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APPLICATIONS OF FRACTIONAL CALCULUS INVOLVING SERIES REPRESENTATION OF THE H-FUNCTION AND ALEPH FUNCTION

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ABSTRACT

The object of the present paper is to discuss about some fractional calculus operators involving many families of bilateral expansions with H-function of several complex variable and Aleph function.

The main result of our paper are unified in nature and capable of generate a very large number of new and known result involving simple special function and polynomial of one or more variable as special cases of our main result.

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1. INTRODUCTION

The series representation of the H-function of several complex variable [26] given by Olkha and Chaurasia [15] as given:

(a) $$\delta_{g_i}^{(i)}[d_j^{(i)}+G_i'] \neq \delta_j^{(i)}[d_{g_i}^{(i)}+G_i], \text{ for } j\neq g_i$$
$$j,\ g_i = 1,\ldots,u^{(i)};G_i',\ G_i = 0,1,2,\ldots \tag{1.1}$$

(b) $$z_i \neq 0$$
$$\nabla_i = \sum_{j=1}^{A}\theta_j^{(i)} - \sum_{j=1}^{C}\psi_j^{(i)} + \sum_{j=1}^{B^{(i)}}\varphi_j^{(i)} - \sum_{j=1}^{D^{(i)}}\delta_j^{(i)} < 0 \quad \forall i \in\{1,\ldots,r\} \tag{1.2}$$

then

$$H\begin{bmatrix} z_1 \\ \vdots \\ z_r \end{bmatrix} = H^{0,\lambda:(u',v');\ldots;(u^{(r)},v^{(r)})}_{A,C:[B',D'];\ldots;(B^{(r)},D^{(r)})}[z_1,\ldots,z_r]$$

$$= \sum_{g_i=1}^{u^{(i)}}\sum_{G_i=0}^{\infty} \frac{\prod_{j=1}^{\lambda}\Gamma\left(1-a_j+\sum_{i=1}^{r}\theta_j^{(i)}\eta_{G_i}\right)}{\prod_{j=\lambda+1}^{A}\Gamma\left(a_j-\sum_{i=1}^{r}\theta_j^{(i)}\eta_{G_i}\right)\prod_{j=1}^{C}\Gamma\left(1-c_j+\sum_{i=1}^{r}\psi_j^{(i)}\eta_{G_i}\right)}$$

$$\cdot\frac{\prod_{\substack{j=1\\ j\neq g_i}}^{u^{(i)}}\Gamma(d_j^{(i)}-\delta_j^{(i)}\eta_{G_i})\prod_{j=1}^{v^{(i)}}\Gamma 1-b_j^{(i)}+\varphi_j^{(i)}\eta_{G_i})\prod_{j=1}^{r}(z_i)\eta_{G_i}(-1)^{\sum_{i=1}^{r}(G_i)}}{\prod_{j=u^{(i)}+1}^{D^{(i)}}\Gamma(1-d_j^{(i)}+\delta_j^{(i)}\eta_{G_i})\prod_{j=v^{(i)}+1}^{B^{(i)}}\Gamma(b_j^{(i)}-\varphi_j^{(i)}\eta_{G_i})\prod_{j=1}^{r}(\delta_{g_i}^{(i)}G_i!)} \tag{1.3}$$

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Suppose as usual that the parameter

$$\begin{matrix} a_j,\ j=1,\dots,A;\ b_j^{(i)},\ j=1,\dots,B^{(i)} \\ c_j,\ j=1,\dots,c\ ;\ d_j^{(i)},\ j=1,\dots,D^{(i)} \end{matrix}\ \forall\, i\in\{1,\dots,r\} \tag{1.4}$$

are complex number and the associated coefficients

$$\begin{matrix} \theta_j^{(i)},\ j=1,\dots,A\ ;\ \varphi_j^{(i)},\ j=1,\dots,B^{(i)} \\ \varphi_j^{(i)},\ j=1,\dots,C;d_j^{(i)},\ j=1,\dots,D^{(i)} \end{matrix}\ \forall\, i\in\{1,\dots,r\} \tag{1.5}$$

are positive real number

where

$$\eta_{G_i}=\frac{d_{g_i}^{(i)}+G_i}{\delta_{g_i}^{(i)}},\ i=1,2,\dots,r \tag{1.6}$$

