International Journal of Mathematical Archive-5(3), 2014, 276-279
Available online through www.ijma.info ISSN 2229 – 5046

ASSOCIATOR IDEAL IN SEMIPRIME WEAKLY STANDARD RINGS

K. Suvarna[1] and K. Chennakesavulu*[2]

[1]*Department of Mathematics, Sri Krishnadevaraya University, Anantapur – 515 003, AP, India.*

[2]*Department of Mathematics, Intel Engineering College, Anantapur, AP, India.*

(Received on: 30-01-14; Revised & Accepted on: 07-03-14)

ABSTRACT

We prove that an associator ideal in a semiprime weakly standard Novikov ring R is anticommutative and alternative. We use this to prove that R is associative.

Key Words: *Commutator, associator, semiprime ring, primering, weakly standard ring, alternative ring, flexible ring, characteristic of a ring.*

1. INTRODUCTION

Kleinfeld and Smith [1] proved that simple finite dimensional weakly Novikov algebras over a field of characteristic zero must be associative. Kleinfeld proved that if R is a prime ring satisfying the Novikov identity x(yz) = y(zx) such that 2x = 0 implies x = 0, then R must be commutative and associative. In [2] it is shown that a semiprime flexible ring with weak Novikov identity is associative. In this paper, we prove that in a weakly standard ring R of characteristic ≠ 2, 3 with Novikov identity (xy)z = (xz)y, the associator ideal I is anticommutative and alternative. Using these results we prove that a semiprime weakly standard ring of characteristic ≠ 2, 3 is associative.

2. PRELIMINARIES

A weakly standard ring R is a nonassociative ring satisfying the identities

(x, y, x) = 0
((w, x), y, z) = 0
((w, (x, y), z) = 0

for all w, x, y, z in R, where the associator (x, y, z) = (xy)z – x(yz) and the commutator (x, y) = xy – yx. We define a ring R to be of characteristic ≠ n if nx = 0 implies x = 0 for all x in R. A ring R is prime if whenever A and B are ideals of R such that AB = 0, then either A = 0 or B = 0 and if ring R is semiprime for any ideal A of R, $A^2 = 0$ implies A = 0.

We define an alternative ring R in which (xx)y = x(xy) = x(xy), y(xx) = (yx)x, for all x, y in R.

Throughout this section R denotes a weakly standard ring of characteristic ≠ 2, 3 satisfying the Novikov identity (xy)z = (xz)y.

Using this identity we obtain

$$\begin{aligned}(w, x, y, z) &= ((w, x)\ y)z – (wx)(yz)\\ &= ((wy)\ z)\ x - (w\ (yz))\ x\\ &= (w, y, z)\ x.\end{aligned}$$

Since the flexible identity (x, y, x) = 0 (1)

holds in R, every commutator is in the nucleus of R and every associator commutes with every element of R. The above equation can be written as

(w, x, yz) = y (w, x, z), (2)

Corresponding author: K. Chennakesavulu[*2]
[2]*Department of Mathematics, Intel Engineering College, Anantapur, AP, India.*
E-mail: venkat_seela@yahoo.com

which is the weak Novikov identity. The commutative center U of R is defined by U = {$u \in R$ / (u, R) = 0}. The associator ideal I consists of al finite sums of associators and left multiples of associators. As a consequence of (2) we observe that the associator ideal I of R consists of all finite sums of associators. We use the Teichmuller identity

$$(wx. y, z) - (w, xy, z) + (w, x, yz) - w(x, y, z) - (w, x, y) z = 0, \quad (3)$$

which is valid in every ring.

A linearization of (1) implies (wx, y, z) = - (z, y, wx).

But use of (2) shows that - (z, y, wx) = - w (z, y, x). From the flexible identity we obtain

$$(wx, y, z) = w(x, y, z). \quad (4)$$

Comparing (3) and (4) implies

$$- (w, xy, z) + (w, x, yz) = (w, x, y) z. \quad (5)$$

Now (z, x, yz) = y (z, x, z) = 0, using (2) and then (1). By substituting w = z in (5), we get

$$(z, x, y)z = 0. \quad (6)$$

A linearization of (6) is (w, x, y)z = - (z, x, y)w. (7)

For arbitrary elements a, b, x, y, z in R, we observe that

$$p = (a, b, (x, y, z)) = (a, b, xy.z) – (a, b, x.yz) = xy. (a, b, z)-x.y (a, b, z) = (x, y, (a, b, z)),$$

Using (2) several times.

