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CAYLEY THEOREM FOR CENTRE OF C-ALGEBRA

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ABSTRACT

In this we study the important properties of ternary operation Γ on C-Algebra, should be viewed as a conditional "if x, then p, else q" and established Cayley's theorem on centre of C-Algebra.

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INTRODUCTION

In [1], Guzman and Squier introduced the concept of C-algebras as the variety generated by the three element algebra C= {T, F, U} which the algebraic form of the three valued conditional logic. They proved that C and the two element Boolean algebra B = {T, F} are the only sub-directly irreducible C-algebras and that the variety of C-algbras is minimal cover of the variety of Boolean algebras. They also define ternary operation $\Gamma_a(p,q) = (a \wedge p) \vee (a' \wedge q)$ on C-Algebra, should be viewed as a conditional "if x, then p, else q" and derived important properties of Γ. In [2], Swany, Rao and Ravi kumar introduced the concept of the centre $B(A)$ of C-algebra A with T as the set of all elements x in A with $x \vee x' = T$ and proved that the centre is a Boolean algebra under the induced operations, which is isomorphic to the Boolean centre of the algebra A.

1. C-ALGEBRA

In this section we recall the definition of C- algebra and some results from [1], [2] and [3]. Let us start with the definition of a C-Algebra.

Definition 1.1: [1] By a C-algebra we mean an algebra of type (2, 2, 1) with binary operations $\wedge, \vee$ and unary operation $'$ satisfying the following identities.

$a)\ x'' = x$ $\qquad b)\ (x \wedge y)' = x' \vee y'$

$c)\ (x \wedge y) \wedge z = x \wedge (y \wedge z)$ $\qquad d)\ x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$

$d)\ (x \vee y) \wedge z = (x \wedge z) \vee (x' \wedge y \wedge z)$ $\qquad f)\ x \vee (x \wedge y) = x$

$g)\ (x \wedge y) \vee (y \wedge x) = (y \wedge x) \vee (x \wedge y)$

Example 1.2: The three element algebra C = $\{T, F, U\}$ with the operations given by the following tables is a C-algebra

$\wedge$	T	F	U
T	T	F	U
F	F	F	F
U	U	U	U

$\vee$	T	F	U
T	T	T	T
F	T	F	U
U	U	U	U

x	$x^{\sim}$
T	F
F	T
U	U

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Note 1.3: [1] The identities 1.1(a), 1.1(b) imply that the variety of C-algebras satisfies all the dual statements of 1.1(c) to 1.1(f). $\wedge$ and $\vee$ are not commutative in C. The ordinary right distributive law of $\wedge$ over $\vee$ fails in C. Every Boolean algebra is a C-algebra.

Lemma 1.4: The following properties of a C-algebra can be verified directly [1], [2], [3], and [4]
(i) $x \wedge x = x$
(ii) $x \wedge y = x \wedge (x' \vee y) = (x' \vee y) \wedge x$
(iii) $x \vee (x' \wedge x) = (x' \wedge x) \vee x = x$
(iv) $(x \vee x') \wedge y = (x \wedge y) \vee (x' \wedge y)$
(v) $x \vee x' = x' \vee x$
(vi) $x \vee y \vee x = x \vee y$
(vii) $x \wedge x' \wedge y = x \wedge x'$
(viii) $x \vee y = F$ if and only if $x = y = F$
(ix) If $x \vee y = T$ then $x \vee x' = T$
(x) $x \vee T = x \vee x'$
(xi) $T \vee x = T$ and $F \wedge x = F$

Definition 1.5: [2] Let A be a C-algebra with T (T is the identity element for $\wedge$ in A). Then the Boolean centre of A is defined as the set $B(A) = \{a \in A / a \vee a' = T\}$. $B(A)$ is known to be a Boolean algebra under the operations induced by those on A.

2. THE TERNARY OPERATION Γ

Definition 2.1: [1] Let A be a C-algebra. If $x, p, q \in A$, define $\Gamma_x(p,q) = (x \wedge p) \vee (x' \wedge q)$ ($\Gamma_x(p,q)$ should be viewed as a conditional " if x, then p ,else q ").

