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# COMMON FIXED POINT THEOREMS FOR OCCASIONALLY WEAKLY COMPATIBLE MAPPINGS IN INTUITIONISTIC FUZZY METRIC SPACE

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***ABSTRACT***

*The objective of this paper is to obtain some common fixed point theorems for occasionally weakly compatible mappings in intuitionistic fuzzy metric space satisfying generalized contractive condition of integral type.*

***Keywords:*** *Occasionally weakly compatible mappings, common fixed point theorem, intuitionistic fuzzy metric space.*

***2000 Mathematics Subject Classification:*** *47H10; 54H25.*

## 1. INTRODUCTION

The study of fixed point theorems, involving four single-valued maps, began with the assumption that all of the maps are commuted. Sessa [15] weakened the condition of commutativity to that of pairwise weakly commuting. Jungck generalized the notion of weak commutativity to that of pairwise compatible [3] and then pairwise weakly compatible maps [4]. Jungck and Rhoades [6] introduced the concept of occasionally weakly compatible maps.

The concept of fuzzy set was developed extensively by many authors and used in various fields. Several authors have defined fuzzy metric space ([7] etc.) with various methods to use this concept in analysis. Recently, Park *et. al.* [8] defined the upgraded intuitionistic fuzzy metric space and Park et. al. ([9],[10],[12],[13]) studied several theories in this space. Also, Park and Kim [11] proved common fixed point theorem for self maps in intuitionistic fuzzy metric space. This paper presents some common fixed point theorems for more general commutative condition i.e. occasionally weakly compatible mappings in intuitionistic fuzzy metric space of integral type.

## 2. PRELIMINARY NOTES

**Definition: 2.1** [14]
A binary operation $*: [0,1]\times[0,1]\to[0,1]$ is a continuous $t-$norm if $*$ is satisfying conditions:
(i) $*$ is commutative and associative;
(ii) $*$ is continuous;
(iii) $a*1=a$ for all $a\in[0,1]$;
(iv) $a*b\le c*d$ whenever $a\le c$ and $b\le d$ and $a,b,c,d\in[0,1]$.

**Definition: 2.2** [14]
A binary operation $\diamond: [0,1]\times[0,1]\to[0,1]$ is a continuous $t-$conorm if $\diamond$ is satisfying conditions:
(i) $\diamond$ is commutative and associative;
(ii) $\diamond$ is continuous;
(iii) $a\diamond 0=a$ for all $a\in[0,1]$;
(iv) $a\diamond b\ge c\diamond d$ whenever $a\le c$ and $b\le d$ and $a,b,c,d\in[0,1]$.

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**Definitions: 2.3** [8] A 5-tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and M, N are fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions, for all $x, y, z \in X, s, t > 0$,

(i) $M(x, y, t) > 0$ ;
(ii) $M(x, y, t) = 1$ if and only if $x = y$;
(iii) $M(x, y, t) = M(y, x, t)$ ;
(iv) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;
(v) $M(x, y, \cdot): (0, \infty) \rightarrow (0,1]$ is continuous;
(vi) $N(x, y, t) > 0$;
(vii) $N(x, y, t) = 0$ if and only if $x = y$;
(viii) $N(x, y, t) = N(y, x, t)$;
(ix) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$;
(x) $N(x, y, \cdot): (0, \infty) \longrightarrow (0,1]$ is continuous.

Note that $(M, N)$ is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non nearness between $x$ and $y$ with respect to $t$, respectively.

**Lemma: 2.4** Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. If there exists $q \in (0,1)$ such that $M(x, y, qt) \geq M(x, y, t)$ and $N(x, y, qt) \leq N(x, y, t)$ for all $x, y \in X$ and $t > 0$, then $x = y$.

**Definition: 2.5** [1] Let $X$ be a set, $f, g$ self maps of $X$. A point $x$ in $X$ is called a coincidence point of $f$ and $g$ iff $fx = gx$. We shall call $w = fx = gx$ a point of coincidence of $f$ and $g$.

**Definition: 2.6** [5] A pair of maps $S$ and $T$ is called weakly compatible pair if they commute at coincidence points.

**Definition: 2.7**[1] Two self maps $f$ and $g$ of a set $X$ are occasionally weakly compatible (owc) iff there is a point $x$ in $X$ which is a coincidence point of $f$ and $g$ at which $f$ and $g$ commute.

Al-Thagafi and Naseer Shahzad [2] shown that occasionally weakly is weakly compatible but converse is not true.

