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b-OPEN SETS AND t-OPEN SETS IN BITOPOLOGICAL SPACES

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ABSTRACT

The purpose of this paper is to characterize b-open sets in bitopological spaces. The concepts of b_t-open sets and t-open sets are also introduced in bitopological spaces and they are studied with existing concepts in bitopological spaces.

Keywords: *Bitopology, b-open sets, t-open sets, p-set, q-set etc.*

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1. introduction and preliminaries:

Abo Khadra and Nasef [1] discussed b-open sets in bitopological spaces. In this paper we further characterize b-open sets in bitopological spaces. We also introduce the notions of b_t-open sets and t-open sets in bitopological spaces and investigate their basic properties. Throughout this paper (X, τ_1, τ_2) denotes a bitopological space, i, j=1, 2 and $i \neq j$. Let A be a subset of X. We use the following notations.

(i) i-clA = the closure of A with respect to the topology τ_i.

(ii) i-intA = the interior of A with respect to the topology τ_i.

(iii) A is open with respect to τ_i if and only if A is i-open in (X, τ_1, τ_2).

(iv) A is closed with respect to τ_i if and only if A is i-closed in (X, τ_1, τ_2).

Definition: 1.1

A is called
(i) ij-semi-open in (X, τ_1, τ_2) if there exists an i-open set U with $U \subseteq A \subseteq j\text{-}cl U$, [8]

(ii) ij-pre-open in(X, τ_1, τ_2) if there exists an i-open set U with $A \subseteq U \subseteq j\text{-}cl A$, [7]

(iii) ij-b-open in (X, τ_1, τ_2) if $A \subseteq j\text{-}cl(i\text{-}int A) \cup i\text{-}int(j\text{-}cl A)$, [1]

(iv) an i-p-set if $i\text{-}cl(i\text{-}int A) \subseteq i\text{-}int(i\text{-}cl A)$, [11]

(v) an ij-p-set if $i\text{-}cl(j\text{-}int A) \subseteq i\text{-}int(j\text{-}cl A)$, [6]

(vi) a contra ij-p-set in (X, τ_1, τ_2) if $i\text{-}cl(j\text{-}int A) \subseteq j\text{-}int(i\text{-}cl A)$, [13]

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(vii) an i-q-set if $i\text{-}int(i\text{-}cl A) \subseteq i\text{-}cl(i\text{-}int A)$, [12]

(viii) an ij-q-set if $i\text{-}int(j\text{-}cl A) \subseteq i\text{-}cl(j\text{-}int A)$, [13]

(ix) a pair wise contra p-set in (X, τ_1, τ_2) if it is a contra 12-p-set and a contra 21-p-set, [13]

(x) a contra ij-q-set in (X, τ_1, τ_2) if $i\text{-}int(j\text{-}cl A) \subseteq j\text{-}cl(i\text{-}int A)$, [13]

(xi) a pair wise contra q-set in (X, τ_1, τ_2) if it is a contra 12-q-set and a contra 21-q-set. [13]

The complement of an ij-b-open set is ij-b-closed. Also ij-semi-closed and ij-pre-closed sets can be analogously defined. The results in the following lemma follow immediately from the definitions.

Lemma: 1.2
Let A be a subset of (X, τ_1, τ_2). Then A is

(a) ij-semi-open if and only if $A \subseteq j\text{-}cl(i\text{-}int A)$,

(b) ij-pre-open if and only if $A \subseteq i\text{-}int(j\text{-}cl A)$,

(c) ij-b-closed if and only if $j\text{-}int(i\text{-}cl A) \cap i\text{-}cl(j\text{-}int A) \subseteq A$.

The concepts of $i\text{-}sint\, A$, $i\text{-}pint A$, $ij\text{-}sint A$, $ij\text{-}scl A$ and $ij\text{-}pcl A$ can be defined in a usual way.