Lemma 2.2: [1] Every C-algebra satisfies the laws:
(a) $\Gamma_x(p,q)' = \Gamma_x(p',q')$
(b)) $\Gamma_x(p,q) \wedge r = \Gamma_x(p \wedge r , q \wedge r)$
(c) $\Gamma_x(p,q) \vee \mathrm{r} = \Gamma_x(p \vee \mathrm{r} , q \vee \mathrm{r})$
(d) $\Gamma_x(\Gamma_y(p,q) ,\Gamma_y(r,s)) = \Gamma_y(\Gamma_x(p,r) ,\Gamma_x(q,s))$

Lemma 2.3: [1] Every C-algebra with the indicated constants satisfies the following laws
(i) $\Gamma_U(p,q) = \mathrm{U}$ (ii) $\Gamma_x(U,U) = U$ (iii) $\Gamma_T(p,q) = p$ (iv) $\Gamma_F(p,q) = q$ (v) $\Gamma_x(T,F) = x$

Lemma 2.4: Every C-algebra with the indicated constants satisfies the following laws.
(a) $\Gamma_{x\vee y}(p,q) = \Gamma_x(p , \Gamma_{\mathrm{y}}(p,q))$ (b) $\Gamma_{x\wedge y}(p,q) = \Gamma_{\mathrm{x}}(\Gamma_{\mathrm{y}}((p,q),q))$ (c) $\Gamma_p(p,p) = p$

Proof:
(a) $\Gamma_x(p , \Gamma_{\mathrm{y}}(p,q))$
$$\begin{aligned}
&= (x \wedge p) \vee (x' \wedge \Gamma_y(p,q)) \\
&= (x \wedge p) \vee [x' \wedge \{(y \wedge p) \vee (y' \wedge q)\}] \\
&= (x \wedge p) \vee [\{x' \wedge (y \wedge p)\} \vee \{x' \wedge (y' \wedge q)\}] \\
&= [(x \wedge p) \vee \{x' \wedge (y \wedge p)\}] \vee \{x' \wedge (y' \wedge q)\}] \\
&= [(x \wedge p) \vee \{x' \wedge y \wedge p)\}] \vee \{x' \wedge (y' \wedge q)\}] \\
&= \{(x \vee y) \wedge p\} \vee \{(x \vee y)' \wedge q\} \\
&= \Gamma_{x\vee y}(p,q)
\end{aligned}$$

Therefore $\Gamma_{x\vee y}(p,q) = \Gamma_x(p , \Gamma_{\mathrm{y}}(p,q))$

(vi) $\Gamma_x(\Gamma_y((p,q), q))$
$$\begin{aligned}
&= (x \wedge \Gamma_y(p,q)) \vee (x' \wedge q) \\
&= [x \wedge \{(y \wedge p) \vee (y' \wedge q)\} \vee (x' \wedge q)]
\end{aligned}$$

$$= [(x \wedge (y \wedge p)) \vee (x \wedge (y' \wedge q))] \vee (x' \wedge q)$$
$$= (x \wedge (y \wedge p)) \vee [(x \wedge (y' \wedge q)) \vee (x' \wedge q)]$$
$$= (x \wedge (y \wedge p)) \vee [(x \wedge y') \wedge q)) \vee (x' \wedge q)]$$
$$= (x \wedge (y \wedge p)) \vee [\{(x \wedge y^{\sim}) \vee x^{\sim}\} \wedge q)]$$
$$= \{(x \wedge y) \wedge p\} \vee \{(x^{\sim} \vee y^{\sim}) \wedge q\}$$
$$= \{(x \wedge y) \wedge p\} \vee \{(x \wedge y)^{\sim} \wedge q\}$$
$$= \Gamma_{x \wedge y}(p, q)$$

Therefore $\Gamma_{x \wedge y}(p,q) = \Gamma_{x}(\Gamma_{y}((p,q),q))$

(vii) $\Gamma_p(p,p) = (p \wedge p) \vee (p' \wedge p) = p \vee (p' \wedge p) = p$

Therefore $\Gamma_p(p,p) = p$.

Observe that $\Gamma_a(p,p) = (a \wedge p) \vee (a' \wedge p) = (a \vee a') \wedge p$, if $a \neq U$, then $a \vee a' = T$ and $\Gamma_a(p,p) = p$ otherwise the value is U. Hence we have the following lemma.

