ON $\delta$-H CONTINUOUS FUNCTIONS IN GTS WITH HEREDITARY CLASSES

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ABSTRACT

In this paper, we introduce a new class of functions called $\delta$-H continuous function. We obtain several characterizations and some of their properties. Also, we investigate its relationship with other types of functions.

Keywords: $\delta$-H cluster points, $R$-$H$-open set, $\theta$-H-continuous, strongly $\delta H$-continuous, almost-H-continuous, $SH$-$R$ space, $AH$-$R$ space with hereditary classes.

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1. INTRODUCTION

In 2007, Csaśzár [3] defined a nonempty class of subsets of a nonempty set, called hereditary class and studied modification of generalized topology via hereditary classes. Also, it is studied in [8]. The aim of the paper is to extend the study of the properties of the generalized topologies via hereditary classes. A subfamily $\mu$ of $\mathcal{P}(X)$ is called a generalized topology (GT) [2] if $\emptyset \in \mu$ and $\mu$ is closed under arbitrary union. The pair $(X, \mu)$ is called a generalized topological space (GTS). Members of $\mu$ are called $\mu$-open sets and its complement is called a $\mu$-closed set.

The largest $\mu$-open set contained in a subset $A$ of $X$ is denoted by $i_\mu(A)$ [1] and is called the $\mu$-interior of $A$. The smallest $\mu$-closed set containing $A$ is called the $\mu$-closure of $A$ and is denoted by $c_\mu(A)$ [1].

A generalized topology $\mu$ is said to be a quasi-topology if $\mu$ is closed under finite intersection. Let $X$ be a nonempty set. A hereditary class $H$ of $X$ is a nonempty collection of subset of $X$ such that $A \subset B$, $B \in H$ implies $A \in H$ [3].

A hereditary class $H$ of $X$ is an ideal [8] if $A \cup B \in H$ whenever $A \in H$ and $B \in H$.

An ideal $I$ in a topological space $(X, \tau)$ is said to be codense if $\tau \cap I = \{\emptyset\}$. With respect to the generalized topology $\mu$ of all $\mu$-open sets and a hereditary class $H$, for each subset $A$ of $X$, a subset $A^* (H)$ or simply $A^*$ of $X$ is defined by $A^* = \{x \in X: M \cap A \in H$ for every $M \in \mu$ such that $x \in M \}$ [3].

In this paper, we introduce the notions of $\delta$-H-open sets and $\delta$-H-continuous functions in GTS with hereditary classes. We obtain several characterizations and some properties of $\delta$-H-continuous functions. Also, we investigate the relationships with other related functions.

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2. \( \delta - H \)-sets

In this section, we introduce \( \delta - H \)-open sets and the \( \delta - H \)-closure of a set in a GTS with hereditary class and investigate their basic properties. It turns out that they have similar properties with \( \delta - \) open and the \( \delta - \) closure [11].

A subset \( A \) of a GTS \((X, \mu)\) with hereditary class \( H \) is said to be an \( R - H \)-open set (resp. regular open set) if \( i_\mu (c^*(A)) = A \) (resp. \( i_\mu (c^*(A)) = A \)). We call a subset \( A \) of \( X \) is \( R - H \)-closed if its complement is \( R - H \)-open.

Let \( A \) be a subset of a GTS \((X, \mu)\) with a hereditary class \( H \). A point \( x \) of \( X \) is called a \( \delta - H \)-cluster point of \( A \) if \( A \cap i_\mu (c*(U )) \neq \emptyset \) for each \( \mu \)-open neighborhood \( U \) of \( x \). The family of all \( \delta - H \)-cluster points of \( A \) is called the \( \delta - H \)-closure of \( A \) and is denoted \( [A] \). The complement of a \( \delta - H \)-closed set of \( X \) is said to be \( \delta - H \)-open.