**Definition: 2.8** [2] Let $R$ be the usual metric space. Define $S, T: R \longrightarrow R$ by $Sx = 2x$ and $Tx = x^2$ for all $x \in R$. Then $Sx = Tx$ for $x = 0,2$ but $ST0 = TS0$, and $ST2 \neq TS2$. Hence $S$ & $T$ are occasionally weakly compatible self maps but not weakly compatible.

**Lemma: 2.9** [4] Let $X$ be a set, $f, g$ owc self maps of $X$. If $f$ and $g$ have a unique point of coincidence, $w = fx = gx$, then $w$ is the unique common fixed point of $f$ and $g$.

**3. MAIN RESULTS**

In this section, we establish several common fixed point theorems for self maps on intuitionistic fuzzy metric space. Define $\varphi: R^+ \longrightarrow R$ is a Lebesgue-integrable mapping which is summable, nonnegative satisfies $\int_0^{\varepsilon} \varphi(t)dt$ for each $\varepsilon > 0$.

**Theorem: 3.1** Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and $A, B, S$ and $T$ be the self-mappings of $X$ and let the pairs $\{A, S\}$ and $\{B, T\}$ be owc. If there exists $q \in (0,1)$ such that

$$\int_0^{M(Ax,By,qt)} \varphi(t)\, dt \geq \int_0^{\alpha_1 M(Sx,Ty,t) + \alpha_2 M(Ax,Ty,t) + \alpha_3 M(By,Sx,t)} \varphi(t)\, dt \quad (1)$$

and

$$\int_0^{N(Ax,By,qt)} \varphi(t)\, dt \leq \int_0^{\alpha_4 N(Sx,Ty,t) + \alpha_5 N(Ax,Ty,t) + \alpha_6 N(By,Sx,t)} \varphi(t)\, dt \quad (2)$$

for all $x, y \in X$, where $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$ are such that $(\alpha_1 + \alpha_2 + \alpha_3) > 1$ and $(\alpha_4 + \alpha_5 + \alpha_6) < 0$. Then there exist a unique point $w \in X$ such that $Aw = Sw = w$ and a unique point $z \in X$ such that $Bz = Tz = z$. Moreover$z = w$, so that there is a unique common fixed point of $A, B, S$ and $T$.

**Proof:** Let the pairs $\{A, S\}$ and $\{B, T\}$ be owc, so there are points $x, y \in X$ such that $Ax = Sx$ and $By = Ty$. We claim that $Ax = By$. If not, by inequality (1) and (2)

$$\int_0^{M(Ax,By,qt)} \varphi(t)\, dt \geq \int_0^{\alpha_1 M(Sx,Ty,t) + \alpha_2 M(Ax,Ty,t) + \alpha_3 M(By,Sx,t)} \varphi(t)\, dt$$

$$= \int_0^{\alpha_1 M(Ax,By,t) + \alpha_2 M(Ax,By,t) + \alpha_3 M(By,Ax,t)} \varphi(t)dt$$

$$= \int_0^{(\alpha_1+\alpha_2+\alpha_3)M(Ax,By,t)} \varphi(t)dt$$

and

$$\int_0^{N(Ax,By,qt)} \varphi(t)\, dt \le \int_0^{\alpha_4 N(Sx,Ty,t) + \alpha_5 N(Ax,Ty,t) + \alpha_6 N(By,Sx,t)} \varphi(t)\, dt$$

$$= \int_0^{\alpha_4 N(Ax,By,t) + \alpha_5 N(Ax,By,t) + \alpha_6 N(By,Ax,t)} \varphi(t)dt$$

$$= \int_0^{(\alpha_4+\alpha_5+\alpha_6)N(Ax,By,t)} \varphi(t)dt$$

a contradiction, since $(\alpha_1 + \alpha_2 + \alpha_3) > 1$ and $(\alpha_4 + \alpha_5 + \alpha_6) < 0$. And by Lemma 2. 4 $Ax = By$, i.e. $Ax = Sx = By = Ty$. Suppose that there is an another point $z$ such that $Az = Sz$ then by (1) and (2) we have $Az = Sz = By = Ty$, so $Ax = Az$ and $w = Ax = Sx$ is the unique point of coincidence of $A$ and $S$. By Lemma 2.9 $w$ is the only fixed point of $A$ and $S$ i.e. $w = Aw = Sw$. Similarly there is a unique point $z \in X$ such that $z = Bz = Tz$.

