International Journal of Mathematical Archive-5(3), 2014, 280-282
IJMA Available online through www.ijma.info ISSN 2229 – 5046

COSMOLOGICAL MODEL IN SAEZ-BALLESTER THEORY OF GRAVITATION

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(Received on: 27-02-14; Revised & Accepted on: 12-03-14)

ABSTRACT

In this paper, we have obtained field equations and their solution in the presence of perfect fluid distribution in a scalar-tensor theory of gravitation proposed by Saez and Ballester (Phy. Lett. 113, 1985, 467) with the aid of Kantowski-Sachs space time. Exact cosmological model is presented with the help of stiff fluid. The physical and kinematical properties of the model are discussed.

Key words: *Perfect fluid, Saez-Ballester theory, stiff fluid.*

1. INTRODUCTION

Saez and Ballester (1985) have proposed a theory in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields. Intriguing results of the scalar field, an antigravity regime appears. This theory suggests a possible way to solve the messing matter problem in non-flat FRW cosmologies.

The field equations given by Saez and Ballester(1985) for the combined Scalar and tensor fields are

$$G_{ij} - \omega\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -T_{ij} \tag{1}$$

$$2\phi^n\phi^i_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{2}$$

Where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is the Einstein tensor, R_{ij} is the Ricci tensor, R is the scalar curvature, n an arbitrary constant, ω is a dimensionless coupling constant and T_{ij} is the matter energy-momentum tensor. Here comma and semicolon denote partial and covariant differentiation respectively (we have chosen the units such that $8\pi G = 1 = C$).

The equation of motion

$$T^{ij}_{;j} = 0 \quad , \tag{3}$$

is a consequence of field equation (1) and (2).

A detailed discussion of Saez and Ballester cosmological model is contained in the work of Singh and Agrawal (1991). Shri Ram and Tiwari (1998). Reddy and Venkateswara Rao(2001), D.R.K. Reddy, CH,C.S.V.V.R. Murthy, R. Venkateswarlu(2006), Adhav *et al.* (2007). The Kantowski-Sachs cosmological models containing a perfect fluid with a zero cosmological constant was analyzed by Collins (1966). Also Lorenz (1984) has obtained exact Kantowski-Sachs vacuum models in Brans-Dicke theory, Gron (1986), Krori *et al.*(1995), Li and Hao (2003) have also studied cosmological model for the Kantowski-Sachs space time. Adhav *et al.*(2008) have studied Kantowski-Sachs cosmological model in general theory of Relativity.

In this paper, we have obtained Kantowski-Sachs cosmological model in a scalar-tensor theory of gravitation proposed by Saez and Ballester in presence of a perfect fluid by using stiff fluid. Some physical and kinematical properties of the cosmological models are also discussed.

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2. METRIC AND FIELD EQUATIONS

We consider the Kantowski-Sachs space time in the form

$$ds^2 = dt^2 - R^2 dr^2 - S^2(d\theta^2 + \sin^2\theta \, d\phi^2) \,, \tag{4}$$

where *R* and *S* are the functions of time *'t'* only.

The energy-momentum tensor for a perfect fluid is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \tag{5}$$

together with $g_{ij}u^i u^j = 1$ (6)

Here u^i is the four velocity vector of the fluid, p and ρ are the proper pressure and energy density of the distribution respectively.

With the help of equations (5) and (6), field equations (1), (2) and (3) for the metric (4) can be written as

$$2\frac{S_{44}}{S} + \left(\frac{S_4}{S}\right)^2 + \frac{1}{S^2} - \frac{\omega}{2}\phi^n {\phi_4}^2 = -p \tag{7}$$

$$\frac{R_{44}}{R} + \frac{S_{44}}{S} + \frac{R_4 S_4}{RS} - \frac{\omega}{2}\phi^n {\phi_4}^2 = -p \tag{8}$$