**Lemma: 2.1** Let \( A \) and \( B \) be subsets of a quasi topological space \((X, \mu)\) with a hereditary class \( H \). Then, the following properties hold,

(a) \( i_\mu (c^*(A)) \) is \( R - H \)-open,
(b) If \( A \) and \( B \) are \( R - H \)-open, then \( A \cap B \) is \( R - H \)-open,
(c) If \( A \) is regular open, then \( A \) is \( R - H \)-open,
(d) If \( A \) is \( R - H \)-open, then \( A \) is \( \delta - H \)-open,
(e) Every \( \delta - H \)-open set is the union of a family of \( R - H \)-open sets.

**Proof:**

(a) Let \( A \) be a subset of \( X \) and \( V = i_\mu (c^*(A)) \). Then, we have
\[
i_\mu (c^*(V)) = i_\mu (c^*(i_\mu (c^*(A)))) \subset i_\mu (c^*(c^*(A))) = i_\mu (c^*(A)) = V \quad \text{and also} \quad V = i_\mu (V) \subset i_\mu (c^*(V)).
\]
Therefore, \( i_\mu (c^*(V)) = V \).

(b) Let \( A \) and \( B \) be \( R - H \)-open. Then,
\[
A \cap B = i_\mu (c^*(A)) \cap i_\mu (c^*(B)) = i_\mu (c^*(A \cap c^*(B)) \supset i_\mu (c^*(A \cap B)) = A \cap B.
\]
Therefore \( A \cap B \) is \( R - H \)-open.

(c) Let \( A \) be regular open. Since \( \mu^* \supset \mu \), we have \( A = i_\mu (A) \subset i_\mu (c^*(A)) \subset i_\mu (c_\mu (A)) = A \) and hence \( i_\mu (c^*(A)) = A \). Therefore, \( A \) is \( R - H \)-open.

(d) Let \( A \) be any \( R - H \)-open set. For each \( x \in A \), \((X - A) \cap A = \emptyset \) and \( A \) is \( R - H \)-open. Hence \( x \notin [X - A] \) for each \( x \in A \). Therefore \( x \notin (X - A) \) implies \( x \notin [X - A] \). Therefore, \([X - A] \subset (X - A) \) since, \( S \subset [S] \) for any subset \( S \) of \( X \). Hence \( A \) is \( \delta - H \)-open.

(e) Let \( A \) be a \( \delta - H \)-open set. Then \( X - A \) is \( \delta - H \)-closed and hence \([X - A] \subset (X - A) \). For each \( x \in A \), there exists an \( \mu \)-open neighborhood \( V_x \) such that \( i_\mu (c^*(V_x)) \cap (X - A) = \emptyset \).

Therefore, \( x \in V_x \subseteq i_\mu (c^*(V_x)) \subset A \), hence \( A = \cup \{i_\mu (c^*(V_x)) \mid x \in A \} \). By (a),
\[
i_\mu (c^*(V_x)) \text{ is } R - H \text{-open for each } x \in A.
\]

**Lemma: 2.2** Let \( A \) and \( B \) be subsets of a quasi topological space \((X, \mu)\) with a hereditary class \( H \). Then, the following properties hold:

(a) \( A \subseteq [A] \); 
(b) If \( A \subseteq B \), then \([A] \subseteq [B] \); 
(c) \([A] = \cap \{F \subseteq X \mid A \subseteq F \text{ and } F \text{ is } \delta - H \text{-closed}\}; 
(d) If \( A \) is a \( \delta - H \)-closed set of \( X \) for each \( \alpha \in \Delta \), then \( \cap \{A_\alpha \mid \alpha \in \Delta \} \) is \( \delta - H \)-closed;
(e) \([A] \subseteq [A] \).
Proof:
(a) For any $x \in A$ and any $\mu$-open neighborhood $V$ of $x$, we have $\emptyset \neq A \cap V \subset A \cap i_\mu (c_\mu^*(V))$ and hence $x \notin [A]_{\delta-H}$. Therefore, $A \subset [A]_{\delta-H}$.

