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WEIGHTED SHARING OF A SMALL FUNCTION OF A MEROMORPHIC FUNCTION AND ITS DERIVATIVES

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ABSTRACT

In this paper, we study the relationship between a meromorphic function and its k-th order derivative which share a small function with weight(multiplicities) l(a positive integer)ignoring multiplicity.

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INTRODUCTION AND RESULTS

In this paper we shall use the standard notations of Nevanlinna theory such as $T(r, f), N(r, f), m(r, f)$ and so on (see[3]), where f is a meromorphic function defined on the whole complex plane. The quantity S(r,f) is defined by S(r,f)=0(T(r,f)) as $r \to \infty$ possibly outside a set of finite linear measure. A meromorphic function a(z) is called a small function with respect to f provided T(r,a)= S(r,f) holds.

Suppose that f and g are two non-constant meromorphic functions, a is a small function with respect to f and g and k be a positive integer. Now f and g share 'a' ignoring multiplicities or IM (counting multiplicities or CM) if f - a and g - a have the same zeros ignoring(counting) multiplicities. We denote by $N_{k)}\left(r, \frac{1}{f-a}\right)$, the counting function for zeros of f - a with multiplicity $\leq k$ (counting multiplicity), and by $\overline{N}_{k)}\left(r, \frac{1}{f-a}\right)$, the corresponding one for which the multiplicity is not counted. Similarly by $N_{(k}\left(r, \frac{1}{f-a}\right)$, we mean the counting function for zeros of f – a with multiplicity at least k(counting multiplicity) and by $\overline{N}_{(k}\left(r, \frac{1}{f-a}\right)$, we mean the corresponding one for which the multiplicity is not counted.

We denote $N_k\left(r, \frac{1}{f-a}\right) = \overline{N}\left(r, \frac{1}{f-a}\right) + \overline{N}_{(2}\left(r, \frac{1}{f-a}\right) + \ldots\ldots + \overline{N}_{(k}\left(r, \frac{1}{f-a}\right)$

where $\overline{N}_{(1}\left(r, \frac{1}{f-a}\right) = \overline{N}\left(r, \frac{1}{f-a}\right)$ and $\delta_p(a, f) = 1 - \limsup_{r\to\infty} \dfrac{N_p\left(r, \frac{1}{f-a}\right)}{T(r, f)}$, where p is a positive integer; then clearly $0 \leq \delta(a, f) \leq \delta_k(a, f) \leq \Theta(a, f) \leq 1$,

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where $\delta(a, f) = 1 - \limsup_{r\to\infty} \dfrac{N\left(r, \dfrac{1}{f-a}\right)}{T(r, f)}$ *and* $\Theta(a, f) = 1 - \limsup_{r\to\infty} \dfrac{\overline{N}\left(r, \dfrac{1}{f-a}\right)}{T(r, f)}$

In [6], Q.C. Zhang proved the following theorem about a meromorphic function and its k-th order derivative.

Theorem: A Let f be a non-constant meromorphic function and let k be a positive integer. Suppose that f and $f^{(k)}$ share 1 CM and $2\overline{N}(r, f) + \overline{N}\left(r, \dfrac{1}{f'}\right) + N\left(r, \dfrac{1}{f^{(k)}}\right) \le (\lambda + o(1))T(r, f^{k})$ *for* $r \in I$, where I is a set of infinite linear measure and λ satisfies $0 < \lambda < 1$ then $\dfrac{f^{(k)} - 1}{f - 1} \equiv c$ for some constant $c \in C - \{0\}$.

In 2003, Kit-wing [5] discussed the problem of a meromorphic function and its k- th derivative sharing one small function and proved the following result.

Theorem: B Let $k \ge 1$. Let f be a non-constant non-entire meromorphic function, $a \in s(f)$ and $a(\ne 0, \infty)$ and f do not have any common pole. If f,$f^{(k)}$ share a CM and $4\delta(0, f) + 2(8 + k)\Theta(\infty, f) > 19 + 2k$, *then* $f = f^{(k)}$.

Two years latter, in 2005, Q.C.Zhang [2] proved the following theorem.

