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α̂g Interior and α̂g Closure in Topological Spaces

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ABSTRACT

In this paper we introduce α̂g interior, α̂g closure and study some of its properties.

Key words: *α̂g open, α̂g closed, α̂g int A, α̂g cl A.*

AMS subject classification: *54C10, 54C08, 54C05.*

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1. INTRODUCTION

Levine [6] introduced generalized closed sets in topology as a generalization of closed sets. Many authors like Arya *et al* [2], Balachandran *et.al* [3], Bhattacharya *et al* [4], Arokiarani[1], Ganambal[5], Malghan[7] and Nagaveni[8] have worked on generalized closed sets. Palaniappan *et al* [9] introduced regular $\hat{\alpha}$ generalized beta (α̂g) closed sets and worked on them. In this paper, α̂g interior, α̂g closure are introduced and their properties are investigated.

Throughout this paper X denote the topological space (X, τ) on which no separation axioms are assumed unless otherwise stated.

2. PRELIMINARIES

Definition: 2.1 A subset A of a topological space X is called
1) A pre open set if A $\subset$ int cl A and a preclosed set if cl int A $\subset$ A.
2) A regular open set if A = int cl A and a regular closed set if A = cl int A.
3) A α open set if A $\subset$ int cl int A and a α closed set if cl int cl A$\subset$A.

The intersection of all α closed subsets of X containing A is called the α closure of A and is denoted by αcl A. αcl A is a α closed set.

Definition: 2.2A subset A of a topological space X is called a $\hat{\alpha}$ generalized closed set (α̂g–closed set) if int cl int A$\subset$U whenever A$\subset$U and U is open in X.

The complement of α̂g closed set in X is α̂g open in X.

The intersection of all α̂g closed sets in X containing A is called $\hat{\alpha}$ generalized closure of A and is denoted by α̂g cl A. In general α̂g cl A is not α̂g closed.

The union of all α̂g open sets contained in A is denoted by α̂g int A. In general α̂g int A is not α̂g open.

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In what follows we assume that X is a topological space in which arbitrary intersections of âg closed sets of X be âg closed in X. Then âg cl A will be a âg closed set in X and âg int A will be a âg open set in X, for $A \subset X$.

Definition: 2.3 Let X be a topological space and let $x \in X$. A subset N of X is said to be a âg neighbourhood of x if and only if there exists a âg open set G such that $x \in G \subset N$

Definition: 2.4 Let X be a topological space and $A \subset X$. A point $x \in X$ is called a âg limit point of X if and only if every âg neighbourhood of x contains point of A other than x. The set of all âg limit points of A is called the âg derived set of A and shall be denoted by $D_{\hat{a}g}$ (A).

Thus x will be a âg limit point of A if and only if $(N-\{x\}) \cap A = \phi$, for every âg neighbourhood N of x.

Definition: 2.5 Let A be a sub set of a topological space X and let $x \in X$. Then x is called an âg adherent point of A if and only if every âg neighbourhood of x contains point of A. The set of all âg adherent points of A is called the âg adherence of A and shall be denoted by âg Adh A.

Definition: 2.6 A point x is said to be an âg isolated point of a subset A of a topological space X if and only if $x \in A$ but is not a âg limit point of A. That is, there exists some âg neighbourhood N of x such that N contains no point of A other than x. A âg closed set which has no âg isolated points is said to be âg perfect.

Remarks: 2.7 An âg adherent point is either âg limit point or âg isolated point.

3. Properties of âg Limit Points

Theorem: 3.1 A set is âg closed in X if and only if it contains all its âg limit pts.

Proof: Let A be âg closed in X. Then A' is âg open. For each $x \in A'$, there exists âg neighbourhood Nx of x such that $N_x \subset A'$. $A \cap A' = \phi$ implies N_X contains no point of A. So, x is not a âg limit point of A. A' contains no âg limit point of A. Hence $D_{\hat{a}g}(A) \subset A$.

Conversely, let $D_{\hat{a}g}(A) \subset A$. Let $x \in A'$. Since $x \notin A$, $x \notin D_{\hat{a}g}(A)$. Therefore, there exises some âg neighbourhood Nx of x such that $Nx \cap A = \varphi$. So $Nx \subset A'$. Hence A' contains a âg neighbourhood of each of its points. That is A' is âg open. So A is âg closed.