Lemma 2.5: Let A be a C-algebra, $a \in B(A)$, then $\Gamma_a(p,p) = p$, $\forall p \in A$

Proof: If $a \in B(A)$, then $\Gamma_a(p,p) = (a \wedge p) \vee (a' \wedge p)$

$$= (a \vee a') \wedge p \quad (\text{since } a \in B(A) \Rightarrow a \vee a' = T)$$
$$= T \wedge p = p$$

Lemma 2.6: Let A be a C-algebra, then $\Gamma_a(\Gamma_a(p,q), \Gamma_a(r,s)) = \Gamma_a(p,s)$

Proof: The result is clear, when $a = U$.

Suppose $a \neq U$ i.e., $a \vee a' = T$

$$\Gamma_a(\Gamma_a(p,q), \Gamma_a(r,s)) = \Gamma_a((a \wedge p) \vee (a' \wedge q), (a \wedge r) \vee (a' \wedge s))$$
$$= [a \wedge \{(a \wedge p) \vee (a' \wedge q)\}] \vee [a' \wedge \{(a \wedge r) \vee (a' \wedge s)\}]$$
$$= \{a \wedge (a \wedge p)\} \vee \{a \wedge (a' \wedge q)\} \vee \{a' \wedge (a \wedge r)\} \vee \{a' \wedge (a' \wedge s)\}$$
$$= \{(a \wedge p)\} \vee \{a \wedge (a' \wedge q)\} \vee \{a' \wedge (a \wedge r)\} \vee \{a' \wedge s)\}$$
$$= \{(a \wedge p)\} \vee \{a \wedge a' \wedge q)\} \vee \{a' \wedge a \wedge r)\} \vee \{a' \wedge s)\}$$
$$= \{(a \wedge p)\} \vee \{a \wedge a'\} \vee \{a' \wedge a)\} \vee \{a' \wedge s)\}$$
$$= \{(a \wedge p)\} \vee 0 \vee 0 \vee \{a' \wedge s)\}$$
$$= (a \wedge p) \vee (a' \wedge s)$$
$$= \Gamma_a(p,s)$$

Therefore $\Gamma_a(\Gamma_a(p,q), \Gamma_a(r,s)) = \Gamma_a(p,s)$

The property on ternary operation $\Gamma_a(\Gamma_a(p,q), \Gamma_a(r,s)) = \Gamma_a(p,s)$ is known as diagonal property hold on C-algebra.

3. CAYLEY'S THEOREM:

The well known Cayley theorem for groups may be summarized as follows: Any group is isomorphic to group of transformations on some set. Stephen L. Bloom, Zoltan Esik and E.G. Manes [5], established Cayley theorem for Boolean algebras. We establish Cayley's theorem for C-algebra in two parts. In Part-I, we construct a C-algebra $(A, \wedge, \vee, ')$, where A is a subset of Ter(X) (ternary functions) on X, called Guard C-algebra on X. In Part-II, we prove that every centre of a C-algebra is isomorphic with guard C-algebra.

Cayley theorem for Groups (part I): Let X be a set and $P(X)$ be the collection of permutations of X. If G is a subset of $P(X)$ satisfying

i) G contains identity function I.

ii) $f, g \in G \Rightarrow fog \in G$.

iii) $f \in G \Rightarrow f^{-1} \in$ G, then G is a group. Such groups are called transformation groups on X.

Cayley theorem for Groups (part II): If G is a group, there is a set X such that G is isomorphic to a transformation group on X.

Theorem 3.1: Cayley theorem for Boolean algebras (part I) 3.3: [5]
Let B be a subset of Bin(X) with the following properties. B contains True and is closed under the operations $\wedge, \vee, (-)'$. Then $(B, \wedge, \vee, (-)'$,True, False) is a Boolean algebra providing each function in B is idempotent and diagonal and any two such functions commute. Then B is a Boolean algebra. Such Boolean algebras on X are called "Guard Algebras" on X.

Theorem 3.2: Cayley theorem for Boolean algebras (part II): [5]
If $(B, \wedge, \vee, (-)'$, 1, 0) is any Boolean algebra, then there is a set X such that B is isomorphic to a Guard algebra on X.

In a similar way, we prove Cayley theorem for C-algebra.

Let X be a set and f be a ternary function on X, satisfying the following properties

(i) $f(x, x) = x$

(ii) $f(f(x, y), f(u, v)) = f(x, v)$ (Diagonal property)

(iii) $f(g(x, y), g(u, v)) = g(f(x, u), f(y, v))$, g is a ternary function on X.