Theorem: C Let f be a non-constant meromorphic function and $k(\ge 1), l(\ge 0)$ be integers. Also let $a \equiv a(z)$ $(not \equiv 0, \infty)$ be a meromorphic function such that T(r,a) = S(r,f). Suppose that f – a and $f^{(k)}$-a share $(0, l)$. If $l \ge 2$ and $(3 + k)\Theta(\infty, f) + 2\delta_{2+k}(0, f) > k + 4$ or, if l =1 and $(4 + k)\Theta(\infty, f) + 3\delta_{2+k}(0, f) > k + 6$, or, if l = 0, i.e., f - a and $f^{(k)}$-a share the value 0 IM and $(6 + k)\Theta(\infty, f) + 5\delta_{2+k}(0, f) > 2k + 10$ *then* $f \equiv f^{(k)}$.

Recently, in 2010, A.Chen, X.Wang and G.Zhang [7] proved the following results.

Theorem: D Let $k(\ge 1), n(\ge 1)$ be integers and f be a non-constant meromorphic function. Also let $a(z)(not \equiv 0, \infty)$ be a small function with respect to f. If f and $[f^n]^{(k)}$ share a(z) IM and

$$4\overline{N}(r, f) + 2\overline{N}\left(r, \frac{1}{\left(\frac{f}{a}\right)'}\right) + 2N_2\left(r, \frac{1}{(f^n)^{(k)}}\right) + \overline{N}\left(r, \frac{1}{(f^n)^{(k)}}\right) \le (\lambda + o(1))T\left(r, (f^n)^{(k)}\right)$$, or, if f and $(f^n)^{(k)}$ share a(z) CM and $2\overline{N}(r, f) + \overline{N}\left(r, \dfrac{1}{\left(\frac{f}{a}\right)'}\right) + N_2\left(r, \dfrac{1}{(f^n)^{(k)}}\right) \le (\lambda + o(1))T\left(r, (f^n)^{(k)}\right)$, for $0 < \lambda < 1$, where $r \in I$ and I is a set of infinite linear measure, then $\dfrac{f - a}{(f^n)^{(k)} - a} = c$, for some constant $c \in C - \{0\}$.

Theorem: E Let $k(\ge 1), n(\ge 1)$ be integers and let f be a non-constant meromorphic function. Also let $a(z)$ $(not \equiv 0, \infty)$ be a small function with respect to f. If f and $(f^n)^{(K)}$ share a(z) IM and $(2k + 6)\Theta(\infty, f) + 3\Theta(0, f) + 2\delta_{k+2}(0, f) > 2k + 10$, or, if f and $(f^n)^{(K)}$ share a(z) CM and $(k + 3)\Theta(\infty, f) + \delta_2(0, f) + \delta_{k+2}(0, f) > k + 4$ then $f \equiv (f^n)^{(k)}$.

In this paper, we will prove the following two theorems which will include the behavior of a meromorphic function and its k th derivative sharing a small function with multiplicity not greater than l, a positive integer.

Theorem: 1 Let k, m and n are three positive integers with $m \le n$ and let f be a non-constant meromorphic function. Also let $a(z)\ (not \equiv 0,\infty)$ be a small function with respect to f. If $\overline{E}_{l)}\left(a, f^{m}(z)\right)=\overline{E}_{l)}\left(a, f^{n\,(k)}\right)$, where l is a positive integer and

$$\overline{N}(r, f)+2N_2\left(r,\frac{1}{f}\right)+2N_2\left(r,\frac{1}{\left(f^{n}\right)^{(k)}}\right)+\overline{N}\left(r,\frac{1}{\left(f^{n}\right)^{(k)}}\right)\le(\lambda+0(1))T\left(r,\left(f^{n}\right)^{(k)}\right), for\ 0<\lambda<1,$$

where $r\in I$ and I is a set of infinite linear measure, then $\dfrac{\left(f^{n}\right)^{(k)}-a}{f^{m}-a}=c$ for some constant $a\in C-\{0\}$ where C is the set of complex numbers.

