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MAGNETOHYDRODYNAMICS FLOW OF AN OSCILLATORY BOUNDARY LAYER FLOW BOUNDED BY TWO HORIZONTAL FLAT PLATES UNDER UNIFORM ROTATION

Yasha Sharma[1] and Vijay Gupta[2*]

[1]Deptt. of Applied Mathematics, Vidisha (M.P.)
[2]Deptt. of Applied Mathematics UIT, RGPV Bhopal (M. P.)

E-mail: vkgupta_12873@rediffmail.com

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ABSTRACT

In This paper an exact solution is developed for an oscillatory boundary layer flow bounded by two horizontal flat plates one of which is oscillating in its own plane & other at rest. Rotating flow of a second grade conducting fluid on an infinite oscillating plate is investigated when the fluid is permeated by a transverse magnetic field and the Hall effects are taken into account. It is once again found that an asymptotic solution exists in the presence of both suction & blowing at the plate. For fixed magnetic field parameter the boundary layer thickness increases with the increase in Hall parameters.

BASIC EQUATIONS:

It is known that an electrical conductor moving in a magnetic field generates an electromotive force that is proportional to its speed of motion and the magnetic field strengths. The coupling between the fluid flow equations and the electromagnetic field equations will take place. The fluid has been electrically conducting. The field of mhd involves the solution of both the momentum equations characterizing fluid flow and Maxwell equations for the magnetic field, so it is complicate. In magneto fluid mechanics, Maxwell equations are presented as follows:

$$\nabla.\mathrm{B} = 0, \tag{1}$$

$$\nabla.\mathrm{E} = 0, \tag{2}$$

$$\nabla\times\mathrm{B} = \mu_e J, \tag{3}$$

$$\nabla\times\mathrm{E} = -\frac{\partial \mathrm{B}}{\partial t}, \tag{4}$$

Where μ_e is the magnetic permeability By Ohm's law the total current flow can be defined as:

$$J = \sigma(\mathrm{E} + V\times B), \tag{5}$$

Where σ is the electrical conductivity. In momentum equation we have to include the electromagnetic force F_m. It is expressed as:

$$F_m = J\times\mathrm{B} = \sigma(V\times\mathrm{B})\times\mathrm{B}\,. \tag{6}$$

FORMULATION OF THE PROBLEM:

Consider unsteady flow of a viscous incompressible electrically conducting second grade fluid bounded by an infinite parallel plates disant h apart, when both fluid & plates rotate with a uniform angular velocity Ω about the Z-axis which is normal to the planes of the plates. It is assumed that Plates are electrically non -conducting and a uniform magnetic field B_0 applied parallel to Z-axis.

Corresponding author: *Vijay Gupta[2], *E-mail: vkgupta_12873@rediffmail.com

Initially (i.e. when time $t \le 0$),fluid as well as lower plate is assumed to be at rest (Z=0) and the upper plate (Z=h) oscillating in its own plane with a velocity $U_1(t) = U_0(1+\varepsilon\cos\omega t)$ {t > 0} about a non –zero uniform mean velocity U_0 along X-direction in its own plane. Since plates of the channel are infinite along X & Y direction and are electrically non-conducting, all the physical quantities except the pressure depend on Z & t only. The condition of incompressibility yields ω = constant = 0, since plates are not porous and thus the velocity field is defined as:

$$V = (u_1(z,t), u_2(z,t), 0) \tag{7}$$

We are considering second grade fluid, then we have:

$$\frac{\partial u_1}{\partial t} = -\frac{1}{\sigma}\frac{\partial p}{\partial x} + (u_2 + \alpha\frac{\partial}{\partial t})\frac{\partial^2 u_1}{\partial z^2} + 2\Omega u_2 - \frac{\eta B_0^2 u_1}{\sigma} \tag{8}$$

$$\frac{\partial u_2}{\partial t} = -\frac{1}{\sigma}\frac{\partial p}{\partial y} + (u_2 + \alpha\frac{\partial}{\partial t})\frac{\partial^2 u_2}{\partial z^2} - 2\Omega u_1 - \frac{\eta B_0^2 u_2}{\sigma} \tag{9}$$

$$0 = -\frac{1}{\sigma}\frac{\partial p}{\partial z} \ , \tag{10}$$

In which $u_2 = \dfrac{\mu}{\sigma}$ is the kinematic viscosity $\alpha = \dfrac{\alpha_1}{\sigma}$. Equation (10) indicates that p is not a function of Z and hence p is at most depend on x, y & t, so considering

$$p = p(x, y, t)$$

The boundary conditions for the problem are:

$u_1 = u_2 = 0$, at $z = 0$

$$u_1 = U_1(t) = U_0(1+\varepsilon\cos\omega t), \quad u_2 = 0 \text{ at } z = h, \tag{11}$$

Where ε is a constant.

