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EFFECT OF VARIABLE VISCOSITY ON THERMAL CONVECTIVE INSTABILITY IN A NANOFLUID SATURATED POROUS LAYER

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ABSTRACT

The effect of nanofluid viscosity varying with temperature on the onset of thermal convection in a layer of nanofluid saturated Darcy porous medium is studied. Both top and bottom-heavy basic nanoparticle distributions are considered and the resulting eigenvalue problem is solved numerically by the Galerkin method. In the case of top-heavy nanoparticle distribution, the onset is hastened with increasing basic volume fraction difference of nanoparticles and the instability sets in only as stationary convection. To the contrary, the onset is delayed and the instability occurs via oscillatory convection in the case of bottom-heavy nanoparticle distribution. Besides, oscillatory convection occurs when the volume fraction difference of nanoparticles exceeds a threshold value depending on the thermophysical properties of nanofluids. It is observed that the viscosity parameter enhances the stability of the system in both stationary and oscillatory modes. The effect of increasing Prandtl number is to hasten the onset of oscillatory convection.

Key words: *nanofluid; oscillatory convection; porous medium; variable viscosity; thermophoresis, Brownian diffusion.*

1. INTRODUCTION

Nanofluids have attracted a great deal of interest recently because of their enhanced thermophysical properties such as thermal conductivity, viscosity and convective heat transfer coefficients compared to those of base fluids. Eastman *et al* (2001) and Choi *et al* (2001) have shown that when a small amount of nanoparticles and nanotubes is added, the enhancement of the thermal conductivity of base fluids (ethylene glycol and oil) reaches up to 160%. For these reasons, the nanofluids are considered to be strong candidates as new heat transfer fluids for thermal management in next-generation microelectronic devices and in many other fields of science and engineering. The review articles by Kakac and Pramuanjaroenkij (2009), Jing and Wang (2011), Yu and Xie (2012) and Mahbubul *et al* (2012) have summarized the recent progress on the study of nanofluids.

Studies have been initiated by several researchers to understand buoyancy driven convection in a nanofluid layer. Buongiorno (2006) has proposed a model for convective transport in nanofluids incorporating the effects of Brownian diffusion and thermophoresis. Vadasz (2006) has studied heat conduction in nanofluid suspensions. Tzou (2008a,b) has studied onset of convection in a horizontal nanofluid layer heated uniformly from below using the model developed by Buongiorno (2006). The corresponding thermal convective instability problem in a nanofluid saturated horizontal porous medium has also attracted equal importance because of its importance in many fields of modern science, engineering and technology, chemical and nuclear industries and bio-mechanics. The problem was first considered by Nield and Kuznetsov (2009) using the Darcy model. Following this formalism several studies have been undertaken subsequently to investigate various additional effects on the problem by the same authors and others. The details can be found in the book of Nield and Bejan (2013).

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It is imperative to note that thermal convective instability is affected by nanofluid properties and specifically by nanofluid viscosity and thermal conductivity. Experiments by Praveen *et al* (2007), Naik *et al* (2010), Nguyen *et al* (2008), Kole and Dey (2010), Turgut (2009), Duangthongsuk and Wongwises (2009) have revealed that the nanoparticles concentration and nanoparticles size influence the nanofluids viscosity under a wide range of temperatures. Zheng *et al* (2012) have shown that with CMC as the base solution, increasing the viscosity of the solution can alleviate the settling velocity of particles and make the suspension better. They have inferred that the viscosity drops with temperature and with concentration of nanoparticles. The onset of convection in a nanofluid saturated porous medium with thermal conductivity and viscosity dependent on the nanoparticle volume fraction is considered by Nield and Kuznetsov (2012). But it is also to be noted that the fluid viscosity varies with temperature and it affects the stability characteristics of the system significantly. For example, the fluidity of Poly-Alpha-Olefin, a synthetic lubricant that is used in cooling electronics in radar equipment, changes almost linearly with the temperature is showed by Nield and Kuznetsov (2003). Similarly, the viscosity of water decreases by about 240% when the temperature increases from 10 to $50^{\circ}C$. Under the circumstances, considering variation in viscosity of the nanofluid with temperature also becomes equally important in order to predict the correct contribution of nanoparticles on thermal convective instability in a nanofluid saturated porous layer.

