International Journal of Mathematical Archive-2(5), May - 2011, Page: 675-680
Available online through www.ijma.info ISSN 2229 – 5046

TAYLOR SERIES APPROACH FOR SOLVING CHANCE- CONSTRAINED BI-LEVEL INTEGER LINEAR FRACTIONAL PROGRAMMING PROBLEM

Omar M. Saad* and Osama E. Emam**

**Department of Mathematics, Faculty of Science, Helwan University, Cairo, Egypt*

***Department of Information Systems, Faculty of Computers & Information, Helwan University, Cairo, Egypt*

****E-mail:** omarsd@gmail.com*

(Received on: 23-04-11; Accepted on: 01-05-11)

ABSTRACT

This paper presents a solution approach to bi-level integer linear fractional programming problem with individual chance constraints (CHBLIFP). We assume that there is randomness in the right-hand side of the constraints only and that the random variables are normally distributed. The basic idea in treating (CHBLIFP) is to convert the probabilistic nature of this problem into a deterministic bi-level integer linear fractional programming problem (BLIFP). A solution of bi-level integer linear fractional programming problem is presented using a Taylor series combined with the cutting- plane algorithm till we obtain a compromise solution. A numerical example is provided to demonstrate the correctness of the proposed approach.

***Keywords**: Fractional programming; Series expansion; Management decision making; Integer programming.*

***2000 MSC No:** 90C29; 90C32; 41A58; 90C10*

1. INTRODUCTION:

Decision problems of chance constrained or stochastic optimization arise when certain coefficient of an optimization model are not fixed or known but are instead, to some extent, probabilistic quantities. In most of the real life problems in mathematical programming, the parameters are considered as random variables. The branch of mathematical programming which deals with the theory and methods for the solution of conditional extreme problems under incomplete information about the random parameters is called stochastic programming [3, 7].

In recent years methods of stochastic optimization have become increasingly important in scientifically based decision-making involved in practical problems arising in economic, industry, health care, transportation, agriculture, military purposes and technology. During the last 30 years, a special case of the multi-level programming problem, the linear bi-level programming (BLP) problem, has been studied with increasing interest in the area of mathematical programming problems [1].

Bi-level programming, a tool for modeling decentralized decisions, consists of the objective(s) of the leader at its first level and that is of the follower at the second level. Bi-level programming is characterized as mathematical programming to solve decentralized planning problems. The decision variables are partitioned among ordered levels [3, 8, and 9]. A decision-maker at one level of the hierarchy may have his own objective function and decision space, but may be influenced by other levels.

Fractional programming problem, which has been used as an important planning tool for the last four decades, is applied to different disciplines such as engineering, business, economics…etc. Fractional programming problem is generally used for modeling real life problems with objective such as profit/cost, inventory/sales, actual cost/standard cost …etc [4-6].

In literature there are many researcher have focused to solve integer linear fractional bi-level programming problem. Some of them presented an algorithm for the integer linear fractional bi-level programming problem has been proposed by Thirwani and Arora in [8]. They examined the case when the objective functions were linear fractional and presented an algorithm for solving the integer case.

***Corresponding author:** Omar M. Saad*, *E-mail:** omarsd@gmail.com

This paper is organized as follows: we start in Section 2 by formulating the model of bi-level integer linear fractional programming problem with random parameters in the right-hand side of the constraints and the solution concept is introduced. In Section 3, an equivalent bi-level linear fractional programming problem associated with the formulated problem is constructed based on finding the convex hull of the feasible integer points. In Section 4, Taylor series approach for bi-level linear fractional programming problem is suggested. In Section 5, an example is provided to illustrate the developed results. Finally, in Section 6, some open points are stated for future research work in the area of stochastic multi-level integer fractional programming optimization problems.