Assume that $w \neq z$ . We have

$$\int_0^{M(w,z,qt)} \varphi(t)\, dt = \int_0^{M(Aw,Bz,qt)} \varphi(t)\, dt$$

$$\ge \int_0^{\alpha_1 M(Sw,Tz,t) + \alpha_2 M(Aw,Tz,t) + \alpha_3 M(Bz,Sw,t)} \varphi(t)\, dt$$

$$= \int_0^{\alpha_1 M(w,z,t) + \alpha_2 M(w,z,t) + \alpha_3 M(z,w,t)} \varphi(t)\, dt$$

$$= \int_0^{(\alpha_1+\alpha_2+\alpha_3)M(w,z,t)} \varphi(t)dt$$

and

$$\int_0^{N(w,z,qt)} \varphi(t)\, dt = \int_0^{N(Aw,Bz,qt)} \varphi(t)\, dt$$

$$\le \int_0^{\alpha_4 N(Sw,Tz,t) + \alpha_5 N(Aw,Tz,t) + \alpha_6 N(Bz,Sw,t)} \varphi(t)\, dt$$

$$= \int_0^{\alpha_4 N(w,z,t) + \alpha_5 N(w,z,t) + \alpha_6 N(z,w,t)} \varphi(t)\, dt$$

$$= \int_0^{(\alpha_4+\alpha_5+\alpha_6)N(w,z,t)} \varphi(t)dt$$

a contradiction, since $(\alpha_1 + \alpha_2 + \alpha_3) > 1$ and $(\alpha_4 + \alpha_5 + \alpha_6) < 0$. And by Lemma 2.9 $z = w$ is the unique common fixed point of $A, B, S$ and $T$. The uniqueness of the fixed point holds from (1) and (2).

**Theorem: 3.2** Let $(X, M, N,*,\diamond)$ be an intuitionistic fuzzy metric space and $A, B, S$ and $T$ be the self-mappings of $X$. Let the pairs $\{A, S\}$ and $\{B, T\}$ be owc. If there exists $q \in (0,1)$ such that

$$\int_0^{M(Ax,By,qt)} \varphi(t)\, dt \ge \int_0^{\min\{M(Sx,Ty,t),M(Ax,Sx,t),M(Ty,Ax,t),M(By,Ty,t),M(By,Sx,t)\}} \varphi(t)\, dt \quad (3)$$

and

$$\int_0^{N(Ax,By,qt)} \varphi(t)\, dt \le \int_0^{\max\{N(Sx,Ty,t),N(Ax,Sx,t),N(Ty,Ax,t),N(By,Ty,t),N(By,Sx,t)\}} \varphi(t)\, dt \quad (4)$$

for all $x, y \in X$. Then there exist a unique point $w \in X$ such that $Aw = Sw = w$ and a unique point $z \in X$ such that $Bz = Tz = z$ . Moreover, $z = w$, so that there is a unique common fixed point of $A, B, S$ and $T$.

**Proof:** Let the pairs $\{A,S\}$ and $\{B,T\}$ be owc, so there are points $x,y \in X$ such that $Ax = Sx$ and $By = Ty$. We claim that $Ax = By$. If not, by inequality (3) and (4)

$$\int_0^{M(Ax,By,qt)} \varphi(t)\,dt \geq \int_0^{\min\{M(Sx,Ty,t),M(Ax,Sx,t),M(Ty,Ax,t),M(By,Ty,t),M(By,Sx,t)\}} \varphi(t)\,dt$$

$$\geq \int_0^{\min\{M(Ax,By,t),M(Ax,Ax,t),M(By,Ax,t),M(By,By,t),M(By,Ax,t)\}} \varphi(t)\,dt$$

$$\geq \int_0^{\min\{M(Ax,By,t),1,M(By,Ax,t),1,M(By,Ax,t)\}} \varphi(t)\,dt$$

$$\geq \int_0^{\mathrm{M(Ax,By,t)}} \varphi(t)\,dt$$

and

$$\int_0^{N(Ax,By,qt)} \varphi(t)\,dt \leq \int_0^{\max\{N(Sx,Ty,t),N(Ax,Sx,t),N(Ty,Ax,t),N(By,Ty,t),N(By,Sx,t)\}} \varphi(t)\,dt$$

$$\leq \int_0^{\max\{N(Ax,By,t),N(Ax,Ax,t),N(By,Ax,t),N(By,By,t),N(By,Ax,t)\}} \varphi(t)\,dt$$

$$\leq \int_0^{\max\{N(Ax,By,t),0,N(By,Ax,t),0,N(By,Ax,t)\}} \varphi(t)\,dt$$

$$\leq \int_0^{N(Ax,By,t)} \varphi(t)\,dt$$

a contradiction , and by Lemma 2.4 $Ax = By$, i.e. $Ax = Sx = By = Ty$. Suppose that there is an another point $z$ such that $Az = Sz$ then by (3) and (4) we have $Az = Sz = By = Ty$, so $Ax = Az$ and $w = Ax = Sx$ is the unique point of coincidence of $A$ and $S$. By Lemma 2.9 $w$ is the only fixed point of $A$ and $S$ i.e. $w = Aw = Sw$. Similarly there is a unique point $z \in X$ such that $z = Bz = Tz$.