The integers λ,A,C, $u^{(i)}$, $B^{(i)}$ and $D^{(i)}$ are constrained by the inequalities $0\le\lambda\le A$, $c\ge 0$, $L\le u^{(i)}\le D^{(i)}$ and $0\le V^{(i)}\le B^{(i)}$ $\forall\ i\in\{1,\dots,r\}$ and the equality in (1.2) hold true for suitable restricted values of the complex variable $z_1,\dots,z_r$ $\sum_{g_i=1}^{u^{(i)}}$ and $\sum_{G_i=0}^{\infty}$ denote the multiple sums $\sum_{g_i=1}^{m}\dots\sum_{g_r=1}^{u^{(r)}}$ and $\sum_{G_1=0}^{\infty}\dots\sum_{G_r=0}^{\infty}$ respectively $\forall\ i\in\{1,\dots,r\}$.

H.M. Srivastava defined and introduced the generalized polynomials [22, p.185, eq. (7)] in the following manner:

$$S_{n_1,\dots,n_R}^{m_1,\dots,m_R}[z_1,\dots,z_R]=\sum_{\alpha_1=0}^{(n_1/m_1)}\dots\sum_{\alpha_R=0}^{(n_R/m_R)}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\dots\frac{(-n_R)_{m_R\alpha_R}}{\alpha_R!}\,B(n_1,\alpha_1;\dots;n_R\alpha_R]Z_1^{\alpha_1}\dots Z_R^{\alpha_R} \tag{1.7}$$

where $n_i^{"}=0,1,2,\dots\forall i''\in(1,\dots,R)\,;m_i^{"}\ne 0\,\forall i''\in(1,\dots,R)$ m_i is an arbitrary positive integer and the coefficients $B[n_1,\alpha_1;\dots;n_R,\alpha_R]$ are arbitrary constant, real or complex.

The Aleph ($\aleph$)-function, introduced by Südland *et al* [18], however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integrals

$$\aleph[z]=\aleph_{p_i,q_i;\tau;r}^{m,n}[z]=\aleph_{p_i,q_i,z_i;r}^{m,n}\left[z\left|\begin{matrix}(a_j,A_j)_{1,n}[\tau_i(a_{ji},A_{ji})_{n+i,p_i,r}\\(b_j,B_j)_{1,m}.[\tau_i(b_{ji},B_{ji})_{m+1,q_i,r}\end{matrix}\right.\right]$$

$$=\frac{1}{2\pi\omega}\int_L\Omega_{p_i,q_i,\tau_i;r}^{m,n}(s)=z^{-s}\,ds \tag{1.8}$$

for $\forall\, z\ne 0$ where $\omega=\sqrt{-1}$ and

$$\Omega_{p_i,q_i,\tau_i;r}^{m,n}(s)=\frac{\prod_{j=1}^{m}\Gamma(b_j+B_js)\prod_{j=1}^{n}\Gamma(1-a_j-A_js)}{\sum_{i=1}^{r}\tau_i\prod_{j=n+1}^{p_i}\Gamma(a_{ji}+A_{ji}s)\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}-B_{ji}s)} \tag{1.9}$$

The integration $L=L_{i\gamma\infty},\gamma\in R$ extends from $\gamma-i\infty$ to $\gamma+i\infty$ and is such that the poles assumed to be simple of $\Gamma(1-a_j-A_js),\ j=1,\dots,n$ do not coincide with the poles of $\Gamma(b_j+B_js),\ j=1,\dots,m$ the parameter p_i, q_i are non-negative integers satisfying $0\le n\le p_i, 1\le m\le q_i, \tau_i>0$ for $i=1,\dots,r.$ The parameter A_j, B_j, A_{ji}, $B_{ji}>0$ and a_j, b_j, a_{ji}, $b_{ji}\in C$. The empty product in (1.2) is interpreted as unity. The existence conditions for the defining integral (1.8) are given below

$$\varphi_\ell > 0 |\arg (z)| < \frac{\pi}{2}\varphi_\ell, \ell = 1,...,r \tag{1.10}$$

