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ON SEMI COMPACT SPACES

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ABSTRACT

In a recent paper [1] some properties of semi compact spaces have been presented. The present paper continuous further study of the properties of semi compact spaces. For example, semi continuous image of a semi compact space is compact.

Subject classification: *54A10, 54D30.*

Key words: *Semi open, Semi continuous, semi compact spaces.*

1. INTRODUCTION

Some important results on the topic are contained in [2] and [3]. Let (X, τ) be a topological space . Let A be a subset of (X, τ). Then A is said to be semi open if $A \subset cl\,(int\,(A))$. A is semi closed if $A \subset int\,(cl\,(A))$.

Note that every open set is semi open. IF every open cover of X has a finite sub cover then X is called a compact space. If every semi open cover has a finite sub cover, then X is a semi compact space. (X, τ) is said to have semi Hausdorff space if $x \neq y$ in X implies existence of semi open neighbourhoods U and V of x and y such that $U \cap V = \phi$.

A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is called a semi continuous function if $f^{-1}(G)$ is a semi open set in X for each open set G in Y. (X, τ) is called an s - normal space if given two disjoint closed sets A and B in X, there exist disjoint semi open neighbourhoods U and V of A and B respectively.

2. MAIN RESULTS

Theorem: 2.1 Every semi closed subsets of a semi compact space is semi compact.

Proof: Suppose that A is a semi closed subset of a semi compact space (X, τ).

We shall show that (A, $\tau/_A$) is semi compact.

Let $\mathcal{C}_A = \{G_\alpha \cap A : \alpha \in \Lambda)$ be any relatively semi open cover of (A, $\tau/_A$).

Then $\mathcal{C} = \{G_\alpha : \alpha \in \Lambda)$ is a semi open cover of (X, τ).

But (X, τ) is semi compact.

Hence $\mathcal{C}$ contains a finite sub over $\{G_{\alpha_1}, G_{\alpha_2}, \dots, G_{\alpha_n}\}$ of (X, τ).

Consequently, $\{G_{\alpha_1} \cap A, \dots, \dots, \dots, G_{\alpha_n} \cap A\}$ is a finite sub cover of A.

Hence (A, $\tau/_A$) is semi compact.

This completes the proof.

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Theorem: 2 Semi continuous image of a semi compact space is compact.

Proof: Suppose that (X,τ) is semi compact.

Let f: (X, τ) $\rightarrow$ (Y, σ) be semi continuous surjection.

We shall show that (Y, σ) is compact.

Let $\mathcal{C}$ = {G_α :α∈Λ} be any open cover of (Y, σ).

Then f $^{-1}$ (G_α) is an semi open set in (X, τ) and $\mathcal{D}$ = {f $^{-1}$ (G_α) : α∈Λ} a semi open cover of X.

But (X, τ) is semi compact.

Accordingly, $\mathcal{D}$ contains a finite sub cover $\{f^{-1}(G_{\alpha_1}), \ldots , f^{-1}(G_{\alpha_n})\}$.

But then $\{G_{\alpha_1}, G_{\alpha_2}, \ldots , G_{\alpha_n}\}$ is a finite open subcover of Y.

Hence (Y, σ) is a compact space.

This finishes the proof.

Theorem: 3 Suppose that A is a semi compact subset of a semi Hausdorff space X. Let x ∈ X – A. Then there exist disjoint semi open neighborhoods U and v of A and x respectively.

Proof: By hypothesis, X is semi Hausdorff.

Let a ∈ A arbitrarily.

Then there exist disjoint semi open neighborhoods U_a and V_x of a and x respectively.

The collection $\mathcal{C}$ = {U_a : a∈A} is a semi open cover of A.

But A is semi compact.

Accordingly, this collection $\mathcal{C}$ has a finite sub cover $\{U_{a_1}, U_{a_2}, \ldots , U_{a_n}\}$.

Let U = $U_{a_1} \cup \ldots \cup U_{a_n}$.

Put V = $V_{a_1} \cap \ldots \cap V_{a_n}$.