Nonetheless, most of the previously mentioned works on this topic have been relied on the viscosity that is not sensitive to fluid temperature. The intent of the present paper is to investigate this aspect by considering the effect of linear variation in the viscosity of nanofluid with temperature for both top and bottom-heavy basic nanoparticle distributions. The eigenvalue problem is solved by higher order Galerkin method and more accurate results are presented. The preferred mode of instability is analyzed in detail. For this purpose, the difference in the basic volume fraction of nanoparticles between the boundaries is taken as an independent parameter and thereby the quantification of basic nanoparticles distribution on the onset is discerned clearly. From the numerical computations it is observed that oscillatory convection occurs if the basic nanoparticle distribution is bottom-heavy and also when the volume fraction difference of nanoparticles exceeds a threshold value.

2. MATHEMATICAL FORMULATION

We consider a nanofluid saturated horizontal porous layer of thickness d. The lower surface is held at constant temperature T_0 and volume fraction ϕ_0, while the upper surface is held at temperature $T_1\,(<T_0)$ and volume fraction ϕ_1. The basic nanoparticle distribution is considered to be either top-heavy or bottom-heavy. A Cartesian coordinate system (x, y, z) is chosen such that the origin is at the bottom of the porous layer and the $z-$ axis vertically upward in the presence of gravitational field. The viscosity of the nanofluid μ is assumed to vary linearly with temperature in the form

$$\mu = \mu_0\left[1-\eta\left(T-T_R\right)\right] \tag{1}$$

where, μ_0, η are positive constants and T_R is a reference temperature. Following Buongiorno (2006) and Nield and Kuznetsov (2009), the governing equations are:

$$\nabla\cdot\vec{q} = 0 \tag{2}$$

$$\frac{\rho_f}{\varepsilon}\frac{\partial\vec{q}}{\partial t} = -\nabla p + \rho\vec{g} - \frac{\mu}{K}\vec{q} \tag{3}$$

$$\left(\rho c\right)_m\frac{\partial T}{\partial t} + \left(\rho c\right)_f\left(\vec{q}\cdot\nabla\right)T = k\nabla^2 T + \varepsilon\left(\rho c\right)_p\left[D_B\nabla\phi\cdot\nabla T + \frac{D_T}{T_R}\nabla T\cdot\nabla T\right] \tag{4}$$

$$\frac{\partial\phi}{\partial t} + \frac{1}{\varepsilon}\left(\vec{q}\cdot\nabla\right)\phi = D_B\nabla^2\phi + \frac{D_T}{T_R}\nabla^2 T \tag{5}$$

$$\rho = \phi\rho_p + \left(1-\phi\right)\rho_f\left\{1-\beta\left(T-T_R\right)\right\} \tag{6}$$

where, $\vec{q} = (u, v, w)$ the velocity vector, p the pressure, ρ the overall density of the nanofluid, ρ_p the density of nanoparticles, ρ_f the density of base fluid, $\vec{g}$ the gravitational acceleration, ϕ the nanoparticle volume fraction, K the permeability of the porous medium, ε the porosity of the porous medium, T the temperature of the nanofluid, β the volumetric coefficient of thermal expansion, c the specific heat, k the thermal conductivity, D_B the Brownian diffusion coefficient, D_T the thermophoretic diffusion coefficient and T_c the bulk temperature.

The basic state is quiescent. The basic temperature and volume fraction distributions satisfying the boundary conditions $T_b = 0$, $\phi_b = 0$ at $z = 0$ and $T_b = -1$, $\phi_b = 1$ at $z = 1$ are respectively given by $T_b = -z$ and $\phi_b = z$. For the bottom-heavy case, the volume fraction distribution is found to be $\phi_b = 1 - z$.

To study the stability of the basic state, we now superimpose perturbations on the basic state in the form $\vec{q} = \vec{q}',\ T = T_b + T',\ \phi = \phi_b + \phi',\ p = p_b + p'$. Following the standard linear stability analysis procedure as outlined in [11], the dimensionless stability equations can then be shown to be

$$\frac{\omega}{Pr}\left(D^2 - a^2\right)W = -\left[\Gamma DW + (1+\Gamma z)\left(D^2 - a^2\right)W\right] + \Delta\phi\, Rn\, a^2\,\Phi - Rt\, a^2\,\Theta + Rt\,\Delta\phi\, a^2 z\left(\Theta - \Phi\right) \tag{7}$$

$$M\omega\Theta = W + \left[D^2 - a^2 + \frac{N_B\left(\Delta\phi - 2N_A\right)}{Le}D\right]\Theta - \frac{\Delta\phi N_B}{Le}D\Phi \tag{8}$$

$$\omega\Phi = -W + \frac{N_A}{\Delta\phi Le}\left(D^2 - a^2\right)\Theta + \frac{1}{Le}\left(D^2 - a^2\right)\Phi \tag{9}$$

where, $D = d/dz$ and $a = \sqrt{l^2 + m^2}$ is the horizontal wave number.