2. PROBLEM FORMULATION AND THE SOLUTION CONCEPT:

The bi-level integer linear fractional programming problem with random parameters in the right-hand side of the constraints (CH-BLIFP) can be stated as follows:

$$\text{(FLDM):}\quad \underset{x_1}{Max}\, F_1 = \frac{c_1^T x + \alpha_1}{d_1^T x + \beta_1} \tag{2.1}$$

where x_2 solves

$$\text{(SLDM):}\quad \underset{x_2}{Max}\, F_2 = \frac{c_2^T x + \alpha_2}{d_2^T x + \beta_2} \tag{2.2}$$

Subject to

$$x \in X, \tag{2.3}$$

where

$$X = \left\{ x \in R^n \middle| P\{\ g_i(x) = \sum_{j=1}^{n} a_{ij} x_j \le b_i\ \} \ge \alpha_i, i = 1, 2, ..., m, .\, x_j \ge 0 \text{ and integer, j} = 1, 2, ..\text{n} \right\} \tag{2.4}$$

Here X is the vector of integer decision variables and F(x) is a vector of k-linear real-valued objective functions to be maximized. Furthermore, P means probability and α_i is a specified probability value. This means that the linear constraints may be violated some of the time and at most 100(1-α_i) % of the time. For the sake of simplicity, we assume that the random parameters b_i, (i =1, 2,…, m) are distributed normally with known means E{b_i} and variances Var {b_i} and independently of each other.

Definition: 1

A point $x^* \in X$ is said to be an optimal solution to problem (CH-BLIFP) with probability $\prod_{i=1}^{m} \alpha_i$ if there does not exist another $x \in X$ such that $F_1(x) \succ F_1(x^*)$ and $F_1(x) \ne F_1(x^*)$.

The basic idea in treating problem (CH-BLIFP) is to convert the probabilistic nature of this problem into a deterministic form. Here, the idea of employing deterministic version will be illustrated by using the interesting technique of chance-constrained programming [3, 7]. In this case, the set of constraints X can be rewritten in the deterministic form as:

$$X' = \left\{ x \in R^n \middle|\ \sum_{j=1}^{n} a_{ij} x_j \le E\{b_i\} + K_{\alpha_i} \sqrt{Var\{b_i\}},\ i = 1, 2, ..., m\ \ x_j \ge 0 \text{ and integer, j} = 1, 2, ..\text{n} \right\} \tag{2.5}$$

where K_{α_i} is the standard normal value such that $\Phi(K_{\alpha_i}) = 1 - \alpha_i$; and $\Phi(a)$ represents the "cumulative distribution function" of the standard normal distribution evaluated at a. Thus, problem (CH-BLIFP) can be understood as the following deterministic version of a bi-level integer linear fractional programming problem (BLIFP):

$$\text{(FLDM):}\qquad \underset{x_1}{Max}\, F_1 = \frac{c_1^T x + \alpha_1}{d_1^T x + \beta_1}\quad , \tag{2.6}$$

where x_2 solves

$$\text{(SLDM):} \quad \underset{x_2}{Max}\, F_2 = \frac{c_2^T x + \alpha_2}{d_2^T x + \beta_2} \quad , \tag{2.7}$$

Subject to

$$x \in X'. \tag{2.8}$$

3. EQUIVALENT BI-LEVEL LINEAR FRACTIONAL PROGRAMMING PROBLEM:

In what follows, an equivalent bi-level linear fractional programming problem (BLFP) associated with problem (2.6)-(2.8) can be stated with the help of cutting-plane technique [7] together with Balinski algorithm [2]. This equivalent BLFP can be written in the following form:

$$\text{(FLDM):} \quad \underset{x_1}{Max}\, F_1 = \frac{c_1^T x + \alpha_1}{d_1^T x + \beta_1} \quad , \tag{3.1}$$

where x_2 solves

$$\text{(SLDM):} \quad \underset{x_2}{Max}\, F_2 = \frac{c_2^T x + \alpha_2}{d_2^T x + \beta_2} \quad , \tag{3.2}$$

Subject to

$$x \in [X']. \tag{3.3}$$

where $[X']$ is the convex hull of the feasible region X' defined by (2.5) earlier. This convex hull is defined by:

$$[X'] = X_R'^{(s)} = \{ x \in R^n \,|\, A^{(s)} x \le b^{(s)}, x \ge 0 \} \tag{3.4}$$

and in addition,

$$A^{(s)} = \begin{bmatrix} A \\ \cdots \\ a_1 \\ \cdot \\ \cdot \\ a_s \end{bmatrix} \quad \text{and} \quad b^{(s)} = \begin{bmatrix} b \\ \cdots \\ b_1 \\ \cdot \\ \cdot \\ b_s \end{bmatrix} \tag{3.5}$$

are the original constraint matrix A and the right-hand side vector b , respectively, with s-additional constraints each corresponding to an efficient cut in the form $a_i x \le b_i$. By an efficient cut, we mean that a cut which is not redundant, to find this convex hull $[X']$.