Lemma: 1.3
Let A be a subset of (X, τ_1, τ_2). Then

(i) $i\text{-}sint\, A = A \cap i\text{-}cl(i\text{-}int A)$, (ii) $i\text{-}pint A = A \cap i\text{-}int(i\text{-}cl A)$. [2]

Lemma: 1.4
Let A be a subset of (X, τ_1, τ_2). Then

(i) $ij\text{-}sint A = A \cap j\text{-}cl(i\text{-}int A)$, (ii) $ij\text{-}scl A = A \cup j\text{-}int(i\text{-}cl A)$. [10]

Definition: 1.5
Let (X, τ) be a topological space. Let A and B be any two subsets of X. We say that (i) A is near to B in (X, τ) if $int A = int B$ and (ii) A is closer to B in (X, τ) if $cl A = cl B$. [9]

Lemma: 1.6
If A is closer to $A \cap i\text{-}int(j\text{-}cl A)$ in (X, τ_j) then $A \cap i\text{-}int(j\text{-}cl A) = ij\text{-}pint A$. [9]

Lemma: 1.7
If A is near to $A \cup i\text{-}cl(j\text{-}int A)$ in (X, τ_j) then $A \cup i\text{-}cl(j\text{-}int A) = ij\text{-}pcl A$. [9]

Definition: 1.8
Let (X, τ_1, τ_2) be a bitopological space. Then

(i) τ_1 is coupled to τ_2 if $1\text{-}cl U \subseteq 2\text{-}cl U$ for every $U \in \tau_1$, [14]

(ii) τ_1 is near τ_2 if $1\text{-}cl U \subseteq 2\text{-}cl U$ for every $U \in \tau_2$. [4,5]

Lemma: 1.9
In a bitopological space (X, τ_1, τ_2), the following are equivalent.

(i) τ_1 is coupled to τ_2.

(ii) $2\text{-}int A \subseteq 1\text{-}int A$ for every 1-closed set A in (X, τ_1, τ_2),

(iii) 1-cl(1-intA) $\subseteq$ 2-cl(1-intA) for every subset A of X,

(iv) 2-int(1-clA) $\subseteq$ 1-int(1-clA) for every subset A of X. [13]

Lemma: 1.10
In a bitopological space (X, τ_1, τ_2), the following are equivalent.

(i) τ_1 is near τ_2.

(ii) 2-intA $\subseteq$ 1-intA for every 2-closed set A in (X, τ_1, τ_2),

(iii) 1-cl(2-intA) $\subseteq$ 2-cl(2-intA) for every subset A of X,

(iv) 2-int(2-clA) $\subseteq$ 1-int(2-clA) for every subset A of X. [13]

Lemma: 1.11
Let B be a subset of (X, τ_1, τ_2). Then B is a contra *ij*-q-set in (X, τ_1, τ_2) if and only if X\B is a contra *ji*-q-set in (X, τ_1, τ_2). [13]

Lemma: 1.12
If A is a contra *ij*-p-set and an *ij*-q- set then *i*-int(*j*-clA) $\subseteq$ *j*-int(*i*-clA). [13]

Lemma: 1.13.
If A is both an *ij*-p-set and a contra *ji*-q-set then *j*-int(*i*-clA) $\subseteq$ *i*-int(*j*-clA). [13]

2. *ij*-b-open sets:

Andrijevic [3] introduced the concept of b-open sets in unital topological spaces and Abo Khadra and Nasef [1] extended this notion to bitopological spaces. In this section we characterize *ij*-b-open sets using contra *ij*-p-sets, contra *ij*-q-sets and the corresponding pair wise sets. The concept of pair wise b-open sets is also introduced and studied in this section.

Proposition: 2.1
Let A be *ij*-b-open and a contra *ji*-p-set in (X, τ_1, τ_2). Then it is *ij*-pre-open.

Proof: Since A is *ij*-b-open in (X, τ_1, τ_2), by Definition **1**.1(iii), A $\subseteq$ *j*-cl(*i*-intA) $\cup$ *i*-int(*j*-clA). Since A is a contra *ji*-p-set, using Definition 1.1(vi), *j*-cl(*i*-intA) $\subseteq$ *i*-int(*j*-clA). This implies that A $\subseteq$ *i*-int(*j*-clA) so that A is *ij*-pre-open.

Corollary: 2.2
If A is *ij*-b-closed and a contra *ij*-p-set in (X, τ_1, τ_2) then it is *ij*-pre-closed.

Proof: Suppose A is *ij*-b-closed and a contra *ij*-p-set in (X, τ_1, τ_2). Then X\A is *ij*-b-open and is a contra *ji*-p-set. Then using Proposition 2.1, X\A is *ij*-pre-open that implies A is *ij*-pre-closed.

Corollary: 2.3
If A is *ij*-b-clopen and a pair wise contra p-set in (X, τ_1, τ_2) then it is *ij*-pre-clopen.

Proof: Follows from Proposition 2.1 and Corollary 2.2.

Proposition: 2.4
If A is *ij*-b-open and a contra *ij*-q-set in (X, τ_1, τ_2) then it is a *ij*-semi-open set.

