectively. Let (φ, ψ) be a fuzzy soft function from $G_1 \to G_2$. Then

(i) The image of (f, A) under the fuzzy soft function (φ, ψ) denoted by $(\varphi, \psi)(f, A)$ is the fuzzy soft set on K_2 defined by $(\varphi, \psi)(f, A) = (\varphi(f), \psi(A))$ where for all $k \in \psi(A)$, $y \in G_2$

$$\varphi(f)_k(y) = \begin{cases} \bigvee_{\varphi(x)=y} \bigvee_{\psi(a)=k} f_a(x) \ if \ x \in \psi^{-1}(y) \\ 0 \qquad otherwise \end{cases}$$

(ii) The preimage of (g, B) under the fuzzy soft function (φ, ψ) denoted by $(\varphi, \psi)^{-1}(g, B)$ is the fuzzy soft set over K_1 defined by $(\varphi, \psi)^{-1}(g, B) = (\varphi^{-1}(g), \psi^{-1}(B))$ where $\varphi^{-1}(g)_a(x) = g_{\psi(a)}(\varphi(x))$ for all $a \in \psi^{-1}(B)$, $x \in G_1$.

Definition: 2.13 Let (φ, ψ) be a fuzzy soft function from $\mathcal{K}_1 \to \mathcal{K}_2$. If φ is a homomorphism from $\mathcal{K}_1$ to $\mathcal{K}_2$ then (φ, ψ) is said to be a fuzzy soft homomorphism, if φ is a isomorphism from $\mathcal{K}_1$ to $\mathcal{K}_2$ and ψ is one-one mapping from A onto B then (φ, ψ) is said to be a fuzzy soft isomorphism.

3. FUZZY SOFT IDEALS OF K-ALGEBRA

In this section we introduce fuzzy soft ideals of K-algebras and study about their properties.

Definition: 3.1 Let (f, A) be a soft set over a K-algebra $\mathcal{K}$. Then (f, A) is called a soft ideal over $\mathcal{K}$ if f(t) is an ideal of K for all $t \in A$. (i.e.,)

i. $e \in f(t)$
ii. $x \odot y \in f(t)$ & $(y \odot (y \odot x)) \in f(t) \Rightarrow x \in f(t)$

Example: 3.2 Consider the K-algebra $\mathcal{K} = (S_3, \cdot, \odot, e)$ on the symmetric group $S_3 = \{e, a, b, x, y, z\}$ where e = (1), a = (1 2 3), b = (1 3 2), x = (1 2), y = (1 3) and z = (2 3) & $\odot$ is given by the Cayley table:

$\odot$	e	x	y	z	a	b
e	e	x	y	z	b	a
x	x	e	a	b	z	y
y	y	b	e	a	x	z
z	z	a	b	e	y	x
a	a	z	x	y	e	b
b	b	y	z	x	a	e

Let (f, A) be a soft set over $\mathcal{K}$ where A = $\mathcal{K}$ and f: A → P($\mathcal{K}$) is a set valued function.

Then f(e) = {e}, f(a) = f(b) = {e, a, b}, f(x) = {e, x}, f(y) = {e, y}, f(z) = {e, z} are ideals of K. Then (f, A) is a soft ideal of $\mathcal{K}$.

Definition: 3.3 Let (f, A) be a soft set over a K-algebra $\mathcal{K}$. Therefore (f, A) is said to be a fuzzy soft ideal over $\mathcal{K}$ if f_ξ is a fuzzy ideal of $\mathcal{K}$ for all ξ ∈ A. (i.e.,)

i. $f_\xi(e) \geq f_\xi(x)$ for all x ∈ G
ii. $f_\xi(x) \geq \min\{f_\xi(x \odot y), f_\xi(y \odot (y \odot x))\}$ for all x, y ∈ G.

Example: 3.4 Consider the K-algebra $\mathcal{K} = (G, \cdot, \odot, e)$ where G = {e, a, a^2, a^3} is the cyclic group of order 4 and ⊙ is given by the following Cayley table:

⊙	e	a	a^2	a^3
e	e	a^3	a^2	a
a	a	e	a^3	a^2
a^2	a^2	a	e	a^3
a^3	a^3	a^2	a	e

Let A = {e_1, e_2, e_3} & f: A → P(G) be a set valued function defined by

$f(e_1)$ = {(e, .7), (a, .3), (a^2, .6), (a^3, .3)};

$f(e_2)$ = {(e, .6), (a, .2), (a^2, .5), (a^3, .2)} and $f(e_3)$ = {(e, .7), (a, .1), (a^2, .3), (a^3, .1)}

Let B = {e_2, e_3} & g : B → P(G) be a set valued function defined by

$g(e_2)$ = {(e, .5), (a, .2), (a^2, .4), (a^3, .2)} and $g(e_3)$ = {(e, .6), (a, .1), (a^2, .3), (a^3, .1)}.