In the above equations, $Rt = dK\beta\rho_f\left(T_0 - T_1\right)g/\mu_0\kappa$ is the thermal Darcy-Rayleigh number, $Rm = Kd\{\phi_0\rho_p + (1-\phi_0)\rho_f\}g/\mu_0\kappa$ and $Rn = Kd\left(\rho_p - \rho_f\right)g/\mu_0\kappa$ are the density Darcy-Rayleigh numbers, $Le = \kappa/\varepsilon D_B$ is the Lewis number, $N_A = D_T\left(T_0 - T_1\right)/D_B T_R$ is the modified diffusivity ratio, $N_B = (\rho c)_p/(\rho c)_f$ is the ratio of heat capacities, $\Gamma = \eta\,\Delta T$ is the viscosity parameter, $Pr = \mu_0\varepsilon^2 d^2/\rho_f\kappa K$ is the Darcy-Prandtl number $M = (\sigma/\varepsilon)$ is the heat capacity ratio and $\Delta\phi = \phi_1 - \phi_0$.

The boundary conditions, on using Eq. (17), now become

$$W = 0,\ \Theta = 0,\ \Phi = 0 \quad \text{at} \quad z = 0,1. \tag{10}$$

3. NUMERICAL SOLUTION

Equations (7) - (9) constitute an eigenvalue problem with variable coefficient and solved numerically by the Galerkin method. The variables are written in a series of basis functions

$$W(z) = \sum_{i=1}^{N} A_i W_i(z), \quad \Theta(z) = \sum_{i=1}^{N} B_i\Theta_i(z), \quad \Phi(z) = \sum_{i=1}^{N} C_i\Phi_i(z) \tag{11}$$

where, A_i, B_i and C_i are unknown coefficients. The basis functions are represented by the power series

$$W_i(z) = \Theta_i(z) = \Phi_i(z) = z^i(1-z). \tag{12}$$

It is noted that $W_i(z)$, $\Theta_i(z)$ and $\Phi_i(z)$ given by Eq.(11) satisfy the corresponding boundary conditions. Multiplying Eq.(7) by $W_j(z)$, Eq.(8) by $\Theta_j(z)$ and Eq.(9) by $\Phi_j(z)$; performing the integration by parts with respect to z between $z = 0$ and 1, and using the boundary conditions , we obtain a generalized eigenvalue problem of the form

$$\Delta_1 X = \omega\Delta_2 X \tag{13}$$

where Δ_1 and Δ_2 are real matrices of order $N \times N$ and X is the eigen vector. By using the subroutine GVLRG of the IMSL library, the complex eigenvalue ω is determined when the other parameters are specified. Then one of the parameters, say Rt, is varied until the real part of ω vanishes. The zero crossing of real part of ω is achieved by

Newton's method for fixed point determination. The corresponding value of Rt and a are the critical conditions for neutral stability. Then the critical Rayleigh number with respect to the wave number is calculated using the golden section search method. The imaginary part of ω indicates whether the instability onsets into steady convection or into growing oscillations. Convergence of the results is achieved by using six terms in the series expansion.

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4. RESULTS AND DISCUSSION

The effect of linear variation in nanofluid viscosity due to temperature on the onset of convection in a nanofluid saturated Darcy porous medium is investigated numerically. Both top-heavy and bottom-heavy basic nanoparticle distributions are considered for discussion. The basic volume fraction difference of nanoparticles $\Delta\phi$ is considered as an independent parameter to know exclusively the quantification of nanoparticles on the stability characteristics of the system.

To have a check on the accuracy of the numerical procedure used, first test computations are carried out under the limiting conditions and compared with the previously published results. It is noted that the convergence of the results is achieved by using six terms in the series expansion. The critical Rayleigh numbers computed for different values of viscosity parameter Γ when $\Delta\phi = 0 = N_A$ (regular fluid) are compared with those of Lebon and Cloot (1986) in Table1 and note that our results are in excellent agreement. Besides, it is instructive to look at the results for various levels of the Galerkin approximation to know the process of convergence and also the accuracy of the results. Table 2 shows the numerically computed values of critical thermal Darcy-Rayleigh number for various values of Γ for the steady case when $\Delta\phi = 0.01$, $N_A = 0.1$, $Rn = 50$, $Le = 10$, $Pr = 100$, $M = 1$ and $N_B = 0.5$. The results exhibited in Table 2 are for top-heavy case and the onset of convection is found to be via stationary mode. From the tabulated values, it is clear that the results converge for six terms in the Galerkin expansion. These comparisons confirm the accuracy of the numerical procedure used.