(b) Suppose that $x \notin [B]_{\delta-H}$. There exists a $\mu$-open neighborhood $V$ of $x$ such that $x \notin i_\mu (c_\mu^*(V)) \cap B$. Hence $i_\mu (c_\mu^*(V)) \cap A = \emptyset$. Therefore, $x \notin [A]_{\delta-H}$.

(c) Suppose that $x \in [A]_{\delta-H}$. For any $\mu$-open neighborhood $V$ of $x$ and any $\delta-H$-closed set $F$ containing $A$, $\emptyset \neq A \cap i_\mu (c_\mu^*(V)) \subset F \cap i_\mu (c_\mu^*(V))$ and hence $x \notin [F]_{\delta-H}$. Therefore $x \notin \cap \{ F \subset X \mid A \subset F \text{ and } F \text{ is } \delta-H\text{-closed} \}$. Conversely, suppose that $x \notin [A]_{\delta-H}$. There exists a $\mu$-open neighborhood $V$ of $x$ such that $i_\mu (c_\mu^*(V)) \cap A = \emptyset$. By Lemma 2.1, $A = i_\mu (c_\mu^*(V))$, which implies that $A$ is $\delta-H$-closed set which contains $A$ and does not contain $x$. Therefore, $x \notin \cap \{ F \subset X \mid A \subset F \text{ and } F \text{ is } \delta-H\text{-closed} \}$.

(d) For each $\alpha \in \Delta [\cap \alpha \in \Delta A]_{\delta-H} \subset [A]_{\delta-H} = A_{\alpha}$ and hence $[\cap \alpha \in \Delta A]_{\delta-H} \subset [\cap \alpha \in \Delta A]_{\delta-H}$. By (a) $[\cap \alpha \in \Delta A]_{\delta-H}$ is $\delta-H$-closed.

(e) This follows immediately from (c) and (d).

A point $x$ of a quasi topological space $(X, \mu)$ with a hereditary class $H$ is called a $\delta$-cluster point of a subset $A$ of $X$ if $i_\mu (c_\mu^*(V)) \cap A \neq \emptyset$ for every $\mu$-open set $V$ containing $x$. The set of all $\delta$-cluster points of $A$ is called the $\delta$-closure of $A$ and is denoted by $c_\delta(A)$. If $c_\delta(A) = A$, then $A$ is said to be $\delta$-closed [6]. The complement of a $\delta$-closed set is said to be $\delta$-open. It is well-known that the family of all regular open sets of $(X, \mu)$ with a hereditary class $H$ is a basis for a quasi topological space which is weaker than $\mu$. This is called the semi-regularization of $\mu$ and is denoted by $\mu_s$.

**Theorem 2.3** Let $(X, \mu)$ be a quasi topological space with a hereditary class $H$ and $\mu_{\delta-H}\{A \subset X \mid A$ is a $\delta-H$-open set of $(X, \mu)\}$. Then $\mu_{\delta-H}$ is a topology such that $\mu_s \subset \mu_{\delta-H} \subset \mu$.

**Proof:** By Lemma 2.1, $\mu_s \subset \mu_{\delta-H} \subset \mu$. Next we show that $\mu_{\delta-H}$ is a topology.

(1) It is obvious that $\emptyset, X \in \mu_{\delta-H}$.

(2) Let $V_\alpha \in \mu_{\delta-H}$ for each $\alpha \in \Delta$. Then $X-V_\alpha$ is $\delta$-H-closed for each $\alpha \in \Delta$. By Lemma 2.2, $\bigcap \alpha \in \Delta (X-V_\alpha)$ is a $\delta$-H-closed and $\bigcap \alpha \in \Delta (X-V_\alpha) = X-\bigcup \alpha \in \Delta V_\alpha$. Hence $\bigcup \alpha \in \Delta V_\alpha$ is $\delta$-H-open.