Assume that $w \neq z$ . We have

$$\int_0^{M(w,z,qt)} \varphi(t)\,dt = \int_0^{M(Aw,Bz,qt)} \varphi(t)\,dt$$

$$\geq \int_0^{\min\{M(Sw,Tz,t),M(Aw,Sw,t),M(Tz,Aw,t),M(Bz,Tz,t),M(Bz,Sw,t)\}} \varphi(t)\,dt$$

$$\geq \int_0^{\min\{M(w,z,t),M(w,w,t),M(z,w,t),M(z,z,t),M(z,w,t)\}} \varphi(t)\,dt$$

$$\geq \int_0^{\min\{M(w,z,t),1,M(z,w,t),1,M(z,w,t)\}} \varphi(t)\,dt$$

$$\geq \int_0^{M(w,z,t)} \varphi(t)\,dt$$

and

$$\int_0^{N(w,z,qt)} \varphi(t)\,dt = \int_0^{N(Aw,Bz,qt)} \varphi(t)\,dt$$

$$\leq \int_0^{\max\{N(Sw,Tz,t),N(Aw,Sw,t),N(Tz,Aw,t),N(Bz,Tz,t),N(Bz,Sw,t)\}} \varphi(t)\,dt$$

$$\leq \int_0^{\max\{N(w,z,t),N(w,w,t),N(z,w,t),N(z,z,t),N(z,w,t)\}} \varphi(t)\,dt$$

$$\leq \int_0^{\max\{N(w,z,t),1,N(z,w,t),1,N(z,w,t)\}} \varphi(t)\,dt$$

$$\leq \int_0^{N(w,z,t)} \varphi(t)\,dt$$

a contradiction. And by Lemma 2.9 $z = w$ is the unique common fixed point of $A,B,S$ and $T$. The uniqueness of the fixed point holds from (3) and (4).

**Theorem: 3.3** Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and $A, B, S$ and $T$ be the self-mappings of $X$. Let the pairs $\{A, S\}$ and $\{B, T\}$ be owc. If there exists $q \in (0,1)$ such that

$$\int_0^{M(Ax,By,qt)} \varphi(t)\, dt \geq \int_0^{\alpha \min\{M(Sx,Ty,t),M(Sx,Ax,t)\}+\beta \min\{M(By,Ty,t),M(Ax,Ty,t)\}+\gamma M(By,Sx,t)} \varphi(t)\, dt \quad (5)$$

and

$$\int_0^{N(Ax,By,qt)} \varphi(t) d \leq \int_0^{\mu \max\{N(Sx,Ty,t),N(Sx,Ax,t)\}+\vartheta \max\{N(By,Ty,t),M(Ax,Ty,t)\}+\partial N(By,Sx,t)} \varphi(t)\, dt \quad (6)$$

for all $x, y \in X$, where $(\alpha + \beta + \gamma) > 1$ $and$ $(\mu + \vartheta + \partial) < 0$. Then there exist a unique point $w \in X$ such that $Aw = Sw = w$ and a unique point $z \in X$ such that $Bz = Tz = z$. Moreover, $z = w$, so that there is a unique common fixed point of $A, B, S$ and $T$.

**Proof:** Let the pairs $\{A, S\}$ and $\{B, T\}$ be owc, so there are points $x, y \in X$ such that $Ax = Sx$ and $By = Ty$. We claim that $Ax = By$. If not, by inequality (5) and (6)

$$\int_0^{M(Ax,By,qt)} \varphi(t)\, dt \geq \int_0^{\alpha \min\{M(Sx,Ty,t),M(Sx,Ax,t)\}+\beta \min\{M(By,Ty,t),M(Ax,Ty,t)\}+\gamma M(By,Sx,t)\}} \varphi(t)\, dt$$