Theorem: 2 Let f be a non-constant meromorphic function and let k and n be two positive integers. If $\overline{E}_{l)}(a, f)\equiv\overline{E}_{l)}(a,\left(f^{n}\right)^{(k)})$, where l is a positive integer and $a(z)\ (not \equiv 0,\infty)$ be a small function of f and $(2k+6)\Theta(\infty, f)+\Theta(0, f)+2\delta_2(0, f)+2\delta_{k+2}(0, f)>2k+10$ then
$f=\left(f^{n}\right)^{(k)}$.

2. LEMMAS

Here we mention some existing lemmas of the literature which will be frequently used to prove the aforementioned theorems.

Lemma 2.1 (see[7]): Let f be a non-constant meromorphic function and k,p be two positive integers. Then
$$N_p\left(r,\frac{1}{f^{(k)}}\right)\le N_{p+k}\left(r,\frac{1}{f}\right)+k\overline{N}(r, f)+S(r, f).$$

Lemma: 2.2 (see[4]) Let f be a non-constant meromorphic function and let n be a positive integer.
$$P(f)=a_n f^{n}+a_{n-1}f^{n-1}+........+a_1 f$$
where a_i is a meromorphic function such that $T(r,a_i)=S(r, f)\,(i=1,2,........,n)$. Then
$$T(r,P(f))=nT(r, f)+S(r, f).$$

3. PROOF OF THE THEOREMS

Proof of Theorem: 1 Let $F=\dfrac{f^{m}}{a}$ and $G=\dfrac{\left(f^{n}\right)^{(k)}}{a}$. Therefore, $F-1=\dfrac{f^{m}-a}{a}$ and $G-1=\dfrac{\left(f^{n}\right)^{(k)}-c}{a}$.
Now, $\overline{E}_{l)}\left(a, f^{m}\right)=\overline{E}_{l)}\left(a,\left(f^{n}\right)^{(k)}\right)$ except the zeros and poles of a(z). Define,
$$H=\left(\frac{F''}{F'}-\frac{2F''}{F-1}\right)-\left(\frac{G''}{G'}-\frac{2G''}{G-1}\right).$$

We now consider two cases:

Case: I Suppose $H\ not \equiv 0$. Then m(r,H) = S(r,f). Now if z_0 is a common simple zero of F-1 and G-1 (except the zeros and poles of a(z)), then after simple calculation, we get $H(z_0)=0$.

So, $\overline{N}_E\left(r,\dfrac{1}{G-1}\right)\le N\left(r,\dfrac{1}{H}\right)+S(r, f)\le T(r,H)+S(r, f)\le N(r,H)+S(r, f)$

Again by analysis, we can deduce that,
$$N(r,H)\le\overline{N}(r, f)+\overline{N}_{(2}\left(r,\frac{1}{F}\right)+\overline{N}_{(2}\left(r,\frac{1}{G}\right)+\overline{N}_L\left(r,\frac{1}{F-1}\right)+\overline{N}_L\left(r,\frac{1}{G-1}\right)+\overline{N}_*\left(r,\frac{1}{F-1}\right)$$
$$+\overline{N}_*\left(r,\frac{1}{G-1}\right)+\overline{N}_0\left(r,\frac{1}{F'}\right)+\overline{N}_0\left(r,\frac{1}{G'}\right)+S(r, f).$$

Also, $\overline{N}\left(r,\frac{1}{G-1}\right)= N_E^{1)} + \overline{N}_E^{(2}\left(r,\frac{1}{G-1}\right)+\overline{N}_L\left(r,\frac{1}{G-1}\right)+\overline{N}_L\left(r,\frac{1}{F-1}\right)+\overline{N}_*\left(r,\frac{1}{G-1}\right)+S(r,f)$.