(3) Let $A,B \in \mu_{\delta-H}$. By Lemma 2.1, $A=\bigcup \alpha \in \Delta_1 A_\alpha$ and $B=\bigcup \beta \in \Delta_2 B_\beta$, where $A_\alpha$ and $B_\beta$ are $R-H$-open sets for each $\alpha \in \Delta_1$ and $\beta \in \Delta_2$. Thus $A \cap B = \bigcup \{ A_\alpha \cap B_\beta \mid \alpha \in \Delta_1, \beta \in \Delta_2 \}$. Since $A_\alpha \cap B_\beta$ is $R-H$-open, $A \cap B$ is $\delta$-H-open set by Lemma 2.1.

The following Example 2.4 shows that the $\delta$-H-open set need not be a $R-H$-open set.

**Example 2.4** Let $X = \{a,b,c,d\}, \mu = \{\emptyset, \{b\}, \{b,c,d\}\} \text{ and } H = \{\emptyset, \{c\}\}$. If $A = \{b, d\}$, then $i_\mu (c_\mu^*(A)) = \{b, c, d\}$ and so $c_\mu^*(i_\mu(A)) = \{b, c, d\}$ which implies that $A$ is $\delta$-H-closed. But $A$ is not $R-H$-open, since $i_\mu (c_\mu^*(A)) = \{b, c, d\}$.

**Proposition 2.5** Let $(X, \mu)$ be a quasi topological space with a hereditary class $H$.

(a) If $H = \{\emptyset\}$ or the hereditary class $N$ of nowhere dense set of $(X, \mu)$, then $\mu_{\delta-H} = \mu_s$.

(b) If $H = P(X)$, then $\mu_{\delta-H} = \mu$.

**Proof:** Let $H = \{\emptyset\}$, then $S^* = c_\mu(S)$ for every subset $S$ of $X$. Let $A$ be $R-H$-open. Then $A = i_\mu (c_\mu^*(A)) = i_\mu (A \cup \mu^*) = i_\mu (c_\mu(A))$ and hence $A$ is regular open. Therefore, every $\delta$-H-open set is $\delta$-open and we obtain $\mu_{\delta-H} \subset \mu_s$. By Theorem 2.1, $\mu_{\delta-H} \subset \mu_s$. Next, let $H = N$. It is well known that $S^* = c_\mu (i_\mu (c_\mu (S)))$ for every subset $S$ of $X$. Let $A$ be any
R–H-open set. Then A is µ-open A = i_\mu (c_\mu^*(A)) = i_\mu (A \cup c_\mu (i_\mu (c_\mu (A)))) = i_\mu (c_\mu (A)). Hence A is regular open. Similarly to the case of H = \emptyset, hence µδ–H = µS.

(b) Let H = P(X). Then S^* = \emptyset for every subset S of X. Now, let A be any µ-open set of X. Then A = i_\mu (A) = i_\mu (A \cup c_\mu (i_\mu (c_\mu (A)))) = i_\mu (c_\mu (A)). Hence A is regular open. Similarly to the case of H = \{\emptyset\}, hence µδ–H = µ.

3. δ–H-continuous functions

A function f: (X, µ1, H) → (Y, µ2, I) is said to be δ–H-continuous if for each x ∈ X and each µ-open neighborhood V of f(x), there exists a µ-open neighborhood U of x such that f(U) ⊂ i_\mu (c_\mu^*(V)).

Theorem: 3.1 For a function f: (X, µ1, H) → (Y, µ2, I), the following properties are equivalent:

(a) f is δ–H-continuous,

(b) For each x ∈ X and each R–H-open set V containing f(x), there exists an R–H-open set containing x such that f(U) ⊂ V.