Proof: Since A is *ij*-b-open in (X, τ_1, τ_2), by Definition 1.1(iii), A $\subseteq$ *j*-cl(*i*-intA) $\cup$ *i*-int(*j*-clA). Since A is a contra *ij*-q-set, by Definition 1.1(x), *i*-int(*j*-clA) $\subseteq$ *j*-cl(*i*-intA). This implies that A $\subseteq$ *j*-cl(*i*-intA). Therefore A is *ij*-semi-open.

Corollary: 2.5
If A is *ij*-b-closed and a contra *ji*-q-set in (X, τ_1, τ_2) then it is a *ij*-semi-closed.

Proof: Follows from Proposition 2.4 and Lemma 1.11.

Corollary: 2.6
If A is *ij*-b-clopen and a pair wise contra-q-set in (X, τ_1, τ_2) then it is a *ij*-semi-clopen.
Proof: Follows from Proposition 2.4 and Corollary 2.5.

Proposition: 2.7
If A is *ij*-b-open in (X, τ_1, τ_2) and A is closer to $A \cap i\text{-}int(j\text{-}cl A)$ in (X, τ_j) then
$A = ij\text{-}sint A \cup ij\text{-}pint A$.

Proof: Suppose A is *ij*-b-open in (X, τ_1, τ_2). Then $A \subseteq j\text{-}cl(i\text{-}int A) \cup i\text{-}int(j\text{-}cl A)$ so that $A = A \cap (j\text{-}cl(i\text{-}int A) \cup i\text{-}int(j\text{-}cl A)) = (A \cap j\text{-}cl(i\text{-}int A)) \cup (A \cap i\text{-}int(j\text{-}cl A))$.
Then by using Proposition 1.4 (i) and lemma 1.6 we see that $A = ij\text{-}sint A \cup ij\text{-}pint A$,

Proposition: 2.8
If A is *ij*-b-closed in (X, τ_1, τ_2) and A is near to $A \cup i\text{-}cl(j\text{-}int A)$ in (X, τ_j) then $A = ij\text{-}scl A \cap ij\text{-}pcl A$.

Proof: Suppose A is *ij*-b-closed in (X, τ_1, τ_2). Then $j\text{-}int(i\text{-}cl A) \cap i\text{-}cl(j\text{-}int A) \subseteq A$. Therefore
$A = A \cup (j\text{-}int(i\text{-}cl A) \cap i\text{-}cl(j\text{-}int A)) = (A \cup j\text{-}int(i\text{-}cl A)) \cap (A \cup i\text{-}cl(j\text{-}int A))$. Then by using Lemma 1.4(ii) and Lemma 1.7 we have $A = ij\text{-}scl A \cup ij\text{-}pcl A$,

Definition: 2.9
A subset B of a bitopological space (X, τ_1, τ_2) is called pair wise b-open in (X, τ_1, τ_2) if B is 12-b-open and 21-b-open.

The next proposition and the subsequent corollaries follow respectively from Proposition 2.4, Corollary 2.5 and Corollary 2.6.

Proposition: 2.10
If A is pair wise b-open and a pair wise contra q-set in (X, τ_1, τ_2) then it is pair wise semi-open.

Corollary: 2.11
If A is pair wise b-closed and a pair wise contra q-set in (X, τ_1, τ_2) then it is pair wise semi-closed.

Corollary: 2.12
If A is pair wise b-clopen and a pair wise contra-q-set in (X, τ_1, τ_2) then it is pair wise semi-clopen.

3. *ij*-b_t-open sets:

In this section the concepts of b_t-open sets and pair wise b_t-open sets in bitopological spaces are introduced and their properties are investigated.

Definition: 3.1
A subset B of a bitopological space (X, τ_1, τ_2) is called ij-b_t-open in (X, τ_1, τ_2) if $B \subseteq j\text{-}cl(i\text{-}int B) \cup j\text{-}int(i\text{-}cl B)$.
The next proposition follows from Lemma 1.2 and Definition 3.1.

Proposition: 3.2

(i) Every *ij*-semi-open set is *ij*-b_t-open.

(ii) Every *ji*-pre-open set is *ij*-b_t-open.

The next lemma can be easily proved.

Lemma: 3.3
$B \subseteq j\text{-}cl(i\text{-}int B) \cup j\text{-}int(i\text{-}cl B)$ if and only if $j\text{-}cl(i\text{-}int(X \setminus B)) \cap j\text{-}int(i\text{-}cl(X \setminus B)) \subseteq X \setminus B$.