$$= \int_0^{\alpha \min\{M(Ax,By,t),M(Ax,Ax,t)\}+\beta \min\{M(By,By,t),M(Ax,By,t)\}+\gamma M(By,Ax,t)\}} \varphi(t)\, dt$$

$$= \int_0^{\alpha \min\{M(Ax,By,t),1\}+\beta \min\{1,M(Ax,By,t)\}+\gamma M(Ax,By,t)\}} \varphi(t)\, dt$$

$$= \int_0^{(\alpha+\beta+\gamma)M(Ax,By,t)} \varphi(t)\, dt$$

$$> \int_0^{M(Ax,By,t)} \varphi(t) dt$$

and

$$\int_0^{N(Ax,By,qt)} \varphi(t) dt \leq \int_0^{\mu \max\{N(Sx,Ty,t),N(Sx,Ax,t)\}+\vartheta \max\{N(By,Ty,t),M(Ax,Ty,t)\}+\partial N(By,Sx,t)} \varphi(t)\, dt$$

$$= \int_0^{\mu \max\{N(Ax,By,t),N(Ax,Ax,t)\}+\vartheta \max\{N(By,By,t),N(Ax,By,t)\}+\partial N(By,Ax,t)\}} \varphi(t)\, dt$$

$$= \int_0^{\mu \max\{N(Ax,By,t),1\}+\vartheta \max\{1,N(Ax,By,t)\}+\partial N(Ax,By,t)\}} \varphi(t)\, dt$$

$$= \int_0^{(\mu+\vartheta+\partial)N(Ax,By,t)} \varphi(t)\, dt$$

$$< \int_0^{N(Ax,By,t)} \varphi(t) dt$$

a contradiction, since $(\alpha + \beta + \gamma) > 1$ and $(\mu + \vartheta + \partial) < 0$. And by Lemma 2.4 $Ax = By, i.e.\ Ax = Sx = By = Ty$. Suppose that there is another point $z$ such that $Az = Sz$ then by (5) and (6) we have $Az = Sz = By = Ty$, so $Ax = Az$ and $w = Ax = Sx$ is the unique point of coincidence of $A$ and $S$. By Lemma 2.9 $w$ is the only fixed point of $A$ and $S$ i.e. $w = Aw = Sw$. Similarly there is a unique point $z \in X$ such that $z = Bz = Tz$.

Assume that $w \neq z$. We have

$$\int_0^{M(w,z,qt)} \varphi(t) dt = \int_0^{M(Aw,Bz,qt)} \varphi(t)\, dt \geq \int_0^{\alpha \min\{M(Sw,Tz,t),M(Sw,Az,t)\}+\beta \min\{M(Bz,Tz,t),M(Aw,Tz,t)\}+\gamma M(Bz,Sw,t)\}} \varphi(t)\, dt$$

$$= \int_0^{\alpha \min\{M(w,z,t),1\}+\beta \min\{1,M(w,z,t)\}+\gamma M(w,z,t)\}} \varphi(t)\, dt$$

$$= \int_0^{(\alpha+\beta+\gamma)M(w,z,t)} \varphi(t)\, dt$$

$$> \int_0^{M(w,z,t)} \varphi(t) dt$$

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and

$$\int_0^{N(Ax,By,qt)} \varphi(t)dt \le \int_0^{\mu \max\{N(Sw,Tz,t),N(Sw,Aw,t)\}+\vartheta \max\{N(Bz,Tz,t),M(Aw,Tz,t)\}+\partial N(Bz,Sw,t)} \varphi(t)\,dt$$

$$= \int_0^{\mu \max\{N(w,z,t),1\}+\vartheta \max\{1,N(w,z,t)\}+\partial N(w,z,t)} \varphi(t)\,dt$$

$$= \int_0^{(\mu+\vartheta+\partial)N(w,z,t)} \varphi(t)\,dt$$

$$< \int_0^{N(w,z,t)} \varphi(t)dt$$

a contradiction, since $(\alpha+\beta+\gamma)>1$ and $(\mu+\vartheta+\partial)<0$. And by Lemma 2.9 $z = w$. Also by Lemma 2.9 $z = w$ is the unique common fixed point of $A, B, S$ and $T$. Therefore the uniqueness of the fixed point holds from (5) and (6).