Theorem: 3.2 Let X be a topological space and $A \subset X$. Then $A = D_{\hat{a}g}(A)$ if and only if A is âg perfect.

Proof: Let A be âg perfect. Then A has no âg isolated point. $x \in A \Rightarrow x$ is not an âg isolated point $\Rightarrow x$ is a âg limit point. $\Rightarrow A \subset D_{\hat{a}g}(A)$

Since A is âg closed, $D_{\hat{a}g}(A) \subset A$

Hence $A = D_{\hat{a}g}(A)$

Conversely, let $A = D_{\hat{a}g}(A)$. Let $x \in X$. $x \in X - A$ implies $x \notin A$. That is $x \notin D_{\hat{a}g}(A)$. This implies there exists âg neighbourhood N of x such that $N \subset X-A$. X-A contains a âg neighbourhood of each of its points. So, X-A is âg open. That is, A is âg closed. Let $y \in A$. So $y \in D_{\hat{a}g}(A)$. Hence y is a âg limit point of A. This implies y is not an âg isolated point of A. That is, no point of A is an âg isolated point of A. A is a âg closed set having no âg isolated point. Hence A is âg perfect.

Let X be any discrete topological space and $A \subset X$. If $x \in X$, $\{x\}$ is âg open which contains no point of $\{x\}$ other than x. So x is not a âg limit point of A. Hence $D_{\hat{a}g}(A) = \varphi$.

Let X be any indiscrete topological space. Let $A \subset X$ containing two or more points. $x \in A$ is a âg limit point of A, since the only âg open set containing x is X, which contains all points of A, other than x. Hence $D_{\hat{a}g}(A) = X$.

Theorem: 3.3 Let A and B be subsets of a topological space X. Then

i) $D_{\hat{a}g}(\varphi) = \varphi$
ii) $A \subset B \Rightarrow D_{\hat{a}g}(A) \subset D_{\hat{a}g}(B)$
iii) $D_{\hat{a}g}(A \cap B) \subset D_{\hat{a}g}(A) \cap D_{\hat{a}g}(B)$

iv) $D_{\hat{\alpha}g}(A \cup B) = D_{\hat{\alpha}g}(A) \cup D_{\hat{\alpha}g}(B)$

Proof:

i) φ is closed. So $D_{\hat{\alpha}g}(\varphi) \subset \varphi$. But $\varphi \subset D_{\hat{\alpha}g}(\varphi)$. Hence $D_{\hat{\alpha}g}(\varphi) = \varphi$.

ii) Let $p \in D_{\hat{\alpha}g}$ (A). Every $\hat{\alpha}g$ neighbourhood of p contains a point of A, other than p. Since $A \subset B$, every $\hat{\alpha}g$ neighbourhood of p contains a point of B, other than p. Hence p is a $\hat{\alpha}g$ limit point of B. So, $D_{\hat{\alpha}g}(A) \subset D_{\hat{\alpha}g}$ (B). $A \cap B \subset A$. Hence $D_{\hat{\alpha}g}(A \cap B) \subset D_{\hat{\alpha}g}(A)$. Similarly, $D_{\hat{\alpha}g}(A \cap B) \subset D_{\hat{\alpha}g}(B)$.

iii) $A \cap B \subset A$. Hence $D\hat{\alpha}g(A \cap B) \subset D\hat{\alpha}g(A)$. Similarly, $D\hat{\alpha}g(A \cap B) \subset D\hat{\alpha}g(B)$. So $D\hat{\alpha}g(A \cap B) \subset D\hat{\alpha}g(A) \cap D\hat{\alpha}g(B)$.

iv) $A \subset A \cup B$. Hence $D_{\hat{\alpha}g}(A) \subset D_{\hat{\alpha}g}(A \cup B)$. Similarly, $D_{\hat{\alpha}g}(B) \subset D_{\hat{\alpha}g}(A \cup B)$.
So $D_{\hat{\alpha}g}(A) \cup D_{\hat{\alpha}g}(B) \subset D_{\hat{\alpha}g}(A \cup B)$.
To prove the other way, we prove the contra positive.
$x \notin D_{\hat{\alpha}g}(A) \cup D_{\hat{\alpha}g}(B) \Rightarrow x \notin D_{\hat{\alpha}g}(A \cup B)$