Proposition: 3.4
If A is a *ji*-q-set and *ij*-b_t-open then it is *ij*-semi-open.

Proof: Suppose A is a *ji*-q-set and *ij*-b_t-open. Then by Definition 1.1(viii) and by Definition 3.1, we get j-$int(i$-clA$)$ $\subseteq$ j-$cl(i$-intA$)$ and A $\subseteq$ j-$cl(i$-intA$)$ $\cup$ j-$int(i$-clA$)$. This implies that A $\subseteq$ j-$cl(i$-intA$)$. Therefore A is *ij*-semi-open.

Proposition: 3.5
If A is a *ji*-p-set and *ij*-b_t-open then it is *ji*-pre-open.

Proof: Suppose A is a *ji*-p-set and *ij*-b_t-open. Then by Definition 1.1(v) and by Definition 3.1, we get j-$int(i$-clA$)$ $\supseteq$ j-$cl(i$-intA$)$ and A $\subseteq$ j-$cl(i$-intA$)$ $\cup$ j-$int(i$-clA$)$. This implies that A $\subseteq$ j-$int(i$-clA$)$. Therefore A is *ji*-pre-open

Proposition: 3.6
Let B be *ij*-b_t-open in (X, τ_1, τ_2) and let B be closer to B $\cap$ j-$int(i$-clB$))$ in (X, τ_i). Then B = *ij*-*sint*B $\cup$ *ji*-*pint*B.

Proof: B = B $\cap$ (j-cl(i-intB) $\cup$ j-int(i-clB)) = (B $\cap$ j-cl(i-intB)) $\cup$ (B$\cap$$j$-$int$($i$-$cl$B)). Then by using Lemma 1.4(i) and Lemma 1.6 we see that B= *ij*-*sint*B $\cup$ *ji*-*pint*B,

Proposition: 3.7
Suppose A is *ij*-b-open, a contra *ij*-p-set and an *ij*-q-set. Then A is *ij*-b_t-open.

Proof: Since A is *ij*-b-open , by Definition 1.1(iii), A $\subseteq$ j-cl(i-intA) $\cup$ i-int(j-clA).Since A is a contra *ij*-p-set and an *ij*-q-set, by Lemma 1.12, i-int(j-clA) $\subseteq$ j-int(i-clA) that gives A $\subseteq$ j-cl(i-intA) $\cup$ j-int(i-clA).

Then by Definition 3.1, A is *ij*-b_t-open.

Proposition: 3.8
Suppose A is *ji*-b-open, an *ij*-p-set and a contra *ji*-q-set. Then A is *ji*-b_t-open.

Proof: Since A is *ji*-b-open , by Definition 1.1(iii), A $\subseteq$$i$-$cl$($j$-$int$A) $\cup$ j-int(i-clA). Since A is an *ij*-p-set and a contra *ji*-q-set, by Lemma 1.13. j-int(i-clA) $\subseteq$ i-int(j-clA) that gives A $\subseteq$ i-cl(j-intA) $\cup$ i-int(j-clA).

Then by Definition 3.1, A is *ji*-b_t-open.

The complement of an *ij*-b_t-open set is *ij*-b_t-closed. It follows from Lemma 3.3 that B is *ij*-b_t-closed if and only if the relation j-cl(i-intB) $\cap$ j-int(i-clB) $\subseteq$ B holds. The next proposition follows from Proposition 3.2, Proposition 3.4 and Proposition 3.5.

Proposition: 3.9
(i) Every *ij*-semi-closed set is *ij*-b_t-closed.

(ii) Every *ji*-pre-closed set is *ij*-b_t-closed.

(iii) If A is a *ji*-q-set and *ij*-b_t-closed then it is *ij*-semi-closed.

(iv) If A is a *ji*-p-set and *ij*-b_t-closed then it is *ji*-pre-closed.

Proposition: 3.10
Suppose B is *ij*-b_t-closed, *ij*-semi-open and *ji*-pre-open in (X, τ_1, τ_2). Then B = j-cl(i-intB) $\cap$ j-int(i-clB).

Proof: Since B is *ij*-b_t-closed, j-cl(i-intB) $\cap$ j-int(i-clB) $\subseteq$ B. Therefore B = B $\cup$ (j-cl(i-intB) $\cap$ j-int(i-clB)) = (B $\cup$ j-cl(i-intB)) $\cap$ (B $\cup$ j-int(i-clB)) = j-cl(i-intB) $\cap$ j-int(i-clB), using Lemma 1.2 (a) and Lemma 1.2(b).