We define the operations $\wedge, \vee, '$ on the set Ter(X) of all ternary functions on X as

$(f \wedge g)(x, y) = f(g(x, y), y)$.

$(f \vee g)(x, y) = f(x, g(x, y))$.

$f'(x,y) = f(y, x)$ and Ter(X) having projections

$\pi_1(x, y) = x$, $\pi_2(x, y) = y$

Theorem 3.3: Cayley's theorem for C-algebras (Part-I):
Let A be a subset of Ter(X) with the properties, A contains projections π_1, π_2 and closed under $\wedge, \vee, '$ then $(A, \wedge, \vee, ')$ is a C-algebra provided each function in A satisfying the properties (i),(ii) and (iii) and operations $\wedge, \vee, '$ are defined above.

Proof: Let $f, g \in A$ and $x, y \in X$

(a) $f(x, y)'' = f'(y, x) = f(x, y)$

Therefore $f'' = f$

(b) $(f \wedge g)'(x,y) = (f \wedge g)(y,x)$

$= f(g(y, x), x)$

$= f'(x, g(y,x))$

$= f'(x, g'(x,y))$

$= (f' \vee g')(x,y)$

Therefore $(f \wedge g)' = g' \vee f'$

(c) $$\begin{aligned}\{f\wedge(g\wedge h)\}(x,y) &= f\{(g\wedge h))(x,y),y)\\ &= f\{g(h(x,y),y),y\}\\ &=(f\wedge g)(h(x,y),y)\\ &=\{(f\wedge g)\wedge h\}(x,y)\end{aligned}$$

Therefore $f\wedge(g\wedge h)=(f\wedge g)\wedge h$

(d) To prove $(f\wedge g)\vee(f\wedge h) = f\wedge(g\vee h)$

Consider

$$\begin{aligned}[(f\wedge g)\vee(f\wedge h)](x,y) &=(f\wedge g)[x,\ (f\wedge h)(x,y)]\\ &=(f\wedge g)[x,\ f(h(x,y),y)]\\ &= f[g\{x,\ f(h(x,y),y)\},\ f(h(x,y),y)]\\ &= f[g\{x,\ f(h(x,y),y)\}, g(f(h(x,y),y),\ f(h(x,y),y)]\\ &=g[f\{x,\ f(h(x,y),y)\},\ f(f(h(x,y),y),\ f(h(x,y),y)]\\ &=g[f\{x,\ f(h(x,y),y)\},\ f(h(x,y),y)]\\ &= f[g\{x,\ h(x,y)\}, g(y,y)]\\ &= f[g\{x,\ h(x,y)\}, y]\\ &= f[(g\vee h)(x,y),y]\\ &=\{f\wedge(g\vee h)\}(x,y)\end{aligned}$$

Therefore $(f\wedge g)\vee(f\wedge h) = f\wedge(g\vee h)$

(e) $(f\vee g)\wedge h=(f\wedge h)\vee(f'\wedge g\wedge h)$

Consider

$$\begin{aligned}[(f\wedge h)\vee(f'\wedge g\wedge h)](x,\mathrm{y}) &=(f\wedge h)\{x,\ (f'\wedge g\wedge h)(x,y)\}\\ &=(f\wedge h)[x,\ f'\{(g\wedge h)(x,y),\mathrm{y}\}]\\ &=(f\wedge h)[x,\ f\{\mathrm{y},(g\wedge h)(x,y)\}]\\ &=(f\wedge h)[x,\ f\{y,g(h(x,y),y\}]\\ &= f[h\{x,\ f(y,g(h(x,y),y),\ f(y,g(h(x,y),y)]\\ &= f[h\{f(x,x),\ f(y,g(h(x,y),y),\ f(y,g(h(x,y),y)]\\ &= f[f\{h(x,\mathrm{y}),h(x,g(h(x,y),y),\ f(y,g(h(x,y),y)]\\ &= f[h(x,\mathrm{y}),g(h(x,y),y)]\\ &=(f\vee g)(h(x,y),y)\\ &=((f\vee g)\wedge h)(x,y)\end{aligned}$$