$$\varphi_\ell \geq 0 \,|\arg (z)| < \frac{\pi}{2}\varphi_\ell \text{ and } \operatorname{Re}\{\xi_\ell\} + 1 < 0 \tag{1.11}$$

where

$$\varphi_\ell = \sum_{j=1}^{n} A_j + \sum_{j=1}^{m} B_j - \tau_\ell \left(\sum_{j=n+1}^{p_\ell} A_{j\ell} + \sum_{j=m+1}^{q_\ell} B_{j\ell} \right) \tag{1.12}$$

$$\xi_\ell = \sum_{j=1}^{m} b_j - \sum_{j=1}^{n} a_j + \tau_\ell \left(\sum_{j=m+1}^{q_\ell} b_{j\ell} - \sum_{j=n+1}^{p_\ell} a_{j\ell} \right) + \frac{1}{2}(p_\ell - q_\ell), \ell = 1,..., r \tag{1.13}$$

for detailed account of the Aleph (ℵ)-function see [18] and [19].

The Riemann-Liouville fractional operator D_x^α defined by [20]

$$D_x^\alpha [F(x)] = \begin{cases} \dfrac{1}{\Gamma(-\alpha)} \displaystyle\int_0^x (x-t)^{-\alpha-1} F(x)\, dt, \ \operatorname{Re}(\alpha) < 0, \\ \dfrac{d^m}{dx^m} D_x^{\alpha - m}\{F(x)\}, \ m - 1 \leq \operatorname{Re}(\alpha) < m, \ r \in N, \end{cases} \tag{1.14}$$

where m is a positive integer and the integral exists.

The Riemann-Liouville fractional operator satisfy generalized Leibnitz rule [20] as

$$D_x^\alpha \{f(x)\, T(x)\} = \sum_{n=-\infty}^{\infty} k \binom{\sigma}{\mu + kn} D_x^{\sigma - \mu - kn}\{F(x)\}\, D_x^{\mu + kn}\{T(x)\}, \ (\sigma, \mu \in C, 0 \leq k \leq 1) \tag{1.15}$$

We also mention that some what different types of fractional derivatives are used in the study of PDES related to physical models.

2. MAIN THEOREMS

The expansion valid under the following conditions:

(a) $\operatorname{Re}(\gamma) + \sum_{i'=1}^{r} q_{i'} \left(\frac{d_j^{(i')}}{\delta_j^{(i')}} \right) > -1$

(b) $\operatorname{Re}(\rho) + \sum_{i'=r+1}^{2r} s_i^{'} \left(\frac{d_j^{(i')}}{\delta_j^{(i')}} \right) > -1,$

(c) $\sigma, \mu \in C, \ 0 \leq k \leq 1,$

(d) $\nabla_i = \sum_{j=1}^{A+M} \theta_j^{(i)} - \sum_{j=1}^{C+N} \psi_j^{(i)} + \sum_{j=1}^{B^{(i)}} \varphi_j^{(i)} - \sum_{j=1}^{D^{(i)}} \delta_j^{(i)} \leq 1; \ \forall \ i = 1,...,2r,$

(e) $a_{i'} > 0, \ \beta_{i''} > 0, \ i' = 1,..., s, \ i'' = 1,..., R; \ M_1,..., M_s$ and $m_1,..., m_R$ are arbitrary positive integers and $A(n_1\alpha_1,\ldots,n_r\alpha_R)$ are arbitrary, real or complex.

$$\sum_{n_1,...,n_R=0}^{\infty} \sum_{m_1,...,m_r=0}^{\infty} \sum_{\alpha_1=0}^{(n_1/m_1)} \cdots \sum_{\alpha_R=0}^{(n_R/m_R)} \sum_{n=-\infty}^{\infty} k \binom{\sigma}{\mu + kn} (-1)^{\sum_{i=1}^{r} S_i \sum_{i=1}^{R} \beta_i \alpha_i + \sum_{i=1}^{r} n_i}$$

$$\cdot \frac{\phi_1(m_1)}{m_1!} \cdots \frac{\phi_r(m_r)}{m_r!} \psi(m_1,...,m_r) \frac{\phi_1^{'}(n_1)}{n_1!} \cdots \frac{\phi_R^{'}(n_R)}{n_R!} \psi'(n_1,...,n_R)$$