Table - 1: Comparison of Rt_c with $\Delta\phi$ for for different orders of approximations in the Galerkin expansion for regular fluid ($\Delta\phi = 0,\ N_A = 0$)

N	Rt_c			
	$\Gamma = 0$	$\Gamma = 0.1$	$\Gamma = 0.3$	$\Gamma = 0.5$
1	40.000000	42.000000	46.000000	50.000000
2	40.000000	41.977832	45.817942	49.535160
3	39.478999	41.435446	45.256327	48.976617
4	39.478999	41.434866	45.251752	48.965679
5	39.478418	41.434284	45.251286	48.965385
6	39.478418	41.434283	45.251279	48.965370
Lebon & Cloot(1986)	39.48	41.45	45.25	49

Table - 2: Comparison of Rt_c with $\Delta\phi$ for for different orders of approximations in the Galerkin expansion for nanofluids with $\Delta\phi = 0.01$, $N_A = 0.1$, $Rn = 50$, $N_B = 0.5$, $Le = 10$ and $M = 1$

N	Rt_c					
	$\Gamma = 0$	$\Gamma = 0.1$	$\Gamma = 0.2$	$\Gamma = 0.3$	$\Gamma = 0.4$	$\Gamma = 0.5$
1	33.519553	35.754190	37.988827	40.223464	42.458101	44.692737
2	33.510964	35.686335	37.815982	39.906056	41.961681	43.987154
3	32.929937	35.087136	37.207820	39.296364	41.356291	43.390495
4	32.929042	35.083835	37.200994	39.285180	41.340180	43.369117
5	32.928403	35.083242	37.200488	39.284782	41.339889	43.368916
6	32.928401	35.083234	37.200474	39.284761	41.339862	43.368885

The results obtained for the two cases namely, (i) top-heavy and (ii) bottom-heavy basic nanoparticle distributions are discussed separately below.

Case (i): Top-heavy nanoparticle distribution ($\phi_1 - \phi_0 = \Delta\phi > 0$)

The variation of critical Darcy-Rayleigh number Rt_c is shown in Figs 1(a), (b), (c) and (d) as a function of $\Delta\phi$ for different values of Γ, Rn, N_A and Le, respectively. The corresponding critical wave number a_c for these values is shown in Figs. 2(a), (b), (c) and (d). The results for $\Gamma = 0$ in Fig. 1(a) correspond to the case of constant fluid viscosity. It is noted that increasing Γ is to increase Rt_c values and hence its effect is to delay the onset of convection in a nanofluid saturated porous medium (Fig. 1a). To the contrary, increase in the values of Rn, N_A and Le is to decrease the values of Rt_c and hence their effect is to hasten the onset of convection. Besides, Rt_c monotonically decreases with increasing $\Delta\phi$ indicating its effect is to hasten the onset of convection. This result is in conformity with