(c) f([A]δ–H) ⊂ [f(A)]δ–H for every A ⊂ X,

(d) [f^{-1}(B)]δ–H ⊂ f^{-1}([B]δ–H) for every B ⊂ Y,

(e) For every δ–H-closed set F of Y, f^{-1}(F) is δ–H-closed in X;

(f) For every δ–H-open set V of Y, f^{-1}(V) is R–H-open in X;

(h) For every δ–H-open set F of Y, f^{-1}(F) is R–H-open in X.

Proof:

(a) ⇒ (b): The proof is obvious.

(b) ⇒ (c): Let x ∈ X and A ⊂ X such that f(x) ∈ f([A] δ–H). Suppose that f(x) ∉ [f(A)] δ–H. Then, there exists an R–H-open neighborhood V of f(x) such that f(A) ∩ V = \emptyset. By (b), there exists an R–H-open neighborhood U of x such that f(U) ⊂ V. Since f(A) ∩ f(U) ⊂ f(A) ∩ V = \emptyset, f(A) ∩ f(U) = \emptyset.

Hence U ∩ A ⊂ f^{-1}(f(U)) ∩ f^{-1}(f(A)) = f^{-1}(f(U) ∩ f(A)) = \emptyset. Hence U ∩ A = \emptyset and x ∉ [A] δ–H.

Therefore f(x) ∉ [f([A] δ–H)]. This is a contradiction. Therefore f(x) ∈ [f([A] δ–H)].

(c) ⇒ (d): Let B ⊂ Y such that A = f^{-1}(B). By (c), f([f^{-1}(B)] δ–H) ⊂ [f(f^{-1}(B)] δ–H) ⊂ [B] δ–H. Therefore [f^{-1}(B)] δ–H ⊂ [(f^{-1}(B)] δ–H) ⊂ [f^{-1}(B)] δ–H).

(d) ⇒ (e): Let F ⊂ Y be δ–H-closed. BY (d), [f^{-1}(F)] δ–H ⊂ f^{-1}([F] δ–H = f^{-1}(F).

Therefore f^{-1}(F) is δ–H-closed.

(e) ⇒ (f): Let V ⊂ Y be δ–H-open. Then Y – V is δ–H–closed. By (e) f^{-1}(Y – V) = X – f^{-1}(V) is δ–H-closed. Therefore, f^{-1}(V) is δ–H–open.

(f) ⇒ (g): Let V ⊂ Y be R–H-open. Since every R–H-open set is δ–H–open, V is δ–H–open, by (f), f^{-1}(V) is δ–H–open.
(g) ⇒ (h): Let $F \subset Y$ be $R-H$ closed. Then $Y - F$ is $R-H$ open. By (g) $f^{-1}(Y - F) = X - f^{-1}(F)$ is $R-H$-open. Therefore $X - f^{-1}(F)$ is $\delta - H$-open. Therefore, $f^{-1}(F)$ is $\delta - H$-closed.

(h) ⇒ (a): Let $x \in X$ and $V$ be a $\mu$-open set containing $f(x)$. Now, $V_0 = i_\mu(c^*(V))$, then by Lemma 2.1 $Y - V_0$ is an $R-H$-closed set. By (8), $f^{-1}(Y - V_0) = X - f^{-1}(V_0)$ is $\delta - H$-closed set. Therefore, $f^{-1}(V_0)$ is $\delta - H$-open. Hence $f(i_\mu(c_\mu(U))) \subset i_\mu(c_\mu(V))$. Hence $f$ is a $\delta - H$-continuous function.

Corollary: 3.2 A function $f : (X, \mu_1, H) \rightarrow (Y, \mu_2, I)$ is $\delta$-H-continuous if and only if $f : (X, \mu_1, H) \rightarrow (Y, \mu_2, I)$ is continuous.

Proof: This is an immediate consequence of Theorem 2.3.

Theorem: 3.3 If $f: (X, \mu_1, H) \rightarrow (Y, \mu_2, I)$ and $g : (Y, \mu_2, I) \rightarrow (Z, \mu_3, J)$ are $\delta$-H-continuous, then so is $g \circ f : (X, \mu_1, H) \rightarrow (Z, \mu_3, J)$.