$$\cdot \frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!} \cdots \frac{(-n_R)_{m_R\alpha_R}}{\alpha_R!} A(n_1, \alpha_1,..., n_R, \alpha_R)(x_1^{s_1} \ldots x_r^{s_r})(x_{r+1}^{n_1} \ldots x_{2r}^{n_R})$$

$$. \ z^{-\sigma+\rho+\nu+\sum_{i=1}^{r} q_i s_i+\sum_{i=1}^{R} B_i \alpha_i+\sum_{i=1}^{R} s_i' n_i'-2}$$

$$. \ \aleph_{p_i+1, q_i+1, \tau_i; r}^{m, n+1}\left[z \left| \begin{matrix} (a_j, A_j)_{1,n}, (1-\rho-\gamma+\sum_{i=1}^{r} q_i s_i-\sum_{i=1}^{R}\beta_i\alpha_i-\sum_{i=1}^{R} s_i' n_i', 1), [\tau_i(a_{ji}, A_{ji})_{n+1}, p_i; r \\ (b_j', \beta_j')_{1,m}, (1-\rho-\nu-\sum_{i=1}^{r} q_i s_i-\sum_{i=1}^{R}\beta_i\alpha_i-\sum_{i=1}^{R} s_i' n_i'+\sigma, 1), [\tau_i(b_{ji}, B_{ji})_{m+1}, q_i, r \end{matrix} \right. \right]$$

$$= \sum_{n_1,\ldots,n_R=0}^{\infty} \sum_{m_1,\ldots,m_r=0}^{\infty} \sum_{\alpha_1=0}^{(n_1/m_1)} \cdots \sum_{\alpha_R=0}^{(n_R/m_R)} \sum_{n=-\infty}^{\infty} k\begin{pmatrix} \sigma \\ \mu+kn \end{pmatrix} (-1)^{\sum_{i=1}^{r} s_i+\sum_{i=1}^{R}\beta_i\alpha_i+\sum_{i=1}^{r} n_i}$$

$$. \ \frac{\phi_1(m_1)}{m_1!} \cdots \frac{\phi_r(m_r)}{m_r!} \psi(m_1,\ldots,m_r) \frac{\phi_1'(n_1)}{n_1!} \cdots \frac{\phi_R'(n_R)}{n_R!} \psi'(n_1,\ldots,n_R)$$

$$. \ \frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!} \cdots \frac{(-n_R)_{m_R\alpha_R}}{\alpha_R!} A(n_1,\alpha_1,\ldots,n_R\alpha_R)(x_1^{s_1}\ldots x_r^{s_r})(x_{r+1}^{n_1}\ldots x_{2r}^{n_R})$$

$$. \ \frac{z^{-\sigma+\rho+\nu+\sum_{i=1}^{r} q_i s_i+\sum_{i=1}^{R} B_i\alpha_i+\sum_{i=1}^{R} s_i' n_i'-2} \ \Gamma\left(\rho+\sum_{i=1}^{R}\beta_i\alpha_i+\sum_{i=1}^{R} s_i' n_i\right)'}{\Gamma\left(\rho+\sum_{i=1}^{R}\beta_i\alpha_i+\sum_{i=1}^{R} s_i' n_i'-\mu+kn\right)}$$

$$. \ \aleph_{p_i+1, q_i+1, \tau_i, r}^{m, n+1}\left[z \left| \begin{matrix} (a_j, A_j)_{1,n}, (1-\nu-\sum_{i=1}^{r} q_i s_i, 1), [\tau_i(a_{ji}; A_{ji})_{n+1}, p_i, r \\ (b_j, B_j)_{1,m}, (1-\nu-\sum_{i=1}^{r} q_i s_i-\sigma-\mu-kn; 1), [\tau_i(b_{ji}, B_{ji})_{m+1, q_i; r}) \end{matrix} \right. \right] \tag{2.1}$$