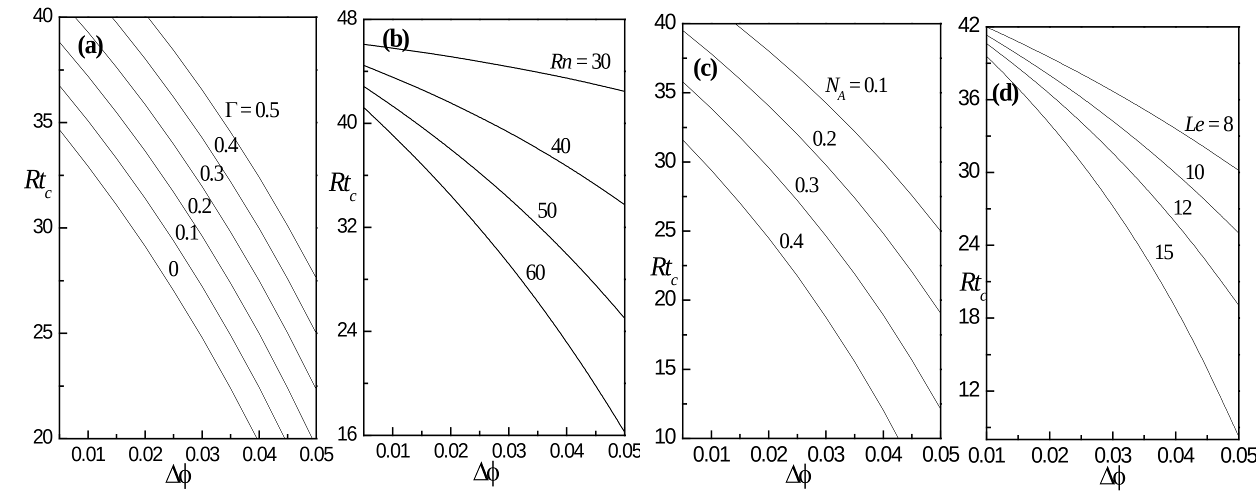


Fig. 1: Variation of Rt_c with $\Delta\phi$ for different values of (a) Γ with $Le = 10$, $Rn = 50$, $N_A = 0.1$, $\Delta\phi = 0.01$, (b) Rn with $Le = 10$, $\Gamma = 0.4$, $N_A = 0.1$, $\Delta\phi = 0.01$, (c) N_A with $Le = 10$, $\Gamma = 0.4$, $Rn = 50$, $\Delta\phi = 0.01$, (d) Le with $N_A = 0.1$, $\Gamma = 0.4$, $Rn = 50$, $\Delta\phi = 0.01$ for $N_B = 0.5$

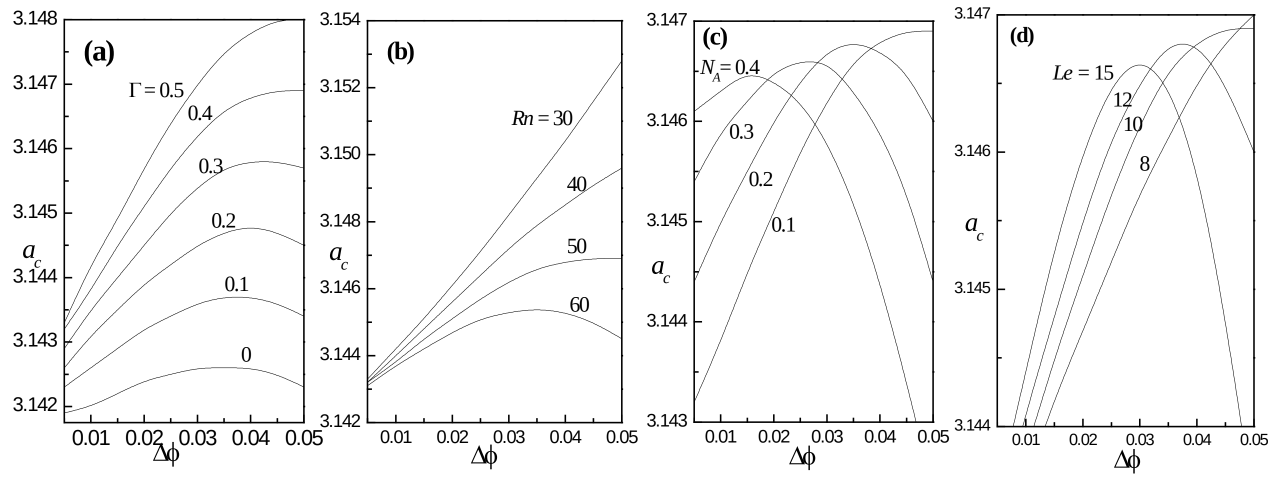
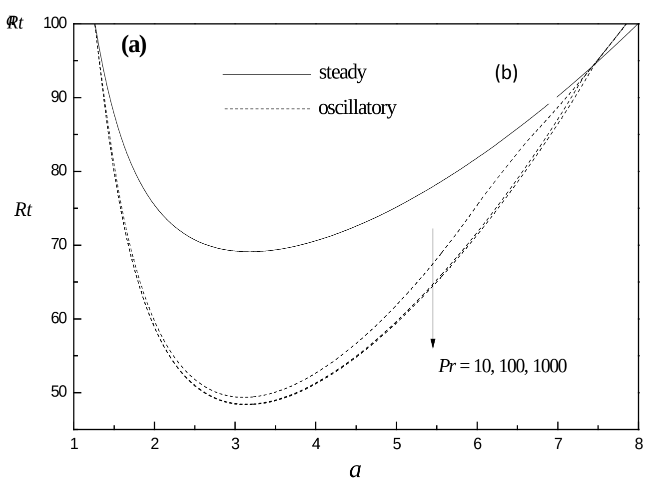