If $x \notin D_{\hat{\alpha}g}(A) \cup D_{\hat{\alpha}g}(B)$, then $x \notin D_{\hat{\alpha}g}(A)$ and $x \notin D_{\hat{\alpha}g}(B)$. That is, x is neither a $\hat{\alpha}g$ limit point of A nor a $\hat{\alpha}g$ limit point of B. Hence, there exist $\hat{\alpha}g$ neighbourhoods N_1 and N_2 of x such that $(N_1 - \{x\}) \cap A = \varphi$ and $(N_2 - \{x\}) \cap B = \varphi$. $N = N_1 \cap N_2$ is a $\hat{\alpha}g$ neighbourhood of x which contains no point of $A \cup B$ other than (possibly) x. So it follows that $x \notin D_{\hat{\alpha}g}(A \cup B)$ as required.

Theorem: 3.4 $\hat{\alpha}g$ cl A = $A \cup D_{\hat{\alpha}g}(A)$

Proof: Let us prove $A \cup D_{\hat{\alpha}g}(A)$ is $\hat{\alpha}g$ closed. That is, $(A \cup D_{\hat{\alpha}g}(A))' = A' \cap D'_{\hat{\alpha}g}(A)$ is $\hat{\alpha}g$ open. Let $x \in A' \cap D'_{\hat{\alpha}g}(A)$. Then $x \in A'$ and $x \in D'_{\hat{\alpha}g}(A)$. So $x \notin A$ and $x \notin D_{\hat{\alpha}g}(A)$. That is, x is not a $\hat{\alpha}g$ limit point of A. Hence, there exists a $\hat{\alpha}g$ neighbourhood N_x of x which contains no point of A. Hence $N_x \subset D'_{\hat{\alpha}g}(A)$. But $N_x \subset A'$. So $N_x \subset A' \cap D'_{\hat{\alpha}g}(A)$ $A' \cap D'_{\hat{\alpha}g}(A)$ contains a $\hat{\alpha}g$ neighbourhood of each of its points and hence $\hat{\alpha}g$ open. We now show that $\hat{\alpha}g$ cl A = $A \cup D_{\hat{\alpha}g}(A)$. $A \cup D_{\hat{\alpha}g}(A)$ is a $\hat{\alpha}g$ closed set containing A.

Hence $\hat{\alpha}g$ cl $A \subset A \cup D_{\hat{\alpha}g}(A)$. $\hat{\alpha}g$ cl A is rgβ closed. Hence $D_{\hat{\alpha}g}(A) \subset A$. But $A \subset \hat{\alpha}g$ cl A

So $D\alpha_g(A) \subset \hat{\alpha}g$ cl A.

Hence $A \cup D_{\hat{\alpha}g}(A) \subset \hat{\alpha}g$ cl A. This completes the proof.

Theorem: 3.5 $\hat{\alpha}g$ cl A = $\hat{\alpha}g$ Adh A.

Proof: $x \in \hat{\alpha}g$ Adh A $\Leftrightarrow$ every $\hat{\alpha}g$ neighbourhood of x intersects A $\Leftrightarrow$ $x \in A$ or every $\hat{\alpha}g$ neighbourhood of x intersects A in a point other than x $\Leftrightarrow$ $x \in A$ or $x \in D_{\hat{\alpha}g}(A) \Leftrightarrow x \in A \cup D_{\hat{\alpha}g}(A) \Leftrightarrow x \in \hat{\alpha}g$ cl A.

Theorem: 3.6 Let X be a topological space and let G be an $\hat{\alpha}g$ open subset of X and $A \subset X$. Then G is disjoint from A if and only if G is disjoint from the $\hat{\alpha}g$ closure of A.

Proof: Let $G \cap \hat{\alpha}g$ cl A = φ.
As $A \subset \hat{\alpha}g$ cl A, $G \cap A = \varphi$.