Proof: We take

$$F(z) = z^{\nu-1} \chi_{p_i, q_i; \tau_i; r}^{m,n}[z] \, H_{A,C:[B',D'+1];\ldots;[B^{(r)},D^{(r)}+1]}^{0,1:(1,\nu');\ldots;(1,\nu^{(r)})}[x_1 z^{q_1},\ldots,x_r z^{q_r}]$$

and

$$T(x) = z^{\rho-1} S_{n_1,\ldots,n_R}^{m_1,\ldots,m_R}[-z^{\beta_1},\ldots,-z^{\beta_R}] \, . \, H_{M,N:(B^{(r+1)},D^{r+1)}+1);\ldots;(B^{(r)},D^{(r)}+1)}^{0,1:(1,\nu^{(r+1)});\ldots;(1,\nu^{(2r)})}[x_{r+1} z^{s_{r+1}},\ldots,x_{2r} z^{s_{2r}}]$$

Using result (1.15), we get

$$\text{LHS } D_z^{\sigma}\left\{ z^{\nu-1} \aleph_{p_i, q_i, \tau_i; r}^{m,n}[z] \, H_{A,C:(B',D'+1),\ldots,(B^{(r)},D^{(r)}+1)}^{0,1:(1,\nu'),\ldots,)1,\nu^{(r)})}[x_1 z^{q_1},\ldots,x_r z^{q_r}] \right.$$

$$. \ z^{\rho-1} \sum_{\alpha_1=0}^{(n_1/m_1)} \cdots \sum_{\alpha_R=0}^{(n_R/m_R)} \frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!} \cdots \frac{(-n_R)_{m_R\alpha_R}}{\alpha_R!}$$

$$. \ A(n_1,\alpha_1,\ldots,n_R,\alpha_R)(-1)^{\sum_{i=1}^{R}\beta_i\alpha_i} (z)^{\sum_{i=1}^{R}\beta_i\alpha_i}$$

$$. \ H_{M,N:(B^{(r+1)},D^{(r+1)}+1),\ldots,(B^{(2r)},D^{(2r)}+1)}^{0,1 \ :(1,\nu^{(r+1)});\ldots;(1,\nu^{(2r)})}[x_{r+1} z^{s_{r+1}},\ldots,x_{2r} z^{s_{2r}}] \tag{2.2}$$

In (2.2) using the result (1.14) and the series representation of the multivariable H-function.

3. PARTICULAR CASES

I. If we take ρ = 1 and $x_{r+1} = \ldots = x_{2r} = 0$ in (2.1), we get the following theorem

Theorem: (a) Under the hypothesis preceding the assertion of the main theorem (2.1)

$$\sum_{n_1,\ldots,n_R=0}^{\infty}\sum_{m_1,\ldots,m_r=0}^{\infty}\sum_{\alpha_1=0}^{(n_1/m_1)}\cdots\sum_{\alpha_R=0}^{(n_R/m_R)}\sum_{n=-\infty}^{\infty} k\binom{\sigma}{m+kn}(-1)^{\sum_{i=1}^{r}s_i+\sum_{i=1}^{r}n_i+\sum_{i=1}^{R}\beta_i\alpha_i}$$

$$.\frac{\phi_1(m_1)}{m_1!}\cdots\frac{\phi_r(m_r)}{m_r!}\psi(m_1,\ldots,m_r)\frac{\phi_1'(n_1)}{n_1!}\cdots\frac{\phi_R'(n_R)}{n_R!}\psi'(n_1,\ldots,n_R)$$

$$.\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_R)_{m_R\alpha_R}}{\alpha_R!}A(n_1,\alpha_1,\ldots,n_R\alpha_R)(x_1^{s_1}\ldots x_r^{s_r})$$

$$.\,z^{-\sigma+\nu+\sum_{i=1}^{r}q_is_i+\sum_{i=1}^{R}B_i\alpha_i+\sum_{i=1}^{R}s_i'n_i'-1}$$