Definition: 3.11
A subset B of a bitopological space (X, τ_1, τ_2) is called pair wise b_t-open in (X, τ_1, τ_2) if B is 12-b_t-open and 21-b_t-open.

The following proposition follows from Proposition 3.2 and Proposition 3.4.

Proposition: 3.12

(i) Every pair wise semi-open set is pair wise b_t-open.

(ii) Every pair wise pre-open set is pair wise b_t-open.
(iii) If A is a pair wise q-set and pair wise b_t-open then it is pair wise semi-open.

Proposition: 3.13
If A is a pair wise p-set and pair wise b_t-open then it is pair wise pre-open.

Proof: Suppose A is a pair wise p-set and pair wise b_t-open set.
Since A is a *ji*-p-set and an *ij*-b_t-open set, by Proposition 3.5, A is *ji*-pre-open. Since A is an *ij*-p-set and a *ji*-b_t-open set, again by Proposition 3.5, A is *ij*-pre-open. Therefore A is pair wise pre-open.

4. *ij*-t-open sets:

In this section the notion of *ij*-t-open sets is introduced in bitopological spaces and their relationships with p-sets, q-sets, b-open sets, semi-open sets and pre-open sets are studied.

Definition: 4.1
A subset A of a bitopological space (X, τ_1, τ_2) is called *ij*-t-open if $A \subseteq i\text{-}cl(i\text{-}int A) \cup j\text{-}int(j\text{-}cl A)$.

Proposition: 4.2
If A is an *ij*-t-open set then A is a union of an *i*-semi-open set and a *j*-pre-open set.

Proof: Suppose A is an *ij*-t-open set. Then by Definition 4.1, $A \subseteq i\text{-}cl(i\text{-}int A) \cup j\text{-}int(j\text{-}cl A)$.
Therefore $A = A \cap (i\text{-}cl(i\text{-}int A) \cup j\text{-}int(j\text{-}cl A)) = (A \cap i\text{-}cl(i\text{-}int A)) \cup (A \cap j\text{-}int(j\text{-}cl A) = i\text{-}sint A \cup j\text{-}pint A$, by Lemma 1.3(i) and Lemma 1.3(ii).

Proposition: 4.3
(i) Every *i*-semi-open set is *ij*-t-open,

(ii) Every *j*-pre-open set is *ij*-t-open.

Proof: Let A be *i*-semi-open. Then $A \subseteq i\text{-}cl(i\text{-}int A)$.Therefore $A \subseteq i\text{-}cl(i\text{-}int A) \cup j\text{-}int(j\text{-}cl A)$. Then by Definition 4.1, A is *ij*-t-open. Now let A be *j*-pre-open. Then $A \subseteq j\text{-}int(j\text{-}cl A)$. Therefore $A \subseteq i\text{-}cl(i\text{-}int A) \cup j\text{-}int(j\text{-}cl A)$. Then by Definition 4.1, A is *ij*-t-open.

Proposition: 4.4
Suppose A is an *i*-p-set and a *j*-q-set. Then if A is *ij*-t-open then it is *ji*-t-open.

Proof: Since A is an *i*-p-set, using Definition 1.1(iv), $i\text{-}cl(i\text{-}int A) \subseteq i\text{-}int(i\text{-}cl A)$. Since A is a *j*-q-set, using Definition 1.1(vii), $j\text{-}int(j\text{-}cl A) \subseteq j\text{-}cl(j\text{-}int A)$. If A is *ij*-t-open, by Definition 4.1, $A \subseteq i\text{-}cl(i\text{-}int A) \cup j\text{-}int(j\text{-}cl A) \subseteq i\text{-}int(i\text{-}cl A) \cup j\text{-}cl(j\text{-}int A)$ that implies A is *ji*-t-open.

Proposition: 4.5
Suppose τ_i is coupled to τ_j and τ_i is near τ_j then every *ij*-t-open set is *ij*-b-open.

Proof: Let A be *ij*-t-open. Then $A \subseteq i\text{-}cl(i\text{-}int A) \cup j\text{-}int(j\text{-}cl A)$. Then by using Lemma 1.9 and by Lemma 1.10 we see that $A \subseteq j\text{-}cl(i\text{-}int A) \cup j\text{-}int(j\text{-}cl A) \subseteq j\text{-}cl(i\text{-}int A) \cup i\text{-}int(j\text{-}cl A))$,

Now by Definition 1.1(iii), A is *ij*-b-open.

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