Clearly (f, A) and (g, B) be two fuzzy soft sets over $\mathcal{K}$. By routine calculation we see that they are fuzzy soft ideals of $\mathcal{K}$.

Definition: 3.5 Let (f, A) and (g, B) be two fuzzy soft ideals of $\mathcal{K}$. Then (f, A) is a fuzzy soft subideal of (g, B) if (i) A ⊂ B (ii) f(x) is a fuzzy subideal of g(x) for all x ∈ A. (i.e.,) if f(x) ≤ g(x) for all x ∈ A. From Example 3.4, we see that (g, B) is a fuzzy soft subideal of (f, A).

Proposition: 3.6 Let (f, A) and (g, B) be two fuzzy soft ideals of $\mathcal{K}$. Then (f, A) ∧ (g, B) is a fuzzy soft ideal over K.

Proposition: 3.7 Let (f, A) and (g, B) be two fuzzy soft ideals of $\mathcal{K}$. If $A \cap B = \Phi$ then (f, A) $\tilde{\cup}$ (g, B) is a fuzzy soft ideal over $\mathcal{K}$.

Theorem: 3.8 Let (f, A) be a soft set over a K-algebra $\mathcal{K}$. Then (f, A) is a fuzzy soft ideal of $\mathcal{K}$ if and only if $(f, A)^t$ is a soft ideal of K for all t ∈ [0, 1]

Proof: Suppose that (f, A) is a fuzzy soft ideal of $\mathcal{K}$. Then f_ξ is a fuzzy ideal of $\mathcal{K}$ for every ξ ∈ A. Then $f_\xi(e) \geq f_\xi(x)$ for all x ∈ G. Therefore $f_\xi(e) \geq t$. Hence e ∈ $(f, A)^t$. Now let x ⊙ y ∈ $(f, A)^t$ & (y ⊙ (y ⊙ x)) ∈ $(f, A)^t$. This implies $f_\xi(x \odot y) \geq t$ & $f_\xi(y \odot (y \odot x)) \geq t$. Since (f, A) is a fuzzy soft ideal $f_\xi(x) \geq \min\{f_\xi(x \odot y), f_\xi(y \odot (y \odot x))\} \geq t$ ⇒ x ∈ $(f, A)^t$.

Conversely let t = $\min\{f_\xi(x \odot y), f_\xi(y \odot (y \odot x))\}$. Then x ⊙ y ∈ $(f, A)^t$ & (y ⊙ (y ⊙ x)) ∈ $(f, A)^t$. Since $(f, A)^t$ is a soft ideal of $\mathcal{K}$ this implies that x ∈ $(f, A)^t$ which implies $f_\xi(x) \geq t$. Thus $f_\xi(x) \geq t = \min\{f_\xi(x \odot y), f_\xi(y \odot (y \odot x))\}$.

Since $(f, A)^t$ is a soft ideal of $\mathcal{K}$ for all t ∈ [0, 1], e ∈ $(f, A)^t$ for all t ∈ [0, 1] ⇒ $f_\xi(e) \geq t$ for all t ∈ [0, 1]

⇒ $f_\xi(e) \geq f_\xi(x)$ for all x ∈ G. Thus (f, A) is a fuzzy soft ideal of $\mathcal{K}$.

Theorem: 3.9 Let (g, B) be a fuzzy soft ideal of $\mathcal{K}_2$. Let (φ, ψ) be a fuzzy soft homomorphism from $\mathcal{K}_1$ to $\mathcal{K}_2$. Then $(\varphi, \psi)^{-1}$ (g, B) is a fuzzy soft ideal of $\mathcal{K}_1$.