$$.\aleph^{m,\,n+1}_{p_i+1,q_i+1,\tau_i;r}\left[z\left|\begin{matrix}(a_j,A_j)_{1,n},(-\nu+\sum_{i=1}^{r}q_is_i-\sum_{i=1}^{R}\beta_i\alpha_i-\sum_{i=1}^{R}s_i'n_i',1),[\tau_i(a_{ji},A_{ji})_{n+1},p_i;r\\(b_j',\beta_j')_{1,m},(-\nu-\sum_{i=1}^{r}q_is_i-\sum_{i=1}^{R}\beta_i\alpha_i-\sum_{i=1}^{R}s_i'n_i'+\sigma,1),[\tau_i(b_{ji},B_{ji})_{m+1},q_i,r\end{matrix}\right.\right]$$

$$=\sum_{n_1,\ldots,n_R=0}^{\infty}\sum_{m_1,\ldots,m_r=0}^{\infty}\sum_{\alpha_1=0}^{(n_1/m_1)}\cdots\sum_{\alpha_R=0}^{(n_R/m_R)}\sum_{n=-\infty}^{\infty} k\binom{\sigma}{\mu+kn}(-1)^{\sum_{i=1}^{r}s_i+\sum_{i=1}^{r}n_i+\sum_{i=1}^{R}\beta_i\alpha_i}$$

$$.\frac{\phi_1(m_1)}{m_1!}\cdots\frac{\phi_r(m_r)}{m_r!}\psi(m_1,\ldots,m_r)\frac{\phi_1'(n_1)}{n_1!}\cdots\frac{\phi_R'(n_R)}{n_R!}\psi'(n_1,\ldots,n_R)$$

$$.\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_R)_{m_R\alpha_R}}{\alpha_R!}A(n_1,\alpha_1,\ldots,n_R\alpha_R)(x_1^{s_1}\ldots x_r^{s_r})$$

$$.\frac{z^{-\sigma+\nu+\sum_{i=1}^{r}q_is_i+\sum_{i=1}^{R}B_i\alpha_i+\sum_{i=1}^{R}s_i'n_i'-1}\,\Gamma 1+\sum_{i=1}^{R}\beta_i\alpha_i+\sum_{i=1}^{R}s_i'n_i'}{\Gamma 1+\sum_{i=1}^{R}\beta_i\alpha_i+\sum_{i=1}^{R}s_i'n_i'-\mu-kn}$$

$$.\aleph^{m,n+1}_{p_i+1,q_i+1,\tau_i,\gamma}\left[z\left|\begin{matrix}(a_j,A_j)_{1,n},(1-\nu-\sum_{i=1}^{r}q_is_i,1),[\tau_i(a_{ji};A_{ji})_{n+1},p_i,r\\(b_j,B_j)_{1,m},(1-\nu-\sum_{i=1}^{r}q_is_i-\sigma-\mu-kn;1),[\tau_i(b_{ji},B_{ji})_{m+1,q_i;r}\end{matrix}\right.\right] \qquad (3.1)$$

If we take $\nu=1$and $x_1,\ldots,x_r = 0$ in (2.1), we find the following theorem

Theorem: (b) Under the hypothesis preceding the assertion of the main theorem (2.1)

$$\sum_{n_1,\ldots,n_R=0}^{\infty}\sum_{m_1,\ldots,m_r=0}^{\infty}\sum_{\alpha_1=0}^{(n_1/m_1)}\cdots\sum_{\alpha_R=0}^{(n_R/m_R)}\sum_{n=-\infty}^{\infty} k\binom{\sigma}{\mu+kn}(-1)^{\sum_{i=1}^{r}s_i+\sum_{i=1}^{R}\beta_i\alpha_i+\sum_{i=1}^{r}n_i}$$

$$.\frac{\phi_1(m_1)}{m_1!}\cdots\frac{\phi_r(m_r)}{m_r!}\psi(m_1,\ldots,m_r)\frac{\phi_1'(n_1)}{n_1!}\cdots\frac{\phi_R'(n_R)}{n_R!}\psi'(n_1,\ldots,n_R)$$

$$.\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_R)_{m_R\alpha_R}}{\alpha_R!}A(n_1,\alpha_1,\ldots,n_R\alpha_R)(\chi_{r+1}^{n_1}\ldots x_{2r}^{n_R})$$

$$.\,z^{-\sigma+\nu+\sum_{i=1}^{r}q_is_i+\sum_{i=1}^{R}B_i\alpha_i+\sum_{i=1}^{R}s_i'n_i'-1}$$