**Theorem: 3.4** Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and $A, B, S$ and $T$ be the self-mappings of $X$. Let the pairs $\{A, S\}$ and $\{B, T\}$ be owc. If there exists $q \in (0,1)$ such that

$$\int_0^{M(Ax,By,qt)} \varphi(t)\,dt \ge \int_0^{\xi\{M(Sx,Ty,t),M(Sx,Ax,t),M(By,Ty,t),M(Ax,Ty,t),M(By,Sx,t)\}} \varphi(t)\,dt \tag{7}$$

and

$$\int_0^{N(Ax,By,qt)} \varphi(t)\,dt \le \int_0^{\psi\{N(Sx,Ty,t),N(Sx,Ax,t),N(By,Ty,t),N(Ax,Ty,t),N(By,Sx,t)\}} \varphi(t)\,dt \tag{8}$$

for all $x, y \in X$, and $\xi: [0,1]^5 \longrightarrow [0,1]$ *and* $\psi: [0,1]^5 \longrightarrow [0,1]$ such that $\xi(t,1,1,t,t) > t$ *and* $\psi(t,0,0,t,t) < t$ for all $0 < t < 1$. Then there exist a unique point $w \in X$ such that $Aw = Sw = w$ and a unique point $z \in X$ such that $Bz = Tz = z$. Moreover, $z = w$, so that there is a unique common fixed point of $A, B, S$ and $T$.

**Theorem: 3.5** Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and A, B, S and T be the self-mappings of X. Let the pairs $\{A, S\}$ and $\{B, T\}$ be owc. If there exists $q \in (0,1)$ such that

$$\int_0^{M(Ax,By,qt)} \varphi(t)\,dt \ge \int_0^{\xi\{M(Sx,Ty,t),M(Ax,Ty,t),M(By,Sx,t)\}} \varphi(t)\,dt \tag{9}$$

and

$$\int_0^{N(Ax,By,qt)} \varphi(t)\,dt \le \int_0^{\psi\{N(Sx,Ty,t),N(Ax,Ty,t),N(By,Sx,t)\}} \varphi(t)\,dt \tag{10}$$

for all x, y∈X, and $\xi: [0,1]^3 \longrightarrow [0,1]$ *and* $\psi: [0,1]^3 \longrightarrow [0,1]$ such that $\xi(t,t,t) > t$ *and* $\psi(t,t,t) < t$ for all $0 < t < 1$. Then there exist a unique point $w \in X$ such that $Aw = Sw = w$ and a unique point $z \in X$ such that $Bz = Tz = z$. Moreover, $z = w$, so that there is a unique common fixed point of $A, B, S$ and $T$.

**Theorem: 3.6** Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and $A, B, S$ and $T$ be the self-mappings of $X$. Let the pairs $\{A, S\}$ and $\{B, T\}$ be owc. If there exists $q \in (0,1)$ such that

$$\int_0^{M(Ax,By,qt)} \varphi(t)\,dt \ge \int_0^{\xi\min\{M(Sx,Ty,t),M(Sx,Ax,t)M(By,Ty,t),M(Ax,Ty,t),M(By,Sx,t)\}} \varphi(t)\,dt \tag{11}$$

and

$$\int_0^{N(Ax,By,qt)} \varphi(t)\,dt \le \int_0^{\psi\max\{N(Sx,Ty,t),N(Sx,Ax,t),N(By,Ty,t)N(Ax,Ty,t),N(By,Sx,t)\}} \varphi(t)\,dt \tag{12}$$

for all $x, y \in X$, and $\xi: [0,1] \longrightarrow [0,1]$ *and* $\psi: [0,1] \longrightarrow [0,1]$ such that $\xi(t) > t$ *and* $\psi(t) < t$ for all $0 < t < 1$. Then there exist a unique point $w \in X$ such that $Aw = Sw = w$ and a unique point $z \in X$ such that $Bz = Tz = z$. Moreover, $z = w$, so that there is a unique common fixed point of $A, B, S$ and $T$.

**Theorem: 3.7** Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and $A, B, S$ and $T$ be the self-mappings of $X$ and let the pairs $\{A, S\}$ and $\{B, T\}$ be owc. If there exists $q \in (0,1)$ such that

$$\int_0^{M(Ax,By,qt)} \varphi(t)\,dt \ge \int_0^{M(Sx,Ty,t)} \varphi(t)\,dt \tag{13}$$

and

$$\int_0^{N(Ax,By,qt)} \varphi(t)\,dt \le \int_0^{N(Sx,Ty,t)} \varphi(t)\,dt \tag{14}$$

for all $x, y \in X$. Then there exist a unique point $w \in X$ such that $Aw = Sw = w$ and a unique point $z \in X$ such that $Bz = Tz = z$. Moreover, $z = w$, so that there is a unique common fixed point of $A, B, S$ and $T$.

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