Proof: Since (g, B) is a fuzzy soft ideal of $\mathcal{K}_2$,
(i) $g_{\psi(\xi)}(e_2) \geq g_{\psi(\xi)}(y)$ for all y ∈ G_2

$g_{\psi(\xi)}\, \varphi\, (e_1) \geq g_{\psi(\xi)}\, \varphi\, (x)$ where $x \in G_1$

$\varphi^{-1}\, g_{\psi(\xi)}(e_1) \geq \varphi^{-1}\, g_{\psi(\xi)}(x)$ for all $x \in G_1$

ii) $g_{\psi(\xi)}(y) \geq \min \{ g_{\psi(\xi)}(y \odot z),\ g_{\psi(\xi)}((z \odot (z \odot y))\}$

$= \min \{ g_{\psi(\xi)}(\varphi(x) \odot \varphi(x')),\ g_{\psi(\xi)}(\varphi(x') \odot (\varphi(x') \odot \varphi(x))\}$

where $y = \varphi(x)$,
$z = \varphi(x') = \min \{ g_{\psi(\xi)}\, \varphi(x \odot x'),$
$g_{\psi(\xi)}\, \varphi(x' \odot (x' \odot x))\} = \min \{ \varphi^{-1} g_{\psi(\xi)}\, (x \odot x'),\ \varphi^{-1} g_{\psi(\xi)}\, (x' \odot (x' \odot x))\}$

Then $\varphi^{-1} g_{\psi(\xi)}\, (x) \geq \min \{ \varphi^{-1} g_{\psi(\xi)}\, (x \odot x'),\ \varphi^{-1} g_{\psi(\xi)}\, (x' \odot (x' \odot x))\}$ which implies $(\varphi,\psi)^{-1}$ (g, B) is a fuzzy soft ideal of $\mathcal{K}_1$.

4. (ε, ε ∨ q)-FUZZY SOFT IDEALS OF THE K-ALGEBRA $\mathcal{K}$

Definition: 4.1 Given a fuzzy set μ in $\mathcal{K}$ and $A \subseteq [0, 1]$, we define two set valued functions $f: A \to P(\mathcal{K})$ and $fq: A \to P(\mathcal{K})$ by $f(t) = \{x \in G / x_t \in \mu\}$ and $f_q(t) = \{x \in G / x_t\ q\ \mu\}$ for all $t \in A$. Then (f, A) is called ε- soft set and (f_q, A) is called q-soft set over $\mathcal{K}$.

Example: 4.2 Consider the K-algebra $\mathcal{K} = \{G, \cdot\, , \odot, e\}$ where $G = \{e, a, a^2, a^3, a^4\}$ is the cyclic group of order 5 & ⊙ is given by the following Cayley table:

$\odot$	e	a	a^2	a^3	a^4
e	e	a^4	a^3	a^2	a
a	a	e	a^4	a^3	a^2
a^2	a^2	a	e	a^4	a^3
a^3	a^3	a^2	a	e	a^4
a^4	a^4	a^3	a^2	a	e

Let μ be a fuzzy set in G defined by $\mu(e) = .7$, $\mu(a) = .8$, $\mu(a^2) = .8$, $\mu(a^3) = \mu(a^4) = .4$. Then μ is an (ε, ε ν q) – fuzzy ideal of $\mathcal{K}$.

Proposition: 4.3 Let μ be a fuzzy set in a K-algebra $\mathcal{K}$. Let (f, A) be an ε- soft set on $\mathcal{K}$ with A = (0, 1]. Then (f, A) is a soft ideal of $\mathcal{K}$ if and only if μ is a fuzzy ideal of $\mathcal{K}$.

Proof: Assume that (f, A) is a soft ideal of $\mathcal{K}$. Then f(t) is an ideal of $\mathcal{K}$ for all $t \in A$. If μ is not a fuzzy ideal of $\mathcal{K}$ then there exists $x, y \in G$ such that $\mu(x) < \min \{\mu(x \odot y), \mu(y \odot (y \odot x))\}$. Take $t \in A$ such that $\mu(x) < t \leq \min \{\mu(x \odot y), \mu(y \odot (y \odot x))\} \Rightarrow \mu(x \odot y) \geq t$ & $\mu(y \odot (y \odot x)) \geq t \Rightarrow x \odot y \in f(t)$ & $(y \odot (y \odot x)) \in f(t)$ implies $x \in f(t)$ (i.e.,) $\mu(x) \geq t$ which is a contradiction.