4. TAYLOR SERIES APPROACH FOR THE BI-LEVEL LINEAR FRACTIONAL:

Programming Problem (3.1)-(3.3)

In the bi-level integer linear fractional programming problem (BLIFP), objective functions are transformed by using Taylor series at first, and then a satisfactory value for the variables of the model is obtained by solving the model, which has a single objective function.

Here, Taylor series obtains polynomial objective functions which are equivalent to fractional objective functions. Then, the BLIFP can be reduced into a single objective combine with the cutting plane algorithm. In the compromised objective function, weight of the first level is more than weight of the second level because the first level decision maker is the center.

The proposed approach to solve a bi-level integer linear fractional programming problem can be explained as:

Step: 1 Determine $x_i^* = (x_{i1}^*, \ldots, x_{in}^*)$ which is the value that is used to maximized the i[th] objective function $F_i(x), (i = 1,2)$ where n is number of the variable.

Step: 2 Transform objective functions by using 1[st] order Taylor series polynomial series.

$$F_i(x) \cong \hat{F}_i(x) = F_i(x_i^*) + \sum_{j=1}^{n} (x_j - x_{ij}^*) \frac{\partial F_i(x_i^*)}{dx_j}, (j = 1,2)$$

Step: 3 Find the satisfactory solution by solving the reduced problem to a single objective. In the compromised objective function, weight of the first level is more than weight of the second level because the first level decision maker is the center.

5. AN ILLUSTRATIVE EXAMPLE:

The following bi-level integer linear fractional programming problem with individual constraints may be formulated as:

(FLDM)**:** $\max_{x_1} F_1 = (\frac{2x_1 + 3x_2}{x_1 + 4x_2 + 6})$,

(SLDM): $\max_{x_2} F_2 = (\frac{3x_1 + 4x_2}{6x_1 + 4x_2 + 3})$,

Subject to

$P\{2x_1 - x_2 \leq b_2\} \geq 0.95$,

$P\{-x_1 + 3x_2 \leq b_3\} \geq 0.90$,

$x_1, x_2 \geq 0$ and integers.

where bi, (i =1, 2, 3) are independent normally distributed random parameters with the following means and variances. E $\{b_1\} = 1$, E $\{b_2\} = 9$, Var $\{b_1\} = 25$, Var $\{b_2\} = 4$.

From standard normal tables, we have: $K_{\alpha 1} = 1.645,\ K_{\alpha 2} = 1.285$.

Therefore, the (CHBLIFP) can be understood as the corresponding bi-level integer linear fractional programming model written as:

(FLDM)**:** $\max_{x_1} F_1 = (\frac{2x_1 + 3x_2}{x_1 + 4x_2 + 6})$,

where x_2 solves

(SLDM)**:** $\max_{x_2} F_2 = (\frac{3x_1 + 4x_2}{6x_1 + 4x_2 + 3})$,

Subject to

$2x_1 - x_2 \leq 9.225$,

$-x_1 + 3x_2 \leq 11.57$,

$x_1, x_2 \geq 0$ and integers.