$$.\aleph^{m,\,n+1}_{p_i+1,q_i+1,\tau_i;r}\left[z\left|\begin{matrix}(a_j,A_j)_{1,n},(-\rho-\sum_{i=1}^{r}q_is_i-\sum_{i=1}^{R}\beta_i\alpha_i-\sum_{i=1}^{R}s_i'n_i',1),[\tau_i(a_{ji},A_{ji})_{n+1},p_i;r\\(b_j,\beta_j)_{1,m},(-\rho-\sum_{i=1}^{r}q_is_i-\sum_{i=1}^{R}\beta_i\alpha_i-\sum_{i=1}^{R}s_i'n_i'+\sigma,1),[\tau_i(b_{ji},B_{ji})_{m+1},q_i,r\end{matrix}\right.\right]$$

$$= \sum_{n_1,\ldots,n_R=0}^{\infty} \sum_{m_1,\ldots,m_r=0}^{\infty} \sum_{\alpha_1=0}^{(n_1/m_1)} \cdots \sum_{\alpha_R=0}^{(n_R/m_R)} \sum_{n=-\infty}^{\infty} k \binom{\sigma}{\mu+kn} (-1)^{\sum_{i=1}^{r} s_i + \sum_{i=1}^{r} n_i + \sum_{i=1}^{R} \beta_i \alpha_i}$$

$$. \frac{\phi_1(m_1)}{m_1!} \cdots \frac{\phi_r(m_r)}{m_r!} \psi(m_1,\ldots,m_r) \frac{\phi_1'(n_1)}{n_1!} \cdots \frac{\phi_R'(n_R)}{n_R!} \psi'(n_1,\ldots,n_R)$$

$$. \frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!} \cdots \frac{(-n_R)_{m_R\alpha_R}}{\alpha_R!} A(n_1,\alpha_1,\ldots,n_R\alpha_R)(x_{r+1}^{n_1} \ldots x_{2r}^{n_R})$$

$$. \frac{z^{-\sigma+\rho+\sum_{i=1}^{r} q_i s_i + \sum_{i=1}^{R} B_i \alpha_i + \sum_{i=1}^{R} s_i' n_i' - 1} \, \Gamma\rho + \sum_{i=1}^{R} \beta_i \alpha_i + \sum_{i=1}^{R} s_i' n_i'}{\Gamma\rho + \sum_{i=1}^{R} \beta_i \alpha_i + \sum_{i=1}^{R} s_i' n_i' - \mu + kn}$$

$$. \aleph_{p_i+1, q_i+1, \tau_i, r}^{m, n+1} \left[z \left| \begin{matrix} (a_j, A_j)_{1,n}, (-\sum_{i=1}^{r} q_i s_i, 1), [\tau_i(a_{ji}; A_{ji})_{n+1, p_i, r} \\ (b_j, B_j)_{1,m}, (-\sum_{i=1}^{r} q_i s_i - \sigma - \mu - kn; 1), [\tau_i(b_{ji}, B_{ji})_{m+1, q_i; r} \end{matrix} \right. \right]. \qquad (3.2)$$