Proof: It follows immediately from Corollary 3.1.

A function $f: (X, \mu_1, H) \rightarrow (Y, \mu_2, I)$ from one GTS $(X, \mu_1)$ with a hereditary class $H$ to another $(Y, \mu_2)$ with a hereditary class $I$ is said to be strongly $\theta$-H-continuous (resp. $\theta$-H-continuous, almost-H-continuous) if for each $x \in X$ and each $\mu$-open neighborhood $V$ of $f(x)$, there exists a $\mu$-open neighborhood $U$ of $x$ such that $f(c_\mu(U)) \subset V$ (resp. $f(c_\mu(U)) \subset c_\mu(V)$, $f(U) \subset i_\mu(c_\mu(V))$). A function $f : (X, \mu_1, H) \rightarrow (Y, \mu_2, I)$ is said to be almost-H-open if for each $R-H$-open set $U$ of $X$, $f(U)$ is $\mu$-open in $Y$.

Theorem: 3.4 (a) If $f : (X, \mu_1, H) \rightarrow (Y, \mu_2, I)$ is strongly $\theta$H-continuous and $g : (Y, \mu_2, I) \rightarrow (Z, \mu_3, J)$ almost-H-continuous, then $g \circ f : (X, \mu_1, H) \rightarrow (Z, \mu_3, J)$ is $\delta$-H-continuous. (b) The following implications hold:

strongly $\theta$-H-continuous $\Rightarrow$ $\delta$-H-continuous $\Rightarrow$ almost-H-continuous.

Proof: (a) Let $x \in X$ and $W$ be any $\mu$-open set of $Z$ containing $(g \circ f)(x)$. Since $g$ is almost-H-continuous, there exists a $\mu$-open neighborhood $V \subset Y$ of $f(x)$ such that $g(V) \subset i_\mu(c_\mu(W))$. Since $f$ is strongly $\theta$H-continuous, there exists a $\mu$-open neighborhood $U \subset X$ of $x$ such that $f(c_\mu(U)) \subset V$. Hence $g(f(c_\mu(U))) \subset g(V)$ and $g(f(i_\mu(c_\mu(U)))) \subset g(f(c_\mu(U))) \subset g(V) \subset i_\mu(c_\mu(U))$. Hence, $g \circ f : (X, \mu_1, H) \rightarrow (Z, \mu_3, J)$ is $\delta$-H-continuous.

(b) Let $f$ be strongly $\theta$-H-continuous. Let $x \in X$ and $V$ be any $\mu$-open neighborhood of $f(x)$. Then, there exists a $\mu$-open neighborhood $U \subset X$ of $x$ such that $f(c_\mu(U)) \subset V$. Also $f(i_\mu(c_\mu(U))) \subset f(c_\mu(U)) \subset V$. Since $V$ is $\mu$-open, $f(i_\mu(c_\mu(U))) \subset i_\mu(c_\mu(U))$. Thus $f$ is $\delta$-H-continuous. Let $f$ be $\delta$-H-continuous.

Now we prove that $f$ is almost H-continuous. Then, for each $x \in X$ and each $\mu$-open neighborhood $V \subset Y$ of $f(x)$, there exists a $\mu$-open neighborhood $U \subset X$ of $x$ such that $f(i_\mu(c_\mu(U))) \subset i_\mu(c_\mu(V))$. Since $U \subset i_\mu(c_\mu(U))$, $f(U) \subset i_\mu(c_\mu(V))$.

Hence $f$ is almost H-continuous. A GTS $(X, \mu)$ with a hereditary class $H$ is said to be SI-R space if for each $x \in X$ and each $\mu$-open neighborhood $V$ of $x$, there exists a $\mu$-open neighborhood $U$ of $x$ such that $x \in U \subset i_\mu(c_\mu(U)) \subset V$. 