Fig. 2: Variation of a_c with $\Delta\phi$ for different values of (a) Γ with $Le = 10$, $Rn = 50$, $N_A = 0.1$, $\Delta\phi = 0.01$, (b) Rn with $Le = 10$, $\Gamma = 0.4$, $N_A = 0.1$, $\Delta\phi = 0.01$, (c) N_A with $Le = 10$, $\Gamma = 0.4$, $Rn = 50$, $\Delta\phi = 0.01$, (d) Le with $N_A = 0.1$, $\Gamma = 0.4$, $Rn = 50$, $\Delta\phi = 0.01$ for $N_B = 0.5$



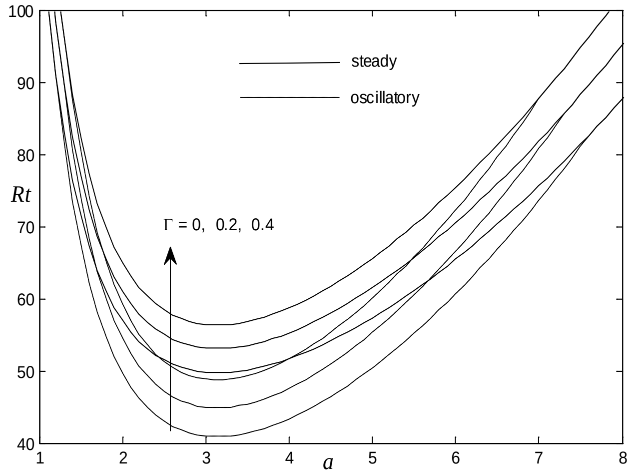


Fig. 3: Neutral curves for different values of (a) Pr with $\Gamma = 0.4$, $Le = 70$, (b) Γ with $Pr = 100$, $Le = 50$ for $\Delta\phi = 0.01$, $N_A = 0.1$, $N_B = 0.5$, $Rn = 50$ and $M = 1$

The one advocated by Nield and Kuznetsov (2009). As can be seen from Figs 2 (a-d), the variation in a_c is found to be negligibly small for all the parametric values chosen. The critical wave number turns out to be approximately π .

Case (ii): Bottom-heavy nanoparticle distribution ($\phi_0 - \phi_1 = \Delta\phi > 0$)

The temperature has a destabilizing effect while the volume fraction of nanoparticle distribution has a stabilizing effect on the system. Due to their opposing contributions one would expect oscillatory convection. The possibility of occurring oscillatory convection was propounded by Nield and Kuznetsov (2009) and this aspect has been explored in the present study. It is found that the onset of convection ceases to be stationary and instability sets in via oscillatory mode for certain choices of parametric values.

Figures 3(a) and (b) exhibit the neutral stability curves in the (Rt, a) - plane for different values of Prandtl number Pr and viscosity parameter Γ , respectively. From these figures it is seen that the steady and oscillatory neutral curves are connected in a topological sense. This connectedness allows the linear stability criteria to be expressed in terms of the critical thermal Darcy-Rayleigh number, below which the system is stable and definitely unstable above.

The points where the oscillatory solutions branch off from the stationary convection can be easily identified from these figures. From Fig.3 (a) it is seen that oscillatory convection is hastened with increasing Pr and moreover for $Pr \geq 100$ all the oscillatory neutral curves coalesce indicating less significance of the fluid conductivity on the onset of oscillatory convection. Figure 3(b) shows that increasing Γ is to increase the region of stability. The critical wave number for the onset of stationary convection is lower than that of oscillatory convection.