Hence $(f\vee g)\wedge h=(f\wedge h)\vee(f'\wedge g\wedge h)$

(f) $(f\wedge g)\vee(g\wedge f)=(\mathrm{g}\wedge f)\vee(f\wedge g)$

$$\begin{aligned}(f\wedge g)\vee(g\wedge f)(x,y) &=(f\wedge g)[x,(g\wedge f)(x,y)]\\ &=(f\wedge g)(x,g[f(x,y),y])\\ &= f\{g(x,g(f(x,y),y),g(f(x,y),y)\}\\ &= f\{g[g(x,x),g(f(x,y),y)],g(f(x,y),y)\}\\ &= f\{g(x,y),g(f(x,y),y)\}\\ &= f\{g(x,y),g(f(x,y),\ f(y,y)\}\\ &= f\{g(x,y),\ f(g(x,y),y)\end{aligned}$$

$$\begin{aligned}(g\wedge f)\vee(f\wedge g)(x,y) &= (g\wedge f)\{x,(f\wedge g)(x,y)\}\\ &= (g\wedge f)\{x,(f[g(x,y),y]\}\\ &= g\{f[x,f(g(x,y),y),f(g(x,y),y)\}\\ &= f\{g[x,g(x,y)],g[f(g(x,y),y),y]\}\\ &= f\{g[g(x,x),g(x,y)],g[f(g(x,y),y),f(y,y)]\}\\ &= f\{g[x,g(x,y)f[g(g(x,y),y)g(y,y)]\}\\ &= f\{g(x,g(x,y)g[f(g(x,y),y),f(y,y)]\}\\ &= f\{g(x,y),f[g(g(x,y),y),g(y,y)]\}\\ &= f(g(x,y),f(g(x,y),y)\end{aligned}$$

Therefore $(f\wedge g)\vee(g\wedge f)=(g\wedge f)\vee(f\wedge g)$

Hence $(A,\wedge,\vee,\,')$ is a C-algebra, such C-algebras on A are called "Guard C-algebras" on X. Observed that on C-Algebra, $\Gamma_a(x,x)\neq x$ but if $a\in B(A)$, then $\Gamma_a(x,x)=x$ (lemma 2.5). So we are confining to Cayley's theorem for centre of C-algebra.

Theorem3 .4: Cayley's theorem for C-algebras (Part-II)
If $(A,\wedge,\vee,\tilde{}\,0,1,2)$ is a C-algebra, there is a set X such that $B(A)$ is isomorphic to guard C-algebra on X.

Proof: Define Γ_a on $B(A)$ such that $\Gamma_a(x,y)=(a\wedge x)\vee(a'\wedge y)$.

Let Ter($B(A)$)= $\{\Gamma_a / a\in B(A)\}$, define $\wedge,\vee,\,'$ as

$$(\Gamma_a\wedge\Gamma_b)(x,y)=(\Gamma_{a\wedge b})(x,y)=\Gamma_a(\Gamma_b(x,y),y)$$
$$(\Gamma_a\vee\Gamma_b)(x,y)=(\Gamma_{a\vee b})(x,y)=\Gamma_a(x,\Gamma_b(x,y))$$
$$(\Gamma_a)'(x,y)=\Gamma_{a'}(x,y)=\Gamma_a(y,x)$$

Now we prove that Ter($B(A)$) is C-algebra.

(a) $(\Gamma_a)''(x,y)=(\Gamma_{a'}(x,y))'=(\Gamma_a(y,x))'=\Gamma_{a'}(y,x)=\Gamma_a(x,y)$

Therefore $(\Gamma_a)''=\Gamma_a$

Therefore $\Gamma_a\wedge\Gamma_a=\Gamma_a$

(b) $$\begin{aligned}(\Gamma_a\wedge\Gamma_b)'(x,y) &= (\Gamma_a\wedge\Gamma_b)(y,x)\\ &= \Gamma_a(\Gamma_b(y,x),x)\\ &= \Gamma_{a'}((x,\Gamma_b(y,x))\\ &= \Gamma_{a'}((x,\Gamma_{b'}(x,y))\\ &= (\Gamma_{a'}\vee\Gamma_{b'})(x,y)\end{aligned}$$