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Theorem: 3.5 For a function $f: (X, \mu_1, H) \rightarrow (Y, \mu_2, I)$, the following are true:

(a) If $Y$ is an SH-R space and $f$ is $\delta$-H-continuous, then $f$ is continuous.
(b) If $X$ is an SH-R space and $f$ is almost H-continuous, then $f$ is $\delta$-H-continuous.

Proof:
(a) Let $Y$ be an SH-R space. Then, for each $\mu$-open neighborhood $V$ of $f(x)$, there exists a $\mu$-open neighborhood $V_\delta$ of $f(x)$ such that $f(x) \in V \cap (\mu_1^{-1}(c_\mu(V)) \cap V)$. Since $f$ is $\delta$-H-continuous, there exists a $\mu$-open neighborhood $U_\delta$ of $x$ such that $f(\mu_1^{-1}(c_\mu(U_\delta))) \subseteq f(U_\delta)$ and $f(U_\delta) \subseteq V$. Thus $f(U_\delta) \subseteq V$, hence $f$ is continuous.

(b) Let $x \in X$ and $V$ be a $\mu$-open neighborhood of $f(x)$. Since $f$ is almost-H continuous, there exists a $\mu$-open neighborhood $U_1$ of $x$ such that $\mu_1(c_\mu(U_1)) \subseteq U$. Thus $f(\mu_1^{-1}(c_\mu(U_1))) \subseteq f(U_1)$ and $f(U_1) \subseteq V$. Therefore, $f$ is $\delta$-H-continuous.

Corollary: 3.6 If $(X, \mu_1)$ with hereditary class $H$ and $(Y, \mu_2)$ with hereditary class $I$ are SH-R spaces, then the following concepts on a function $f: (X, \mu_1, H) \rightarrow (Y, \mu_2, I)$: $\delta$-H-continuity, continuity, almost-H-continuity are equivalent.

Proof: The proof follows from Theorem 3.7. A quasi topological space $(X, \mu)$ with a hereditary class $H$ is said to be an AH-R space if for each $R$-$H$-closed set $F \subseteq X$ and each $x \notin F$, there exist disjoint $\mu$-open sets $U$ and $V$ in $X$ such that $x \in U$ and $F \subseteq V$.

Theorem: 3.7 A quasi topological space $(X, \mu)$ with a hereditary class $H$ is an AH-R space if and only if each $x \in X$ and each $R$-$H$-open neighborhood $V$ of $x$, there exists an $R$-$H$-open neighborhood $U$ of $x$ such that $x \in U \cap c^*_\mu(U) \subseteq c_\mu(U) \subseteq V$.

Proof: Suppose $(X, \mu)$ with a hereditary class $H$ is an AI-R space. Let $x \in V$ and $V$ be $R$-$H$-open. Then $\{x\} \cap (X - V) = \emptyset$. Since $X$ is an AI-R space, there exist $\mu$-open sets $U_1$ and $U_2$ containing $x$ and $X - V$ respectively, such that $U_1 \cap U_2 = \emptyset$. Then $c_\mu(U_1) \cap c_\mu(U_2) = \emptyset$ and hence $c_\mu(U_1) \subseteq c_\mu(U_1) \subseteq (X - U_2) \subseteq V$. Thus $x \in U_1 \subseteq c^*_\mu(U_1) \subseteq c_\mu(U_1) \subseteq V$ and we have $U_1 \subseteq c_\mu(U_1) \subseteq U_2$. Let $i_\mu(c_\mu(U_1)) = U$. Thus $c\mu(U_1) = c\mu(i_\mu(U_1)) \subseteq c\mu(c\mu(U_1)) < c\mu(c\mu(U_1)) < c\mu(U_1) \subseteq c_\mu(U_1) \subseteq (X - c^*_\mu(U_1)) \subseteq V$. Therefore, there exists an $R$-$H$-open set $U$ such that $x \in U \subseteq c^*_\mu(U) \subseteq c_\mu(U) \subseteq V$. Conversely, let $x \in X$ and an $R$-$H$-closed set $F$ such that $x \notin F$. Then, $X - F$ is an $R$-$H$-open neighborhood of $x$.