$$2\frac{R_4 S_4}{RS} + \left(\frac{S_4}{S}\right)^2 + \frac{1}{S^2} + \frac{\omega}{2}\phi^n {\phi_4}^2 = \rho \tag{9}$$

$$\phi_{44} + \left(\frac{R_4}{R} + 2\frac{S_4}{S}\right)\phi_4 + \frac{n}{2}\frac{{\phi_4}^2}{\phi} = 0 \tag{10}$$

$$\rho_4 + (\rho + p)\left(\frac{R_4}{R} + 2\frac{S_4}{S}\right)\phi_4 - \frac{p}{S^2}\cot\theta = 0 \tag{11}$$

3. SOLUTIONS OF THE MODEL

The set of field equation (7) to (11) are highly nonlinear in matter, in general, it is difficult to obtain the explicit solution of the field equation, for this we consider two cases

i] Relation between metric coefficient *R* and *S*

i.e. $R = AS^n$, where *'A'* and *'n'* are both constant (12)

ii] Stiff fluid can be regarded as perfect fluid having the energy-momentum tensor given by (5) characterized by the equation of state

$$p = \rho \tag{13}$$

Using these cases, an exact solution of the field equation are

$$R = M\,(k_1 t + k_2)^{\frac{n}{n+2}}, \quad S = N(k_1 t + k_2)^{\frac{n}{n+2}}, \tag{14}$$

where, $M = AN^n$ and $N = (n+2)^{\frac{1}{n+2}}$.

$$p = \rho = (2n+1)\left(\frac{nk_1}{n+2}\right)^2 \frac{1}{(k_1 t + k_2)^2} + \frac{1}{N^2\,(k_1 t + k_2)^{\frac{2n}{n+2}}} + \frac{\omega}{2}\frac{{k_1}^2}{A^2 N^{2n+4}}\frac{1}{(k_1 t + k_2)^{2n}} \tag{15}$$

Using equation (14) and a proper choice of co-ordinates and constants the metric corresponding to our solution can be written as

$$ds^2 = -\frac{dT^2}{k_1^{\,2}} - M^{\,2}(T)^{\frac{2n}{n+2}} dr^2 - N^2(T)^{\frac{2}{n+2}}(d\theta^2 + \sin^2\theta d\phi^2), \tag{16}$$

Which represents a non-static Kantowski-Sachs Zel'dovich universe in Saez-Ballester scalar tensor theory of gravitation.

The scalar field in the universe is given by

$$\phi_4^{\,2}\phi^n = \frac{k_1^{\,2}}{A^2(N)^{2n+4}}\frac{1}{(k_1 t + k_2)^{2n}} \tag{17}$$

4. PHYSICAL AND KINEMATICAL PROPERTIES

The physical and kinematical quantities for the model (16) have the following expression

Proper volume $V^3 = \sqrt{-g} = MN^2T$ (18)

Expansion Scalar $(\theta) = \dfrac{1}{2T}$ (19)

Shear scalar $(\sigma^2) = \dfrac{1}{18T^2}$ (20)

Deceleration parameter $(q) = 10$ (21)

The model (16) has no initial singularity, while the energy density and pressure given by (15) possess initial singularities. However, as T increases these singularities vanish. The proper volume of the model given by (18) shows the anisotropic expansion of the universe (16) with time. For the model (16), the expansion scalar θ and shear scalar σ^2 tends to zero as $T \to \infty$. The positive values of the deceleration parameter indicates that the model decelerates in the standard way.

Also, since $\lim_{T\to\infty}\left(\dfrac{\sigma}{\theta}\right) \neq 0$

The model does not approach isotropy for large values of *T*.

5. CONCLUSION

We have considered Kantowski-Sachs cosmological model in Saez-Ballester scalar tensor theory of gravitation in the presence of perfect fluid. For solving the field equations we have assumed that pressure and density are equal. The model is free from singularities and it is expanding and decelerates in the standard way.

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Source of support: Nil, Conflict of interest: None Declared