REFERENCES

1. T.M.Atanackovie, M., Bundincevie and S. Pilipovie, On a fractional distributed-order oscalator, J.Phys.A38 (2005), 6705-6713.
2. H.A. Biagioni and T. Gramchev, Fractional derivative estimates in Ceurey Spaces, global regularity and decay for solutions to semilinear equations in IR, J. of differential Equations,194 (2003),140-165.
3. P.Biler,T.Funaki and W. Woyczyski, Fractal Burgers equations, J. of Differential Equations, 148 (1998), 9-46.
4. V.B.L. Chaurasia, R.Patni and V.K. Singhal, Applications of the fractional calculus pertaining to multivariable H-function. (Communicated for publication).
5. V.B.L. Chaurasia and Yudhveer Singh, Lambert's law and Aleph function, International Journal of Modern Mathematical Science, 4(2),(2014), 84-88.
6. T.Gramehev, Global in time Spatial analyticity of solutions of fractional Burgers, equations, Mat.Contemp., 27 (2004), 75-100.
7. R.N. Kalia, Editor, Recent Advances in Fractional Calculus, Global Publishing Company, SankRapids, MI, 1993.
8. R.N. Kalia, Integral Transforms, Fractional Calculus and Special functions, In transform method and special functions, Proceeding of the first international workshop on Transform method and special function held at Bankya [Sofia];August 1994, 12-17),(Edited by P. Ruseu, I.Dimovski and V. Kiryakova), Science Culture Technology Publishing Company, Singapore, 1995, 82-92).
9. R.N. Kalia and H.M. Srivastava, Fractional calculus and its applications involving functions of several variables, Applied Mathematics letters, 12(1999), 19-23.
10. A.A. Kilbas and M. Saigo, Fractional calculus of the H-function, Fukuoka Univ. Sci. rep., 28 (1998), 41-51.
11. J.L. Lavoie, T.J. Osler and R. Trembly, Fractional derivatives and special functions, SIAM Rev.,18 (1976), 240-268.
12. H.L. Manocha and B.L. Sharma, Fractional derivatives and summation, J. Indian Math. Soc. (N.S.), 38 (1974), 371-382.
13. H.L. Manocha and B.L. Sharma, Summation of infinite series, J. Austral. Math. Soc., 6 (1966), 470-476.
14. A.M. Mathai and R.K. Saxena, The H-function with application in statistics and other disciplines, Wiley Eastern, New Delhi, 1978.
15. G.S. Olkha and V.B.L. Chaurasia, Series representation for the H-function of several complex variables, Math. Edu., 19 :1(1985), 38-40.
16. T.J. Osler, Afurther extension of the Lebnitz rule to fractional derivatives and its relation to Parseval's formula,SIAM J. Math. Anal., 13 (1972), 1-16.
17. R.K. Raina and C.L. Koul,Fractional derivatives of the H-functions, Jnānābha (Professor Charles Fox Memorial Volume) 7, (1977),97-105.
18. N. Sudland,B. Baumann and T.F. Nonnenmacher, Open problem : who knows about the Aleph (ℵ)-function? Fract. Calc., Appl. Anal. 1(4) (1998): 401-402.
19. N. Sudland, B.Baumann and T.F. Nonnenmacher, Fractional diftless, Fokker-Plank equation with power law diffusion coefficients. In V.G. Gangha, E.W.Mayr, W.G.Vorozhtsov, editions, Computer Algebra in Scientific Computing (CASC Konstanz 2001), Springer, Berlin (2001): 513-525.

20. S.G. Samko, A.A. Kilbas and O.I. Marichev, Fractional Integrals and Derivatives: Theory and Applications, Gordon and Breach Science Publishers, Reading, PA, 1993.
21. H.S.P. Shrivastava, Fractional derivatives of multivariable generalized hypergeometric functions, J. Indian Acad. Math., 21 (1999),89-106.
22. H.M. Srivastava, Some Applications of Fractional Calculus Involving the Gamma and Related Functions, In Recent Advances in Fractional Calculus, (Edited by R.N. Kalia), Global Publishing Company, Sauk Rapids, MI, 1993, 179-202.
23. H.M. Srivastava, A multilinear generating function for the Konhauser sets of biorthogonal polynomials suggested by the Laguerre polynomials, Pacific J. Math., 117 (1985), 183-191.
24. H.M. Srivastava and S.P. Goyal, Fractional derivatives of the H-function of several variables, J. Math. Anal. Appl., 112 (1985), 641-651.
25. H.M. Srivastava and M.A. Hussain, Fractional integration of the H-function of several variables, Comput. Math. Appl., 30 (1995), 73-85.
26. H.M. Srivastava and P.W. Karlsson, Multiple Gaussian Hypergeometric Series, Halsted Press (Ellis Horwood Limited)/John Wiley and Sons, Chichester/New York, 1985.
27. H.M.Srivastava and R. Panda, Some bilateral generating functions for the class of generalized hypergeometric ppolynomials, J. Reine Math., 283/284 (1976), 265-274.
28. H.M. Srivastava, R.C. Singh Chandel and P.K. Vishwakarma, Fractional derivatives of certain generalized hypergeometric functions of several variables, J. Math. Anal. Appl., 184 (1994), 560-572.

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