Conversely, let $G \cap A = \varphi$. Let $x \in G \cap \hat{\alpha}g$ cl A
$\hat{\alpha}g$ cl A = $A \cup D_{\hat{\alpha}g}(A)$. Hence $x \in D_{\hat{\alpha}g}(A)$.

As G is $\hat{\alpha}g$ neighbourhood of x, it intersects A, a contradiction. This completes the proof.

4. PROPERTIES OF $\hat{\alpha}g$ CLOSURE

Theorem: 4.1 Let X be a topological space and A and B be subsets of X.

i) $\hat{\alpha}g$ cl $\varphi = \varphi$
ii) $A \subset \hat{\alpha}g$ cl A.
iii) $A \subset B \Rightarrow \hat{\alpha}g$ cl $A \subset \hat{\alpha}g$ cl B
iv) $\hat{\alpha}g$ cl $(A \cup B) = \hat{\alpha}g$ cl $A \cup \hat{\alpha}g$ cl B.
v) $\hat{\alpha}g$ cl $(A \cap B) \subset \hat{\alpha}g$ cl $A \cap \hat{\alpha}g$ cl B.
vi) $\hat{\alpha}g$ cl ($\hat{\alpha}g$ cl A) = $\hat{\alpha}g$ cl A.

Proof:

i) Since φ is $\hat{\alpha}$g closed, $\hat{\alpha}$g cl $\varphi = \varphi$

ii) By definition of $\hat{\alpha}$g cl A, $A \subset \hat{\alpha}$g cl A.

iii) $A \subset B \subset \hat{\alpha}$g cl B. Hence $\hat{\alpha}$g cl $A \subset \hat{\alpha}$g cl B.

iv) $A \subset A \cup B$. Hence $\hat{\alpha}$g cl $A \subset \hat{\alpha}$g cl $(A \cup B)$
Similarly $\hat{\alpha}$g cl $B \subset \hat{\alpha}$g cl $(A \cup B)$
So $\hat{\alpha}$g cl $A \cup \hat{\alpha}$g cl $B \subset \hat{\alpha}$g cl $(A \cup B)$
$\hat{\alpha}$g cl $A \cup \hat{\alpha}$g cl B is a $\hat{\alpha}$g closed set containing $A \cup B$.
Hence $\hat{\alpha}$g cl $(A \cup B) \subset \hat{\alpha}$g cl $A \cup \hat{\alpha}$g cl B.This completes the proof.

v) $A \cap B \subset A$, $A \cap B \subset B$
Hence $\hat{\alpha}$g cl $(A \cap B) \subset \hat{\alpha}$g cl $A \cap \hat{\alpha}$g cl B.

vi) $\hat{\alpha}$g cl A is $\hat{\alpha}$g closed.
Hence $\hat{\alpha}$g cl ($\hat{\alpha}$g cl A) = $\hat{\alpha}$g cl A.

5. $\hat{\alpha}$g interior points and $\hat{\alpha}$g interior of a set

Definition: 5.1 Let X be a topological space and $A \subset X$. A point $x \in A$ is said to be $\hat{\alpha}$g interior point of A if and only if A is a $\hat{\alpha}$g neighbourhood of x. That is, there exists an $\hat{\alpha}$g open set G such that $x \in G \subset A$. The set of all $\hat{\alpha}$g interior points of A is called the $\hat{\alpha}$g interior A and is denoted by $\hat{\alpha}$g int A.

Theorem: 5.2 $\hat{\alpha}$g int A = $\cup$ {G: G is $\hat{\alpha}$g open, $G \subset A$}

Proof: $x \in \hat{\alpha}$g int A $\Leftrightarrow$ A is a $\hat{\alpha}$g neighbourhood of x. $\Leftrightarrow$ there exists an $\hat{\alpha}$g open set G such that $x \in G \subset A \Leftrightarrow x \in \cup${ G:G is $\hat{\alpha}$g open, $G \subset A$}. Thus A = $\cup$ {G: G is $\hat{\alpha}$g open, $G \subset A$}

Theorem: 5.3 Let X be a topological space and $A \subset X$. Then

i) $\hat{\alpha}$g int A is $\hat{\alpha}$g open
ii) $\hat{\alpha}$g int A is the largest $\hat{\alpha}$g open set contained in A.