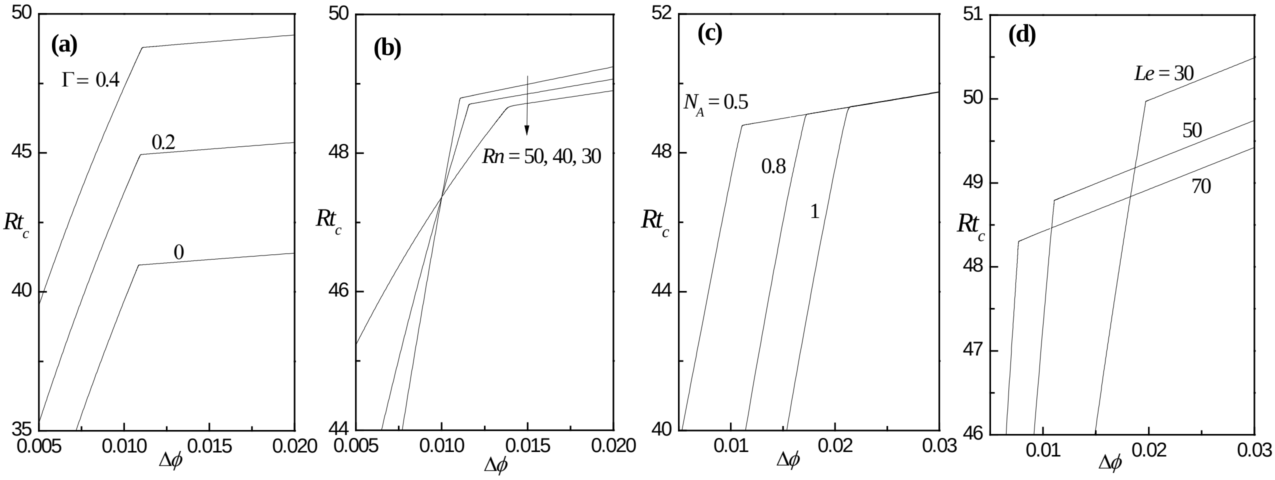


Fig. 4: Variation of Rt_c with $\Delta\phi$ for different values of (a) Γ with $N_A = 0.5 = N_B$, $Rn = 50 = Le$, (b) Rn with $N_A = 0.5 = N_B$, $Le = 50$, $\Gamma = 0.4$, (c) N_A with $\Gamma = 0.4$, $N_B = 0.5$, $Rn = 50 = Le$, (d) Le with $N_A = 0.5 = N_B$, $Rn = 50$, $\Gamma = 0.4$ for $Pr = 100$ and $M = 1$

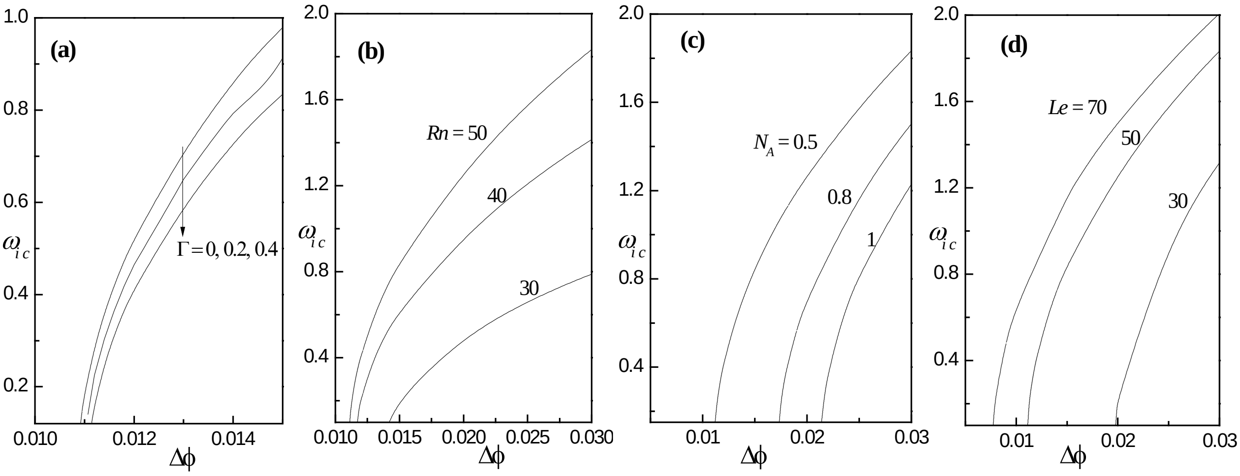


Fig. 5: Variation of ω_{ic} with $\Delta\phi$ for different values of (a) Γ with $N_A = 0.5 = N_B$, $Rn = 50 = Le$, (b) Rn with $N_A = 0.5 = N_B$, $Le = 50$, $\Gamma = 0.4$, (c) N_A with $\Gamma = 0.4$, $N_B = 0.5$, $Rn = 50 = Le$, (d) Le with $N_A = 0.5 = N_B$, $Rn = 50$, $\Gamma = 0.4$ for $Pr = 100$ and $M = 1$