Therefore,

$$\begin{aligned}\overline{N}\left(r,\frac{1}{G-1}\right) \le \overline{N}(r,f)+\overline{N}_{(2}\left(r,\frac{1}{F}\right)+\overline{N}_{(2}\left(r,\frac{1}{G}\right)+2\overline{N}_L\left(r,\frac{1}{F-1}\right)+2\overline{N}_L\left(r,\frac{1}{G-1}\right)\\ +2\overline{N}_L\left(r,\frac{1}{G-1}\right)+\overline{N}_E^{(2}\left(r,\frac{1}{G-1}\right)+2\overline{N}_*\left(r,\frac{1}{G-1}\right)+\overline{N}_*\left(r,\frac{1}{F-1}\right)\\ +\overline{N}_0\left(r,\frac{1}{F'}\right)+\overline{N}_0\left(r,\frac{1}{G'}\right)+S(r,f)\end{aligned} \tag{1}$$

Since, $\overline{E}_{l)}(1,F)=\overline{E}_{l)}(1,G)$,

Therefore, $2\overline{N}_L\left(r,\frac{1}{G-1}\right)+2\overline{N}_*\left(r,\frac{1}{G-1}\right)+\overline{N}_E^{(2}\left(r,\frac{1}{G-1}\right)\le 2\,\overline{N}_{(2}\left(r,\frac{1}{G-1}\right)$.

From (1), we have,

$$\begin{aligned}\overline{N}\left(r,\frac{1}{G-1}\right)\le\overline{N}(r,f)+\overline{N}_{(2}\left(r,\frac{1}{F}\right)+\overline{N}_{(2}\left(r,\frac{1}{G}\right)+2\overline{N}_{(2}\left(r,\frac{1}{G-1}\right)\\ +2\overline{N}_L\left(r,\frac{1}{F-1}\right)+\overline{N}_*\left(r,\frac{1}{F-1}\right)+\overline{N}_0\left(r,\frac{1}{F'}\right)+\overline{N}_0\left(r,\frac{1}{G'}\right)+S(r,f)\end{aligned} \tag{2}$$

We also have,

$$\overline{N}_2\left(r,\frac{1}{F}\right)+2\overline{N}_L\left(r,\frac{1}{F-1}\right)+\overline{N}_*\left(r,\frac{1}{F-1}\right)+\overline{N}_0\left(r,\frac{1}{F'}\right)\le 2\overline{N}\left(r,\frac{1}{F'}\right) \tag{3}$$

Now by the second fundamental theorem we get,

$$T(r,G)\le \overline{N}(r,G)+\overline{N}\left(r,\frac{1}{G}\right)+\overline{N}\left(r,\frac{1}{G-1}\right)-N_0\left(r,\frac{1}{G'}\right)+S(r,G) \tag{4}$$

From (4) using (2) and (3) we get,

$$T(r,G)\le 2\overline{N}(r,f)+2\overline{N}\left(r,\frac{1}{F'}\right)+2\overline{N}\left(r,\frac{1}{G'}\right)+\overline{N}\left(r,\frac{1}{G}\right)+S(r,f) \tag{5}$$

By lemma(2.1) we have,

$$T\left(r,\left(f^n\right)^{(k)}\right)\le 6\overline{N}(r,f)+2N_2\left(r,\frac{1}{f}\right)+2N_2\left(r,\frac{1}{\left(f^n\right)^{(k)}}\right)+\overline{N}\left(r,\frac{1}{\left(f^n\right)^{(k)}}\right)+S(r,f)$$

which contradicts the given conditions of the theorem.

Case: II Suppose $H(z)\equiv 0\, i.e., \frac{F''}{F'}-\frac{2F'}{F-1}=\frac{G''}{G'}-\frac{2G''}{G-1}$. Integrating we get,

$\log F'-2\log(F-1)=\log G'-\log(G-1)+\log A.$ where A is a constant $\neq 0$.

That is, $\log\frac{F'}{(F-1)^2}=\log\frac{AG'}{(G-1)^2}$.

Again integrating we get,

$$\frac{1}{F-1} = \frac{A}{G-1} + B \tag{6}$$

Now if z_0 is a pole of f with multiplicity p which is not the poles and the zeros of a(z), then z_0 is the pole of F with multiplicity mp and the pole of G with multiplicity np+k($\neq$mp). This contradicts (6). This implies f has no pole, that is f is an entire function.