Conversely suppose μ is a fuzzy ideal of $\mathcal{K}$. Take $t \in A$, $x \odot y \in f(t)$ and $(y \odot (y \odot x)) \in f(t)$ implies $\mu(x \odot y) \geq t$ & $\mu(y \odot (y \odot x)) \geq t$. Since $\mu(x) \geq \min \{\mu(x \odot y), \mu(y \odot (y \odot x))\} \geq t \Rightarrow x \in f(t)$. Now $\mu(e) \geq \mu(x)$ for all $x \in G$, $\mu(e) \geq t \Rightarrow e \in f(t)$. Hence f(t) is an ideal for all $t \in (0, 1]$. Thus (f, A) is a soft ideal of $\mathcal{K}$.

Proposition: 4.4 Let μ be a fuzzy set in a K-algebra $\mathcal{K}$ and let (f_q, A) be a q-soft set over $\mathcal{K}$ with A = (0, 1]. Then (f_q, A) is a soft ideal of $\mathcal{K}$ if and only if μ is a fuzzy ideal of $\mathcal{K}$.

Proof: Suppose that μ is a fuzzy ideal of $\mathcal{K}$. Let $t \in A$ & $x, y \in G$ be such that $\mu(x \odot y) + t > 1$ & $\mu(y \odot (y \odot x)) + t > 1$. We have

$\mu(x) \geq \min \{\mu(x \odot y), \mu(y \odot (y \odot x))\}$

$\Rightarrow \mu(x) + t \geq \min \{\mu(x \odot y), \mu(y \odot (y \odot x))\} + t$
$= \min \{\mu(x \odot y) + t, \mu(y \odot (y \odot x)) + t\}$
> 1

implies $x \in f_q(t)$.

Since $\mu(e) \geq \mu(x)$ for all $x \in G$, in particular $\mu(e) \geq \mu(x)$ for all $x \in f_q(t)$. Therefore $\mu(e) \geq \mu(x) > 1 - t \Rightarrow \mu(e) + t > 1 \Rightarrow e \in fq(t)$.

Conversely if μ is not a fuzzy ideal of $\mathcal{K}$ then there exists t Є A such that $\mu(x) < 1 - t \leq \min\{\mu(x \odot y), \mu(y \odot (y \odot x))\} \Rightarrow \mu(x \odot y) \geq 1 - t$ & $\mu(y \odot (y \odot x)) \geq 1 - t \Rightarrow \mu(x \odot y) + t > 1$ & $\mu(y \odot (y \odot x)) + t > 1 \Rightarrow x \in f_q(t)$ (i.e.,) $\mu(x) + t > 1 \Rightarrow \mu(x) > 1 - t$ a contradiction.

Theorem: 4.5 Let μ be a fuzzy set in a K-algebra $\mathcal{K}$. Let (f, A) be an ε- soft set on K with A = (0.5, 1]. Then the following assertions are equivalent:

i. (f, A) is a soft ideal of $\mathcal{K}$
ii. $\max(\mu(x), .5) \geq \min\{\mu(x \odot y), \mu(y \odot (y \odot x))\}$

Proof:
(i) ⇒ (ii): Let (f, A) be a soft ideal of $\mathcal{K}$. If one of $\mu(x \odot y)$ or $\mu(y \odot (y \odot x))$ or both ≤ .5, then $\min\{\mu(x \odot y), \mu(y \odot (y \odot x))\} \leq .5$.

If $\mu(x) \leq .5$ then $\max\{\mu(x),.5\} \geq \min\{\mu(x \odot y), \mu(y \odot (y \odot x))\}$.

If $\mu(x) > .5$ then $\max\{\mu(x),.5\} = \mu(x)$ then $\max\{\mu(x),.5\} \geq \min\{\mu(x \odot y), \mu(y \odot (y \odot x))\}$.

If $\mu(x \odot y) > .5$ & $\mu(y \odot (y \odot x)) > .5$ then $\min\{\mu(x \odot y), \mu(y \odot (y \odot x))\} > .5$.

Suppose $\mu(x) < t \leq \min\{\mu(x \odot y), \mu(y \odot (y \odot x))\}$ for all t > .5. Hence $(x \odot y)_t \in \mu$, $(y \odot (y \odot x))_t \in \mu \Rightarrow x_t \in \mu$. (i.e.,) $\mu(x) \geq t$ which is a contradiction. Thus $\max(\mu(x), .5) \geq \min\{\mu(x \odot y), \mu(y \odot (y \odot x))\}$.

