SPHERICAL SYMMETRIC COSMOLOGICAL MODEL WITH COSMIC STRINGS COUPLED WITH PERFECT FLUID IN BIMETRIC RELATIVITY

1V. Mahurpawar* and 2A. K. Ronghe

1Asstt. Professor of Mathematics Govt. P. G. College, Chhindawara (India)
2Professor of Mathematics S. S. L. C. Jain P. G. College, Vidisha (India)

*E-mail: vm_pawar12@rediffmail.com

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ABSTRACT

Spherical symmetric Kantowski–Sachs space-time is studied in Rosen’s bimetric relativity, considering the source of gravitation as cosmic strings coupled with perfect fluid distribution. It is shown that a macro cosmological model represented by cosmic string coupled with perfect distribution does not exist and only a vacuum model can be constructed.

Keywords: Spherical symmetric, Kantowski-Sachs model, cosmic string.

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INTRODUCTION:

The spherical symmetry has its own importance in general relativity theory by virtue of its comparative simplicity. Many noteworthy space-time, such as the Schwarzschild solutions (exterior and interior), the Robertson-Walker model of expanding universe etc. are all spherically symmetric. Rosen N. (6, 7, 8, 9, 10) introduced a new theory known as bimetric theory of relativity. It is based on two metric tensors $g_{ij}$ and $\gamma_{ij}$. The first metric tensor described the curved space-time and thereby the gravitational field. The second metric tensor refers to the flat space-time and described the inertial force associated with acceleration of the frame of reference. Israelit (1, 2) studied several aspects of bimetric theory of gravitation. Recently Mohanty et-al. (4, 5) constructed some physical viable models in this theory.

In this paper we have shown the spherical symmetric cosmological model with cosmic string does not exist. Mahurpawa and Ronghe (3) have shown that cosmic strings do not exist in this model in the context of bimetric relativity.

1. FIELD EQUATIONS:

Accordingly, at each space-time point one has two line elements

$$ds^2 = g_{ij} dx^i dx^j$$

and

$$d\sigma^2 = \gamma_{ij} dx^i dx^j$$

(1.1)

(1.2)

This theory is based on a simple form of Lagrangian and has a simpler mathematical structure than that of the general relativity.

The field equations of the bimetric theory of gravitation formulated by Rosen N. are

*Corresponding author: 1V. Mahurpawar*, *E-mail: vm_pawar12@rediffmail.com

\[ K_{ij} = N_{ij} - \frac{1}{2} N_{g_{ij}} = -8\pi\kappa T_{ij} \]  

(1.3)

where

\[ N_{ij} = \frac{1}{2} \gamma^{\alpha\beta} (g_{hi} g_{bj} | \alpha) \beta \]  

(1.4)

\[ \kappa = \left( \frac{g}{\gamma} \right)^{1/2} \]  with \( g \)-the determinant of \( g_{ij} \) and \( \gamma \)-determinant of \( \gamma_{ij} \).

The vertical bar ( | ) stands for \( \gamma \)-differentiation and \( T_{ij} \) is the energy-momentum tensor. We considered here the spherically symmetric Kantowski-Sachs space-time in the form

\[ ds^2 = dt^2 - \lambda dr^2 - k^2 (d\theta^2 - \sin^2 \theta d\phi^2) \]  

(1.5)

where \( \lambda \) and \( k \) are functions of “t” only

the background metric corresponding to the metric (1.5) is

\[ d\sigma^2 = dr^2 - d\theta^2 - \sin^2 \theta d\phi^2 \]  

(1.6)

In this case we have taken the source of gravitation cosmic strings coupled with perfect fluid distribution. The energy-momentum tensor for cosmic string coupled with perfect fluid distribution is given

\[ T_{ij} = T_{ij}^{\text{string}} + (\epsilon + p)v_{j}v_{i} + pg_{ij} \]  

(1.7)

where

\[ T_{ij}^{\text{string}} = \rho v_{j}v_{i} - \lambda x_{i}x_{j} \]  

(1.8)

Here \( \rho \) is energy density for a cloud of cosmic strings with particle attached to them, \( \lambda \) the string tensor density, \( v_{i} \) are four velocity vector of cosmic string distribution, \( x^{i} \) is an anisotropic direction or say direction of strings, and \( p \) are \( \epsilon \) proper pressure and matter density. The particle density associated with configuration is given by

\[ \rho = \rho_{p} + \lambda \]  

(1.9)

where \( \rho_{p} \) is the particle density in the string cloud

\[ -x_{i}x^{i} = v_{j}v^{i} = 1, v_{i}x^{i} = 0 \text{ if } i \neq j \]

We considered the anisotropic direction along x direction

\[ x_{i}x^{i} = -1, v_{j}v_{i} = 1 \]

So \( T_{1} = p + \lambda, T_{2} = p = T_{3} = T_{4} = \epsilon + 2p + \rho \)  

(1.10)

Using equations (1.1) to (1.10) the field equations are

\[ \left( \frac{\lambda_{i}}{\lambda} \right)_{4} - 2\left( \frac{k_{i}}{k} \right)_{4} = 16\pi\kappa(p + \lambda) \]  

(1.11)

\[ \left( \frac{\lambda_{i}}{\lambda} \right)_{4} = 16\pi\kappa(\rho) \]  

(1.12)
\[
\left( \frac{\lambda_t}{\lambda} \right)_4 + 2 \left( \frac{k_t}{k} \right)_4 = 16\pi\kappa (\varepsilon + 2p + \rho) \tag{1.13}
\]

Here suffix “4” following an unknown function denotes an ordinary differentiation with respect to time “t”. Equation (1.11) and (1.13) with help of (1.12) gives

\[
p + \varepsilon + \lambda + \rho = 0 \tag{1.14}
\]

Since \( p \geq 0, \lambda \geq 0, \rho \geq 0 \)

So \( p = \lambda = \varepsilon = \rho = 0 \) \tag{1.15}

Using equation (1.14) in the field equations (1.11) to (1.13), we have

\[
\left( \frac{\lambda_t}{\lambda} \right)_4 - 2 \left( \frac{k_t}{k} \right)_4 = 0 \tag{1.16}
\]

\[
\left( \frac{\lambda_t}{\lambda} \right)_4 = 0 \tag{1.17}
\]

\[
\left( \frac{\lambda_t}{\lambda} \right)_4 + 2 \left( \frac{k_t}{k} \right)_4 = 0 \tag{1.18}
\]

Equations (1.17) and (1.18), gives

\[
\left( \frac{\lambda_t}{\lambda} \right)_4 = \left( \frac{k_t}{k} \right)_4 = 0 \tag{1.19}
\]

We have

\( \lambda = e^{\tau_t} \) \tag{1.20}

And \( k = e^{2\tau_t} \) \tag{1.21}

Using equations (1.20) and (1.21) the line element (1.5) becomes

\[
ds^2 = dt^2 - e^{2\tau_t} dr^2 - e^{2\tau_t} \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \tag{1.22}
\]

By proper choice of coordinates this metric can be transform

\[
ds^2 = d\tau^2 - e^{2\tau} \left( d\rho^2 + d\theta^2 + \sin^2 \theta d\phi^2 \right) \tag{1.23}
\]

Which is free from singularity at \( \tau = 0 \)

2. CONCLUSION:

We have studied spherically symmetric Kantowaski-Sachs cosmological model with cosmic string coupled with perfect fluid as energy-momentum tensor and observed that the cosmic string coupled with perfect fluid does not accommodate in this model only a vacuum model can be constructed. There is no singularity at \( \tau = 0 \).

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