So, $\overline{N}(r.F) = S(r, f)$ and $\overline{N}(r.G) = S(r, f)$. Now we prove that B = 0.

We first assume that B $\neq$ 0, then $\dfrac{1}{F-1} = \dfrac{B\left(G-1+\frac{A}{B}\right)}{G-1}$.

Therefore, $\overline{N}\left(r, \dfrac{1}{G-1+\frac{A}{B}}\right) = \overline{N}(r, F) = S(r, f)$

Now we assume $\dfrac{A}{B} \neq 1$.

By the second fundamental theorem,

$$\begin{aligned}
T(r,G) &\leq \overline{N}(r,G) + \overline{N}\left(r, \frac{1}{G}\right) + \overline{N}\left(r, \frac{1}{G-1+\frac{A}{B}}\right) + S(r,G) \\
&\leq \overline{N}\left(r, \frac{1}{G}\right) + S(r, f) \\
&\leq T(r,G) + S(r, f)
\end{aligned}$$

Hence $T(r,G) = \overline{N}\left(r, \dfrac{1}{G}\right) + S(r, f)$ i.e., $T(r, (f^n)^{(k)}) = \overline{N}\left(r, \dfrac{1}{(f^n)^{(k)}}\right) + S(r, f)$

This contradicts the given condition of the theorem.

Next, we assume $\dfrac{A}{B} = 1$. Then, (AF – A – 1) G = - 1.

So, $\dfrac{a^2}{f^n(Af^m - Aa - A)} = -\dfrac{(f^n)^{(k)}}{f^n}$

Now by lemma (2.1) and (2.2), we get,

$$\begin{aligned}
(n+m)T(r, f) &= T\left(r, \frac{(f^n)^{(k)}}{f^n}\right) + S(r, f) \\
&\leq n \ N\left(r, \frac{1}{f}\right) + k\,\overline{N}(r, f) + S(r, f) \\
&\leq nT(r, f) + S(r, f)
\end{aligned}$$

i.e., $T(r, f) = S(r, f)$. This is not true.

Hence our assumption is not true and therefore B =0. So, $\dfrac{G-1}{F-1} = A$

This proves the theorem.

Proof of the theorem: 2

Let $F=\dfrac{f(z)}{a(z)}$ and $G=\dfrac{(f^n)^{(k)}}{a(z)}$. So, $\overline{E}_{l)}(a,f)=\overline{E}_{l)}(a,(f^n)^{(k)})$ implies, $\overline{E}_{l)}(1,F)=\overline{E}_{l)}(1,G)$, except the zeros and poles of a(z).

We define, $H=\left(\dfrac{F''}{F'}-\dfrac{2F'}{F-1}\right)-\left(\dfrac{G''}{G'}-\dfrac{2G'}{G-1}\right)$.

Now we consider two cases:

Case: I Suppose H not ≡ 0.

Then (5) of the proof in theorem1 still holds. Writing (5) for the function F, we get,

$$T(r,F)\le 2\,\overline{N}(r,f)+2\,\overline{N}\left(r,\frac{1}{G}\right)+2\,\overline{N}\left(r,\frac{1}{F'}\right)+\overline{N}\left(r,\frac{1}{G}\right)+S(r,f)$$

i.e. $T(r,f)\le 2\,\overline{N}(r,f)+2\,\overline{N}_2\left(r,\dfrac{1}{(f^n)^{(k)}}\right)+2\,\overline{N}(r,f)+\overline{N}_2\left(r,\dfrac{1}{f}\right)+2\,\overline{N}(r,f)+\overline{N}\left(r,\dfrac{1}{f}\right)+S(r,f)$

$$\le (2k+6)\overline{N}(r,f)+2N_{K+2}\left(r,\frac{1}{f}\right)+2\,\overline{N}_2\left(r,\frac{1}{f}\right)+\overline{N}\left(r,\frac{1}{f}\right)+S(r,f)$$

i.e., $(2k+6)\Theta(a,f)+\Theta(0,f)+2\delta_2(0,f)+2\delta_{k+2}(0,f)\le 2k+10$

This contradicts the given condition of the theorem.