(ii) ⇒ (i): To prove (f, A) is a soft ideal where A = (.5, 1], it is enough to prove that μ is a fuzzy ideal of $\mathcal{K}$. If not there exists t Є A such that $\mu(x) < t \leq \min\{\mu(x \odot y), \mu(y \odot (y \odot x))\}$.

Now $\max(\mu(x), .5) \geq \min\{\mu(x \odot y), \mu(y \odot (y \odot x))\} \geq t$ where t Є (.5, 1] ⇒ $\mu(x) \geq t$ a contradiction. Thus (f, A) is a soft ideal of $\mathcal{K}$.

Definition: 4.6 Let (f, A) be a fuzzy soft set over $\mathcal{K}$. Then (f, A) is said to be an (ε, ε ⋁ q)-fuzzy soft ideal of $\mathcal{K}$ if f_ξ is an (ε, ε ⋁ q)-fuzzy ideal of $\mathcal{K}$ for all ξ Є A.

Lemma: 4.7 A fuzzy soft set μ in a K-algebra is an (ε, ε ⋁ q)-fuzzy soft ideal of $\mathcal{K}$ if and only if

i. $\mu(e) \geq \min\{\mu(x), .5\}$
ii. $\mu(x) \geq \min\{\mu(x \odot y), \mu(y \odot (y \odot x)), .5\}$ for all x, y Є G.

Proposition: 4.8 Let μ be a fuzzy set in a K-algebra K. Let (f, A) be an ε- soft set on $\mathcal{K}$ with A = (0.5, 1]. Then the following assertions are equivalent:

(i) μ is an (ε, ε ⋁ q)-fuzzy soft ideal of $\mathcal{K}$
(ii) (f, A) is a soft ideal of $\mathcal{K}$.

Proof:
(i) ⇒ (ii) Let t Є A, $x \odot y$ Є f(t) & $y \odot (y \odot x)$ Є f(t). Therefore $\mu(x \odot y) \geq t$ & $\mu(y \odot (y \odot x)) \geq t$ where t Є (0, .5). Now $x_t \in \vee q\, \mu \Rightarrow \mu(x) \geq t$ or $\mu(x) + t > 1 \Rightarrow \mu(x) \geq \min\{\mu(x \odot y), \mu(y \odot (y \odot x)), .5\} \geq \min\{t, .5\} = t \Rightarrow x_t \in \mu$. Similarly e Є f(t). Hence (f, A) is a soft ideal of $\mathcal{K}$ for all t Є A.

(ii) ⇒ (i): If there exists $x \odot y$ Є G & $y \odot (y \odot x)$ Є G such that $\mu(x) < \min\{\mu(x \odot y), \mu(y \odot (y \odot x)), .5\}$, Take t Є (0, .5) such that $\mu(x) < t \leq \min\{\mu(x \odot y), \mu(y \odot (y \odot x)), .5\}$. Thus t≤ .5 & $(x \odot y)_t \in \mu$, $(y \odot (y \odot x))_t \in \mu \Rightarrow x_t \in \mu$(since f(t) is an ideal for all t ≤ .5). (i.e.,) $\mu(x) \geq t$ which is a contradiction. Thus $\mu(x) \geq \min\{\mu(x \odot y), \mu(y \odot (y \odot x)), .5\}$for all x, y Є G. Likewise $\mu(e) \geq \min\{\mu(x), .5\}$ for all x Є G. Hence μ is an (ε, ε ⋁ q)-fuzzy soft ideal of $\mathcal{K}$.

REFERENCES

1. M.Akram and K.H.Dar., Generalized fuzzy K-algebras, VDM verlag, 2010.
2. M.Akram and K.H.Dar., Fuzzy ideals of K- algebras, Annals of University of Craiova, Math.comp.Sci.Ser 34(2007) 3 – 12.
3. K.H.Dar and M.Akram., On a K-algebra built on a group, Southeast Asian Bulletin Of Mathematics 29(1) (2005) 41-49.
4. P.K.Maji, R.Biswas and R.Roy., Sift set theory, Comput. Math. Appl. 45 (2003) 555-562.
5. D.Molodsov., Soft set theory first results, Comput.Math. Appl. 37 (1999) 19-31.
6. Muhammed Akram., Fuzzy soft K-algebras, Utilitas Mathematica 90 (2013), pp. 307-325.
7. L.A.Zadeh., Fuzzy sets, Inform. And control 8 (1965) 338-353.

Source of support: Nil, Conflict of interest: None Declared