The critical thermal Darcy-Rayleigh number computed numerically for both stationary and oscillatory modes for various values of physical parameters are compared in Figs 4(a)-(d), while the corresponding critical frequency of oscillations is illustrated in Figs. 5(a)-(d) with $\Delta\phi (= \phi_0 - \phi_1)$. In Figs 4(a)-(d) the portion of each stability boundary lying to the left of the discontinuity in slope corresponds to steady onset, while to the right the onset is of the oscillatory type. Initially, stationary onset occurs up to certain values of $\Delta\phi$. As $\Delta\phi$ is increased further beyond a critical value, the convection is bifurcated into the oscillatory mode. The value of $\Delta\phi$ at which the preferred mode of instability changes depends on the parametric values chosen. It is affected very little for various values of Γ (Fig. 4a). Moreover, the critical thermal Darcy-Rayleigh number for both stationary and oscillatory mode is found to increase with increasing Γ. Therefore, the viscosity parameter enhances the stability of the system in both stationary and oscillatory modes. The value of $\Delta\phi$ at which the preferred mode of instability changes is found to be significant with increasing Rn, N_A and Le and the same is evident from Figs. 4(b), (c) and (d), respectively. Although the effect of increasing Rn is to delay the onset of oscillatory convection, it shows a dual trend on the onset of stationary convection. It is seen that increasing Rn is to hasten the onset of stationary convection till $\Delta\phi = 0.01$ and exceeding this value the trend reverses. Increasing N_A is to advance the onset of stationary convection but shows no influence on the onset of oscillatory convection. The effect of Le on the stationary and oscillatory onset is found to be different. The oscillatory convection is hastened with increased values of Le while an opposite trend is noticed on the stationary convection. In this case also increasing N_B shows insignificant effect on the stability of the system.

The critical frequency of oscillations ω_{ic} increases with $\Delta\phi$ as seen from Figs. 5 (a)-(d). Also, increase in the value of Γ, N_A as well as decrease in the value of Rn and Le is to decrease ω_{ic}. From these figures it is obvious that the oscillatory convection sets in only when $\Delta\phi$ exceeds certain value which depends on the choice of other parametric values.

5. CONCLUSIONS

The onset of convection in a nanofluid saturated Darcy porous medium is studied by considering variation in nanofluid viscosity with temperature. The eigenvalue problem is solved numerically using the Galerkin method. The onset of convection is hastened and via stationary mode if the basic nanoparticle distribution is top-heavy. However, the instability is via oscillatory mode depending on the choices of physical parameters and also the onset is delayed if the basic nanoparticle distribution is bottom-heavy. Moreover, oscillatory convection occurs when the volume fraction difference of nanoparticles exceeds a threshold value which in turn depends on the choices of physical parameters. The effect of increasing viscosity parameter Γ is to delay the onset of convection irrespective of the arrangement of basic volume fraction of nanoparticles. An increase in the value of density Darcy - Rayleigh number Rn, modified diffusivity ratio N_A and the Lewis number Le is to hasten the onset of convection when the basic nanoparticle distribution is top-heavy. In the bottom-heavy case, the effect of increasing Prandtl number Pr and Le is to hasten while increasing Rn is to delay the onset of oscillatory convection but N_A shows no influence on the onset of oscillatory convection. Moreover, increase in the value of Γ, N_A as well as decrease in the value of Rn and Le is to decrease the critical frequency of oscillations. The critical wave number for the onset of stationary convection is slightly higher than that of oscillatory convection.

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NOMENCLATURE

a	wave number
D_B	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
d	depth of the porous layer
k	thermal conductivity of the nanofluid
K	permeability of the porous medium
Le	Lewis number
N_A	modified diffusivity ratio
N_B	ratio of heat capacities

p	pressure
$\vec{q}=(u,v,w)$	nanofluid velocity
Rm, Rn	density Darcy - Rayleigh numbers
Rt	thermal Darcy - Rayleigh number
(x,y,z)	Cartesian coordinates
t	time
T	nanofluid temperature
T_0	temperature at the lower boundary
T_1	temperature at the upper boundary
T_R	reference temperature
W	amplitude of perturbed vertical component of velocity

GREEK SYMBOLS

β	volumetric coefficient of thermal expansion
ε	porosity of porous media
σ	heat capacity ratio
Γ	viscosity parameter
μ	viscosity of the fluid
η	thermal expansion coefficient of viscosity
ρ	nanofluid density
Θ	amplitude of perturbed temperature
ϕ	nanoparticle volume fraction
ϕ_0	nanoparticle volume fraction at the lower boundary
ϕ_1	nanoparticle volume fraction at the upper boundary
Φ	amplitude of perturbed nanoparticle volume fraction
κ	thermal diffusivity of the fluid

SUPERSCRIPTS

*	dimensionless variable
'	perturbed variable

SUBSCRIPTS

b	basic state
f	fluid
p	particle

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