Therefore $(\Gamma_a\wedge\Gamma_b)'=(\Gamma_{a'}\vee\Gamma_{b'})$

(c) $$\begin{aligned}\{(\Gamma_a\wedge\Gamma_b)\wedge\Gamma_c\}(x,y) &= (\Gamma_a\wedge\Gamma_b)\{\Gamma_c(x,y),y\}\\ &= \Gamma_a[\Gamma_b\{\Gamma_c(x,y),y),y\}]\\ &= \Gamma_a\{(\Gamma_b\wedge\Gamma_c)(x,y),y\}\\ &= \{\Gamma_a\wedge(\Gamma_b\wedge\Gamma_c)\}(x,y)\end{aligned}$$

Therefore $(\Gamma_a\wedge\Gamma_b)\wedge\Gamma_c=\Gamma_a\wedge(\Gamma_b\wedge\Gamma_c)$

(d) To prove $\{\Gamma_a \wedge (\Gamma_b \vee \Gamma_c)\} = \{(\Gamma_a \wedge \Gamma_b) \vee (\Gamma_a \wedge \Gamma_c)\}$

Consider

$$
\begin{aligned}
&\{(\Gamma_a \wedge \Gamma_b) \vee (\Gamma_a \wedge \Gamma_c)\}(x,y) \\
&= (\Gamma_a \wedge \Gamma_b)\{x, (\Gamma_a \wedge \Gamma_c)(x,y)\} \\
&= (\Gamma_a \wedge \Gamma_b)[x, \Gamma_a\{\Gamma_c(x,y), y\}] \\
&= \Gamma_a[\Gamma_b[x,\ \{\Gamma_a(\Gamma_c(x,y),y)\}],\ \Gamma_a\{\Gamma_c(x,y),y\}] \\
&= [a \wedge \{\Gamma_b[x,\ \{\Gamma_a(\Gamma_c(x,y),y)\}\}] \vee\ [a' \wedge \{\Gamma_a\{\Gamma_c(x,y),y\}\}] \\
&= (a \wedge [\Gamma_b[x, a \wedge \{\Gamma_c(x,y)\} \vee (a' \wedge y)]]) \vee\ [a' \wedge \{(a \wedge \Gamma_c(x,y)) \vee (a' \wedge y)\}] \\
&= (a \wedge [(b \wedge x) \vee b' \wedge \{a \wedge \Gamma_c(x,y) \vee (a' \wedge y)\}]) \vee [a' \wedge a \wedge \Gamma_c(x,y) \vee (a' \wedge a' \wedge y)\}] \\
&= (a \wedge [(b \wedge x) \vee (b' \wedge a \wedge \Gamma_c(x,y)) \vee (b' \wedge (a' \wedge y))]) \vee\ [a \wedge a' \vee (a' \wedge y)\}] \\
&= (a \wedge [(b \wedge x) \vee (b' \wedge a \wedge \Gamma_c(x,y)) \vee (b' \wedge a' \wedge y)]) \vee (a' \wedge y) \\
&= (a \wedge (b \wedge x) \vee (b' \wedge a \wedge \Gamma_c(x,y)) \vee F]) \vee (a' \wedge y) \\
&= a \wedge (b \wedge x) \vee (b' \wedge a \wedge \Gamma_c(x,y)) \vee (a' \wedge y) \\
&= a \wedge \{(b \wedge x) \vee (b^{\sim} \wedge \Gamma_c(x,y)\} \vee (a' \wedge y) \\
&= a \wedge \{(\Gamma_b(x, \Gamma_c(x,y)\} \vee (a' \wedge y) \\
&= \Gamma_a\{(\Gamma_b(x, \Gamma_c(x,y)\}, y\} \\
&= \Gamma_a[(\Gamma_b \vee \Gamma_c)(x,y), y] \\
&= \{\Gamma_a \wedge (\Gamma_b \vee \Gamma_c)\}(x,y)
\end{aligned}
$$

Therefore $\{\Gamma_a \wedge (\Gamma_b \vee \Gamma_c)\} = \{(\Gamma_a \wedge \Gamma_b) \vee (\Gamma_a \wedge \Gamma_c)\}$