Proof:
i) Let $x \in \hat{\alpha}$g int A. So there exists a $\hat{\alpha}$g open set G such that $x \in G \subset A$. Since G is $\hat{\alpha}$g open, it is a $\hat{\alpha}$g neighbourhood of each of its points. So A is also a $\hat{\alpha}$g neighbourhood of each of the points of G. It follows that every point of G is a $\hat{\alpha}$g interior point of A. Hence $G \subset \hat{\alpha}$g int A. $\hat{\alpha}$g int A contains a $\hat{\alpha}$g neighbourhood of each of its points. Hence $\hat{\alpha}$g int A is $\hat{\alpha}$g open.

ii) Let G be any $\hat{\alpha}$g open set such that $G \subset A$. Let $x \in G$. A is $\hat{\alpha}$g neighbourhood of x. Therefore $x \in \hat{\alpha}$g int A. Hence $G \subset \hat{\alpha}$g int A. So $\hat{\alpha}$g int A is the largest $\hat{\alpha}$g open set contained in A.

Remark: 5.4 If X be any discrete topological space, then every subset of X coincides with its $\hat{\alpha}$g interior.

Theorem: 5.5 Let X be a topological space. Then $\hat{\alpha}$g int A equals the set of all points of A which are not $\hat{\alpha}$g limit points of A'

Proof: Let $x \in A$, which is not a $\hat{\alpha}$g limit point of A' Then, there exists a $\hat{\alpha}$g neighbourhood N of x, which contains no point of A'. So $N \subset A$. This implies A is also $\hat{\alpha}$g neighbourhood of x. Hence $x \in \hat{\alpha}$g int A. Let $x \in \hat{\alpha}$g int A. Since $\hat{\alpha}$g int A is $\hat{\alpha}$g open, $\hat{\alpha}$g int A is a $\hat{\alpha}$g neighbourhood of x. Also $\hat{\alpha}$g int A contains no point of A'. It follows x is not a $\hat{\alpha}$g limit point of A'. Thus no point of $\hat{\alpha}$g int A can be a $\hat{\alpha}$g limit point of A'. So $\hat{\alpha}$g int A consists precisely those points of A which are not $\hat{\alpha}$g limit points of A'.

6. Properties of $\hat{\alpha}$g interior

Theorem: 6.1 Let X be a topological space and let A,B be subsets of X

i) $\hat{\alpha}$g int X=X, $\hat{\alpha}$g int $\varphi = \varphi$
ii) $\hat{\alpha}$g int $A \subset A$
iii) $A \subset B \Rightarrow \hat{\alpha}$g int $A \subset \hat{\alpha}$g int B
iv) $\hat{\alpha}$g int $(A \cap B)$ = $\hat{\alpha}$g int $A \cap \hat{\alpha}$g int B
v) $\hat{\alpha}$g int $A \cup \hat{\alpha}$g int $B \subset \hat{\alpha}$g int $(A \cup B)$
vi) $\hat{\alpha}$g int ($\hat{\alpha}$g int A) = $\hat{\alpha}$g int A

Proof:

i) obvious
ii) obvious
iii) Let x ∈ α̂g int A. A is a α̂g neighbourhood of x. As A⊂ B, B is a α̂g neighbourhood of x.
This implies x∈ α̂g int B.
Hence α̂g int A ⊂ α̂g int B.
iv) A∩B⊂A, A∩B⊂B.
Hence α̂g int (A∩B) ⊂ α̂g int A ∩ α̂g int B.
Let x ∈ α̂g int A ∩ α̂g int B.
x ∈ α̂g int A and x ∈ α̂g int B
A and B are α̂g neighbourhoods of x. Hence A∩B is a α̂g neighbourhood of x.
So x∈ α̂g int (A∩B).
Therefore α̂g int A∩ α̂g int B ⊂ α̂g int (A∩B). This complets the proof.
v) A ⊂ A∪B, B ⊂ A∪B
Hence α̂g int A∪ α̂g int B⊂ α̂g int (A∪B).
vi) α̂g int A is α̂g open.
Hence α̂g int (α̂g int A) = α̂g int A.

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