Thus (a, b, (x, y, z)) = (x, y, (z, b, z)). (8)

Let x = z in (8). As a consequence of (1), it follows that

$$(z, y,(a, b, z)) = 0. \quad (9)$$

By linearization of (9) together with (1), we have

$$((w^{\pi}, y, (a^{\pi}, b, z^{\pi})) = (\text{sgn } \pi) (w, y,(a, b, z)), \quad (10)$$

where π is any permutation on the set {w, a, z}. Combining (10) with (8) we obtain

$$(a, b, (x, y, z)) = -(x, b, (a, y, z)) = -(a, y, (x, b, z)).$$

Thus $(w, y^{\sigma}, (a, b^{\sigma}, z)) = (\text{sgn } \sigma) (w, y,(a, b, z))$, (11)

where σ is any permutation on the set {y, b}.

Now (x, y, z) (a, b, c) = -((a, b, c), y, z)x using (7). Then flexible implies

$$– ((a, b, c), y, z)x = (z, y, (a, b, c))x.$$

Let c = z. Since (z, y, (a, b, z)) x = - (a, y, (z, b, z)) x = 0, using (10) and (1) it follows that

$$(x, y, z)(a, b, z) = 0. \quad (12)$$

A linearization of (12) leads to

$$(x, y, z)(a, b, c) = -(x, y, c)(a, b, z). \quad (13)$$

Now, (13) and (1) imply

$$(x^{\alpha}, y, z^{\alpha}) (a^{\alpha}, b, c^{\alpha}) = (\text{sgn } \alpha) (x, y, z) (a, b, c), \quad (14)$$

where α stands for any permutation on the set {x, z, a, c}. Also we note that

q = (x, b, z) (a, b, c) = - ((a, b, c), b, z)x = (z, b, (a, b, c))x,

By combining (7) with the flexible identity. But then q = (z, b, (a, b, c)) x =-(z, b, (a, b, c)) x, using (11). Thus 2q = 0.

Using characteristic different from 2, we get q = 0, so that

(x, b, z)(a, b, c) = 0. (15)

By linearization of (15) we have

(x, y, z)(a, b, c) = -(x, b, z) (a, y, c). (16)

Then the combination of (16) and (14) gives

(x, y, z)(a, b, c) = - (a, b, c) (x, y, z). (17)

This implies the following result.

Lemma: 1 *A is anticommutative.*

MAIN RESULTS

Now we prove the main results.

Lemma: 2 *A is alternative.*

Proof: Let q be an arbitrary element in A and w, x, y, z arbitrary elements in R. Then (z, x, y).qw = - (qw, x, y) z using (7). But – (qw, x, y) z = -q(w, x, y).z, using (4). Then (2) implies –q(w, x, y).z = -(w, x, qy)z. Again (7) implies

- (w, x, qy)z = (z, x, qy) and (2)

Implies (z, x, qy) w = q(z, x, y).w. Then

Lemma (1) yields q (z, x, y).w = - (z, x, y) q.w.

By taking these equalities together we obtain (z, x, y).qw = - (z, x, y) q.w.

In other words p.qw = -pq.w (18)

for all p,q $\in$ A and all w in R. We also assume that r is an element of A.

Then (p, q, r) + (p, r, q) = pq.r – p.qr + pr.q-p.rq = -2p.qr – 2p.rq,

Using (18). However –2p.qr – 2p.rq = -2p (qr + 2q).

Then lemma (1) implies that qr + rq = 0, so that (p, q, r) + (p, r, q) = 0. At this point I is both flexible and right alternative, hence alternative.

Lemma: 3 *If S is an anti-commutative alternative ring of characteristic $\neq 2$ then (S^2) (S^2) = 0.*

Proof: For arbitrary elements w, x, y, z in S, we have (xy)(zx) = x(yz)x = x^2(yz) = 0, using alternative identities and anti-commutativity.

By linearizing this identity, we get (wy) (zx) = - (xy) (zw). Applying this in conjunction with anti-commutativity leads to (wy) (zx) = (zx) (wy). However wy also anti-commutes with zx, so that 2(wy) (zx) = 0. Since R is of characteristic $\neq$ 2, we have (wy) (zx) = 0. So that (S^2) (S^2) = 0.

Using the above results, we prove the following:

Theorem: 1 *A semiprime weakly standard Novikov ring R of characteristic $\neq 2$ is associative.*

Proof: Let p, q be arbitrary elements of R, I the associative ideal of R, and z an arbitrary element of R. Then pq.z = - p.qz, using (18). Thus I^2 is a right ideal of R. Also (z, p, q) = -(q, p, z) = - qp.z + q.pz is an element of I^2. But zp.q is in I^2. Hence z.pq = - (z, p, q) + zp.q is also in I^2. Thus I^2 is an ideal of R. Then lemmas (1), (2) and (3) imply that the ideal I^2 of R squares to 0. Since R is semiprime, it follows that I = 0. So R must be associative.

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Source of support: Nil, Conflict of interest: None Declared