Then A ⊂ U and x ∈ V.

Also U and V are semi open.

Since $U_{a_i} \cap V_{a_i} = \phi \, for \; 1 \le i \le n.$

We obtain that U ∩ V = ϕ.

We have proved the result.

Theorem: 4 Every semi compact subset of a semi Hausdorff space is semi closed.

Proof: Let X be a semi Hausdorff space.

Suppose that A is a semi compact subset of X.

Let x ∈ X – A.

By theorem 3, there exist disjoint semi open sets U_x and V_x containing x and A respectively.

Therefore, x ∈ U_x⊂ X – V_x⊂ X – A.

It follows that X – A is semi open.

Consequently, A is semi closed.

This proves the theorem.

Theorem: 5 Every semi compact, semi Hausdorff space is s - normal.

Proof: Suppose that X is a semi compact, semi Hausdorff space.

Let A and B be two disjoint closed subsets of X.

But then A is semi compact.

It follows , by theorem 3, that for each $x \in B$ there exist disjoint semi open sets U_x and V_x such that $x \in U_x$ and $B \subset V_x$.

The collection $\mathcal{C} = \{U_x : x \in B\}$ is a semi open cover of B.

But B is semi compact.

Hence the collection $\mathcal{C}$ has a sub cover $\{U_{x_1}, U_{x_2}, \ldots, U_{x_n}\}$.

Let $U = U_{x_1} \cup \ldots \cup U_{x_n}$.

Put $V = V_{x_1} \cap \ldots \cap V_{x_n}$.

Then U and V are semi open with $A \subset U$ and $B \subset V$.

Since $U_{x_i} \cap V_{x_i} = \phi \, for \; 1 \leq i \leq n$.

It follows that $U \cap V = \phi$.

Hence theorem holds.

Theorem: 6 Let X and Y be non empty topological spaces. The product space X×Y is semi compact if both X and Y are semi compact.

Proof: Suppose that X and Y are semi compact.

Let $\mathcal{C}$ be a semi open cover of X × Y, consisting of basic semi open sets of the form U × V, where U is a semi open set in X and V is a semi open set in Y.

Let $x \in X$.

Then for each $y \in Y$ there exists a set $(U_y \times V_y)$ in $\mathcal{C}$ containing (x, y).

The collection $\{V_y : y \in Y\}$ is a semi open cover of Y.

But Y is semi compact.

Consequently, this collection has a finite sub cover $\{V_{y_1}, V_{y_2}, \ldots, V_{y_n}\}$.

Consider, the corresponding sets $U_{y_1}, U_{y_2}, \ldots, U_{y_n}$.

Put $U_x = U_{y_1} \cap \ldots \cap U_{y_n}$.

$$
\begin{aligned}
\text{Then } \{x\} \times Y &= U_x \times Y \\
&= U_x \times \{ V_{y_1} \cup \ldots \cup V_{y_n} \} \\
&= (U_{y_1} \times V_{y_1}) \cup \ldots \ldots \ldots \cup (U_{y_n} \times V_{y_n}).
\end{aligned}
$$

Thus for each x in X there is a set U_x such that $\{x\} \times Y \subset U_x \times Y$ and that $U_x \times Y$ is contained in a finite number of sets in $\mathcal{C}$.

But the collection $\{U_x : x \in X\}$ covers X.

Since X is semi compact, this collection has a finite sub cover $\{U_{x_1}, U_{x_2}, \dots, U_{x_m}\}$.

Then $X \times Y = (U_{x_1} \cup \dots \cup U_{x_m}) \times Y$
$= (U_{x_1} \times Y) \cup \dots \dots \dots \cup (U_{x_m} \times Y)$

But $(U_{x_i} \times Y) \subset$ union of a finite number of sets in $\mathcal{C}$ for each i with $1 \le i \le m$.

It follows that $X \times Y$ = union of a finite number of sets in $\mathcal{C}$.

Hence $\mathcal{C}$ has a finite sub cover.

Therefore, $X \times Y$ is semi compact

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