Then, the (BLIFP) can be understood as the corresponding bi-level linear fractional programming model written as:

(FLDM) **:** $\max_{x_1} F_1 = (\frac{2x_1+3x_2}{x_1+4x_2+6})$,

where x_2 solves

(SLDM): $\max_{x_2} F_2 = (\frac{3x_1+4x_2}{6x_1+4x_2+3})$,

Subject to

$$2x_1 - x_2 \leq 9.225,$$

$$-x_1 + 3x_2 \leq 11.57,$$

$$x_1 \leq 4,$$

$$x_2 \leq 3,$$

$$x_1,\ x_2 \geq 0\ .$$

If the problem is solved for each membership functions one by one then $F_1(4{,}0) = 0.8$ and $F_2(0{,}3) = 0.75$. Then, the objective functions are transformed by using 1st order Taylor polynomial series.

$$F_1(x) \cong \hat{F}_1(x) = 0.1x_1 + 0.02x_2 + 0.37 \quad , F_2(x) \cong \hat{F}_2(x) = -0.096x_1 + 0.023x_2 + 0.76 \quad .$$

Now, the final form of the BLLFP is denoted by:

$$\text{Max } 0.6\hat{F}_1(x) + 0.4\hat{F}_2(x) = 0.022x_1 + 0.021x_2 + 0.522$$

Subject to

$$2x_1 - x_2 \leq 9.225,$$

$$-x_1 + 3x_2 \leq 11.57,$$

$$x_1 \leq 4,$$

$$x_2 \leq 3,$$

$$x_1,\ x_2 \geq 0\ .$$

So, the compromise solution is $(x_1^*, x_2^*) = (4{,}0),\ F_1^* = 0.8,\ F_2^* = 0.444444$ with probability 0.855.

6. CONCLUSIONS:

In this paper, a powerful approach is based on Taylor series to solve bi-level integer linear fractional problem with individual chance constraints (CH-BLIFP). We assume that there is randomness in the right-hand sides of the constraints only and that the random variables are normally distributed. The basic idea in treating (CH-BLIFP) is to convert the probabilistic nature of this problem into a bi-level integer linear fractional problem (BLIFP). A solution of bi-level integer linear fractional problem is presented using a Taylor series combine with the cutting plane algorithm.

Certainly, there are many other points for future research in this area of stochastic multi-level integer fractional programming and should be studied. One may have to tackle the following open points for future research:

(i) Taylor series approach for solving chance constrained bi-level multiobjective integer linear fractional programming problem.

(ii)Taylor series approach for solving chance constrained multi-level multiobjective integer linear fractional programming problem.

(iii) Taylor series approach for solving chance constrained bi-level multiobjective mixed integer non-linear fractional programming problem.

REFERENCES:

[1] M. A. Abo-Sinna and I. A. Baky, Interactive balance space approach for solving multi-level multi-objective programming problems: Information Sciences. 177 (2007) 3397-3410.

[2] M. Balinski , An Algorithm For Finding All Vertices of Convex Polyhedral Sets: SIAM Journal. 9 (1961) 72-88.

[3] E. Roghanian, S.J. Sadjadi and M.B. Aryanezhad, A probabilistic bi-level linear multi-objective programming problem to supply chain planning: Applied Mathematics and Computation. 188 (1 May 2007) 786-800.

[4] O. M. Saad, A. H. Amer and E. F. Abdellah, On solving single-objective fuzzy integer linear fractional programs, Appl. Math. Inf. Sci. 4 (2010) 447-457.

[5] O. M. Saad, M. S. Biltagy and T. B. Farag, On the solution of fuzzy multiobjective integer linear fractional programming problem, Int. J. Contemp. Math. Sci. 5 (2010) 2003-2018.

[6] O. M. Saad, M. Sh. Biltagy and T. B. Farag, An algorithm for multiobjective integer nonlinear fractional problem under fuzziness , Annals of Fuzzy Mathematics and Informatics,1(2) (April 2011) 207-220.

[7] O. M. Saad, O. E. Emam, On the solution of stochastic multiobjective integer linear programming problems with a parametric study, Journal of the Advances in Computer Science, El-shorouk Institute, Cairo, Egypt, 3 (June 2009) 71-82.

[8] D. Thirwani and S.R. Arora, An algorithm for the integer linear fractional bi-level programming problem: Optimization. 39 (1997) 53-67.

[9] M. D. Toksari, Taylor series approach for bi-level linear fractional programming problem, Seluck Journal of Applied Mathematics. 11 (2010) 63-69.

****