(e) To prove $(\Gamma_a \vee \Gamma_b) \wedge \Gamma_c) = (\Gamma_a \wedge \Gamma_c) \vee (\Gamma_a' \wedge \Gamma_b \wedge \Gamma_c)$

$$
\begin{aligned}
&(\Gamma_a \wedge \Gamma_c) \vee (\Gamma_{a'} \wedge \Gamma_b \wedge \Gamma_c)\ (x,y) \\
&= \Gamma_a \wedge \Gamma_c)\ \{x, \Gamma_{a'} \wedge \Gamma_b \wedge \Gamma_c)(x,y)\} \\
&= (\Gamma_a \wedge \Gamma_c)\ \{\mathrm{x}, \Gamma_{a'}[(\Gamma_b \wedge \Gamma_c)(x,y), y]\} \\
&= (\Gamma_a \wedge \Gamma_c)\ \{\mathrm{x}, (\Gamma_a[y, (\Gamma_b \wedge \Gamma_c)(x,y)] \\
&= (\Gamma_a \wedge \Gamma_c)\ \{\mathrm{x}, (\Gamma_a[y, \Gamma_b(\Gamma_c(x,y), y)]\} \\
&= (\Gamma_a\{\Gamma_c[\mathrm{x}, \Gamma a[y, \Gamma_b(\Gamma_c(x,y), y)], \Gamma_a[y, \Gamma_b(\Gamma_c(x,y), y)]\} \\
&= (a \wedge \{\Gamma_c[\mathrm{x}, \Gamma_a[y, \Gamma_b(\Gamma_c(x,y), y)]\}) \vee \{a' \wedge \Gamma_a[y, \Gamma_b(\Gamma_c(x,y), y)]\} \\
&= (a \wedge \{(c \wedge x) \vee [c' \wedge (a \wedge y) \vee (a' \wedge (\Gamma_b(\Gamma_c(x,y), y)]\}) \vee (a' \wedge \{(a \wedge y) \vee (a' \wedge [(b \wedge \Gamma_c(x,y)) \vee (b' \wedge y)]\}) \\
&= (a \wedge c \wedge x) \vee (a \wedge c' \wedge a \wedge y) \vee (a \wedge c' \wedge a' \wedge \Gamma_b(\Gamma_c(x,y), y)) \vee (a' \wedge a \wedge y) \vee (a' \wedge a' \wedge (b \wedge \Gamma_c(x,y)) \vee (a' \wedge a' \wedge b' \wedge y) \\
&= (a \wedge c \wedge x) \vee (a \wedge c' \wedge y) \vee (a' \wedge (b \wedge (\Gamma_c(x,y) \vee (a' \wedge b' \wedge y) \\
&= \{a \wedge [(c \wedge x) \vee (c' \wedge y)]\} \vee \{a' \wedge (b \wedge \Gamma_c(x,y)) \vee (b' \wedge y)\} \\
&= [a \wedge (\Gamma_c(x,y)] \vee [a' \wedge (\Gamma_b(\Gamma_c(x,y), y)] \\
&= \Gamma_a(\Gamma_c(x,y),\ \Gamma_b(\Gamma_c(x,y), y) \\
&= (\Gamma_a \vee \Gamma_b)(\Gamma_c(x,y), y) \\
&= \{(\Gamma_a \vee \Gamma_b) \wedge \Gamma_c\}(x,y)
\end{aligned}
$$

Therefore $(\Gamma_a \vee \Gamma_b) \wedge \Gamma_c) = (\Gamma_a \wedge \Gamma_c) \vee (\Gamma_a' \wedge \Gamma_b \wedge \Gamma_c)$

f) To prove $(\Gamma_a \wedge \Gamma_b) \vee (\Gamma_b \wedge \Gamma_a) = (\Gamma_b \wedge \Gamma_a) \vee (\Gamma_a \wedge \Gamma_b)$