Case: II Suppose H ≡ 0.

So $\dfrac{1}{F-1}=\dfrac{A}{G-1}+B,$ where A ≠ 0, B are constant. By the same argument of the proof of theorem 1, we get,

$\overline{N}(r,F)=S(r,f)$ and $\overline{N}(r,G)=S(r,f)$.

So, $\Theta(\infty,f)=1.$

Assume that, B ≠ 0, then $\dfrac{B\left(F-1-\dfrac{1}{B}\right)}{F-1}=-\dfrac{A}{G-1}$

So, $\overline{N}\left(r,\dfrac{1}{F-1+\dfrac{1}{B}}\right)=\overline{N}(r,G)=S(r,f)$.

If B ≠ -1, then by the second fundamental theorem for F, we have

$$T(r,F)\le \overline{N}(r,F)+\overline{N}(r,\frac{1}{F})+\overline{N}\left(r,\frac{1}{F-1+\dfrac{A}{B}}\right)+S(r,f)$$

$$\le \overline{N}\left(r,\frac{1}{F}\right)+S(r,f)$$

$$\le T(r,F)+S(r,f)$$

So $T(r,F)\le \overline{N}\left(r,\dfrac{1}{F}\right)+S(r,f)$ i.e., $T(r,f)\le \overline{N}\left(r,\dfrac{1}{f}\right)$. Hence, $\Theta(0,f)=0.$

Putting $\Theta(\infty, f)=1; \Theta(0, f)=0$ and also $\delta(0, f)\le \Theta(0, f)=0$ in the given condition of the theorem we have, $\delta_{k+2}(0, f)>2,$ which is not true. Hence B = - 1.

So $\overline{N}\left(r,\frac{1}{F}\right)=S(r, f)$, i.e., $\overline{N}\left(r,\frac{1}{f}\right)=S(r, f)$. Therefore, $\frac{F}{F-1}=\frac{A}{G-1},$

i.e., F(G-1-A) = - A that is $F=\frac{A}{-G+(1+A)}.$

So, $f=\frac{A}{-(f^n)^{(k)}+(1+A)}.$ Therefore, $\overline{N}\left(r,\frac{1}{(f^n)^{(k)}+(1+A)}\right)=\overline{N}(r, f)=S(r, f).$

Hence $T(r, f)=T(r,(f^n)^{(k)})=S(r, f)$. which is also not true. Thus B = 0.

So $\frac{1}{F-1}=\frac{A}{G-1}$, i.e., G – 1 = A (F – 1).

If A ≠ 1 then $G=A\left(F-1+\frac{1}{A}\right).$ So, $N\left(r,\frac{1}{G}\right)=N\left(r,\frac{1}{F-1+\frac{1}{A}}\right).$

By the second fundamental theorem, we have,

$$T(r,F)\le \overline{N}(r,F)+\overline{N}\left(r,\frac{1}{F}\right)+\overline{N}\left(r,\frac{1}{F-1+\frac{1}{A}}\right)+S(r, f).$$

i.e.,

$$\begin{aligned}T(r, f)&\le \overline{N}\left(r,\frac{1}{F}\right)+\overline{N}\left(r,\frac{1}{G}\right)+S(r, f).\\
&=\overline{N}\left(r,\frac{1}{f}\right)+\overline{N}\left(r,\frac{1}{(f^n)^{(k)}}\right)+S(r, f).\\
&\le \overline{N}\left(r,\frac{1}{f}\right)+N_{k+1}\left(r,\frac{1}{f}\right)+k\,\overline{N}(r, f)+S(r, f)\\
&=\overline{N}\left(r,\frac{1}{f}\right)+N_{k+1}\left(r,\frac{1}{f}\right)+S(r, f)\end{aligned}$$

So,

$$\Theta(0, f)+\delta_{k+1}(0, f)\le 1 \tag{7}$$

Now by the given condition of the theorem and by (7) we have, $\Theta(0, f)>2.$ This is not possible.

So, A = 1 and hence F = G i.e., $f=(f^n)^{(k)}$.

This proves the theorem.

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