By hypothesis, there exists an $R$-$H$-open neighborhood $V$ of $x$ such that $x \notin V \subseteq c^*_\mu(V) \subseteq c_\mu(V) \subseteq X - F$. Thus $F \subseteq X - c_\mu(V) \subseteq (X - c^*_\mu(V))$. Hence $F \subseteq X - c_\mu(V) \subseteq (X - c^*_\mu(V)) \subseteq c_\mu(U) \subseteq c_\mu(U_2) \subseteq X - V$. Therefore, $V \cap (X - c_\mu(V)) = \emptyset$ and $V$ is $\mu$-open. Therefore, $X$ is an AH-R space.

Theorem: 3.8 For a function $f: (X, \mu_1, H) \rightarrow (Y, \mu_2, I)$, the following are hold:

(a) If $Y$ is an AI-R space and $f$ is $\theta$-H-continuous, then $f$ is $\delta$-H-continuous.
(b) If $X$ is an AI-R space, $Y$ is an SH-R space and $f$ is $\delta$-H-continuous, then $f$ is strongly $\theta$-H-continuous.

Proof:
(a) Let $Y$ be an AH-R space. Then, for each $x \in X$ and each $R$-$H$-open neighborhood $V$ of $f(x)$, there exists an $R$-$H$-open neighborhood $V_\theta$ of $f(x)$ such that $f(x) \in V_\theta \subseteq c_\theta(V) \subseteq V$. Since $f$ is $\theta$-H-continuous, there exists a $\mu$-open neighborhood $U$ of $x$ such that $f(U) \subseteq c_\mu(V_\theta)$. Hence $f(i_\mu(c_\mu(U))) \subseteq f(c_\mu(U)) \subseteq c_\mu(V) \subseteq V$ and thus $f(i_\mu(c_\mu(U))) \subseteq V$. By Theorem 3.1, $f$ is $\delta$-H-continuous.
(b) Let $X$ be an AHR space, $Y$ an SH-R space. For each $x \in X$ and each $\mu$-open neighborhood $V$ of $f(x)$, there exists a $\mu$-open set $V_\circ$ such that $f(x) \in V_\circ \subset i_\mu(c_\mu*(V))$. Since $Y$ is an SH-R space, since $f$ is $\delta$-H continuous, there exists a $\mu$-open set $U$ of $x$ such that $f(i_\mu(c_\mu(U))) \subset i_\mu(c_\mu*(V))$. By Lemma 2.1, $i_\mu(c_\mu(U))$ is R-H-open and since $X$ is an AI-R space, by Theorem 3.7, there exists an R-H-open set $U_\circ$ such that $x \in U_\circ \subset c_\mu*(U) \subset i_\mu(c_\mu(U))$. But every R-H-open set is $\mu$-open, hence $U$ is $\mu$-open. Also, $f(c_\mu(U)) \subset V$. Hence $f$ is strongly $\theta$-H-continuous.

**Theorem: 3.9** If a function $f: (X, \mu_1, H) \rightarrow (Y, \mu_2, I)$ is $\theta$-H-continuous and almost- H-open, then $f$ is $\delta$-H-continuous.

**Proof:** Let $x \in X$ and $V$ be a $\mu$-open neighborhood of $f(x)$. Since $f$ is $\theta$-H-continuous, there exists a $\mu$-open neighborhood $U$ of $x$ such that $f(c_\mu(U)) \subset c_\mu*(V)$. Hence $f(i_\mu(c_\mu(U))) \subset c_\mu*(V)$. Since $f$ is almost- H-open, $f(i_\mu(c_\mu(U))) \subset i_\mu(c_\mu*(V))$. This shows $f$ is strongly $\theta$-H-continuous.

**REFERENCES**


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