Consider

$$\begin{aligned}[\Gamma_a \wedge \Gamma_b) \vee (\Gamma_b \wedge \Gamma_a)](x,y) &= (\Gamma_a \wedge \Gamma_b)[x,(\Gamma_b \wedge \Gamma_a)(x,y)]\\
&= (\Gamma_a \wedge \Gamma_b)[x,\Gamma_b(\Gamma_a(x,y),y]\\
&= \Gamma_a\{\Gamma_b(x,\Gamma_b(\Gamma_a(x,y),y),\Gamma_b(\Gamma_a(x,y),y)\}\\
&= \Gamma_a\{\Gamma_b[\Gamma_b(x,x),\Gamma_b(\Gamma_a(x,y),y)],\Gamma_b(\Gamma_a(x,y),y)\}\\
&= \Gamma_a\{\Gamma_b(x,y),\Gamma_b(\Gamma_a(x,y),y)\}\\
&= a \wedge \{[(b \wedge x) \vee (b' \wedge y)] \vee [a' \wedge ((b \wedge \Gamma_a(x,y) \vee (b' \wedge y))\}\\
&= (a \wedge b \wedge x) \vee (a \wedge b' \wedge y) \vee [a' \wedge b \wedge (a \wedge x)] \vee [a' \wedge b \wedge a' \wedge y] \vee [a' \wedge b' \wedge y]\\
&= (a \wedge b \wedge x) \vee (a \wedge b' \wedge y) \vee (a' \wedge b \wedge y)\end{aligned}$$

Now

$$\begin{aligned}(\Gamma_b \wedge \Gamma_a) \vee (\Gamma_a \wedge \Gamma_b)(x,y) &= (\Gamma_b \wedge \Gamma_a)\{x,(\Gamma_a \wedge \Gamma_b)(x,y)\}\\
&= (\Gamma_b \wedge \Gamma_a)\{x,(\Gamma_a(\Gamma_b(x,y),y)\}\\
&= \Gamma_b\{\Gamma_a[x,\Gamma_a(\Gamma_b(x,y),y],\Gamma_a(\Gamma_b(x,y),y)\}\\
&= \Gamma_a\{\Gamma_b(x,\Gamma_b(x,y)),\Gamma_b(\Gamma_a(\Gamma_b((x,y),y),y)\}\\
&= \Gamma_a\{\Gamma_b[\Gamma_{b'}(x,x),\Gamma_b(x,y)]\Gamma_b[\Gamma_a(\Gamma_b(x,y),y),\Gamma_a(y,y)]\}\\
&= \Gamma_a\{\Gamma_b(x,y),\Gamma_a[\Gamma_b(\Gamma_b((x,y),y),\Gamma_b(y,y)]\\
&= \Gamma_a\{\Gamma_b(x,y),\Gamma_a[\Gamma_b((x,y),y)]\}\\
&= \Gamma_a\{\Gamma_b(x,y),\Gamma_b[\Gamma_a((x,y),y)]\}\\
&= (a \wedge b \wedge x) \vee (a \wedge b' \wedge y) \vee [(a' \wedge b \wedge a \wedge x)] \vee [a' \wedge b \wedge a' \wedge y] \vee (a' \wedge b' \wedge y)\\
&= (a \wedge b \wedge x) \vee (a \wedge b' \wedge y) \vee (a' \wedge b' \wedge y)\end{aligned}$$

Therefore $(\Gamma_a \wedge \Gamma_b) \vee (\Gamma_b \wedge \Gamma_a) = (\Gamma_b \wedge \Gamma_a) \vee (\Gamma_a \wedge \Gamma_b)$

Hence Ter($B(A)$) is a C-algebra , which is called guard C-algebra.

Define $f : B(A) \to Ter(B(A))$ by $f(a) = \Gamma_a$, $\forall a \in B(A)$

Now we prove that f is homomorphism

(i) $f(a') = \Gamma_{a'}(x,y) = (\Gamma_a)'(x,y) = (f(a))'$

Therefore $f(a') = (f(a))'$

(ii) $f(a) \wedge f(b) = (\Gamma_a \wedge \Gamma_b)(x,y) = \Gamma_a(\Gamma_b(x,y),y) = \Gamma_{a \wedge b}(x,y) = f(a \wedge b)$

Therefore $f(a \wedge b) = f(a) \wedge f(b)$

(iii) $f(a) \vee f(b) = (\Gamma_a \vee \Gamma_b)(x,y) = \Gamma_a(y,\Gamma_b(x,y)) = \Gamma_{a \vee b}(x,y) = f(a \vee b)$

Therefore $f(a \vee b) = f(a) \vee f(b)$.

Therefore f is homomorphism.

f is one-one:

Let $f(a) = f(b)$

$\Rightarrow \Gamma_a(x,y) = \Gamma_b(x,y)$, $\forall x, y$

$\Rightarrow \Gamma_a(1,0) = \Gamma_b(1,0)$

$\Rightarrow a = b$

This shows that f is one-one:

Hence $B(A) \cong Ter(B(A))$

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