Elimination of p from equation (8) – equation (10) by cross differentiation gives:

$$\frac{\partial^2 u_1}{\partial z\partial t} = (u_2 + \alpha\frac{\partial}{\partial t})\frac{\partial^3 u_1}{\partial z^3} + 2\Omega\frac{\partial u_2}{\partial z} - \frac{\eta B_0^2}{\sigma}\frac{\partial u_1}{\partial z} \tag{12}$$

$$\frac{\partial^2 u_2}{\partial z\partial t} = (u_2 + \alpha\frac{\partial}{\partial t})\frac{\partial^3 u_2}{\partial z^3} - 2\Omega\frac{\partial u_2}{\partial z} - \frac{\eta B_0^2}{\sigma}\frac{\partial u_2}{\partial z} \tag{13}$$

Integrating above equation gives:

$$\frac{\partial u_1}{\partial t} = (u_2 + \alpha\frac{\partial}{\partial t})\frac{\partial^2 u_1}{\partial z^2} + 2\Omega u_2 - \frac{\eta B_0^2 u_1}{\sigma} + A(t) \tag{14}$$

$$\frac{\partial u_2}{\partial t} = (u_2 + \alpha\frac{\partial}{\partial t})\frac{\partial^2 u_2}{\partial z^2} - 2\Omega u_1 - \frac{\eta B_0^2 u_2}{\sigma} + B(t) \tag{15}$$

Where A & B are functions of integration. The resulting boundary layer equations of equation (13) & equation (14) can be combined into following partial differential equation.

$$\frac{\partial r}{\partial t} = (u_2 + \alpha \frac{\partial}{\partial t})\frac{\partial^2 r}{\partial z^2} + \frac{\partial U_1}{\partial t} - 2i\Omega(r - U_1) - \frac{\eta B_0^2}{\sigma}(r - U_1) \tag{16}$$

And the corresponding boundary conditions (11) are:

$$r = 0 \quad \text{At} \quad z = 0$$

$$r = U_1(t) \quad \text{At} \quad z = h. \tag{16'}$$

Where $r = u_1 + iu_2$ (17)

Is the fluid velocity in the complex form.
It should be noted that equation (16) includes the Newtonian fluid as a special case for $\alpha = 0$.
If $\Omega = 0$, the equation reduces to that of second grad fluid in an inertial frame. Moreover if $B_0 = 0$, the equation governing the flow of a non –conducting second grade fluid is obtained.

SOLUTION:

In order to solve equation (16) subject to the boundary conditions (16'), we look for the solution of the form:

$$r(\xi, t) = U_0[r_0(\xi) + \frac{\varepsilon}{2}\{r_1(\xi)e^{i\omega t} + r_2(\xi)e^{-i\omega t}\}] \tag{18}$$

Where $\xi = \frac{z}{h}$, $r_0(\xi) = u_{10}(\xi) + iu_{20}(\xi)$ and

$$r_1(\xi)e^{i\omega t} + r_2(\xi)e^{-i\omega t} = u_{11}(\xi, t) + iu_{21}(\xi, t) \tag{19}$$

Using equation (18) into equation (16) & boundary conditions (16') and then collecting harmonic and nonharmonic terms, we obtain:

$$\frac{d^2 r_0}{d\xi^2} - (2ik + M^2)r_0 = -(2ik + M^2) \tag{20}$$

$$\frac{d^2 r_1}{d\xi^2} - (1 + \lambda\beta)^{-1}[2ik + M^2 + i\lambda]r_1 = -(1 + \lambda\beta)^{-1}[2ik + M^2 + i\lambda] \tag{21}$$

$$\frac{d^2 r_2}{d\xi^2} - (1 + \lambda\beta)^{-1}[2ik + M^2 - i\lambda]r_2 = -(1 + \lambda\beta)^{-1}[2ik + M^2 - i\lambda] \tag{22}$$

$$r_0 = r_1 = r_2 = 0, \quad \text{at} \quad \xi = 0$$
$$r_0 = r_1 = r_2 = 1, \quad \text{at} \quad \xi = 1 \tag{23}$$

From equation (20):

Auxiliary equtn -

$$m^2 - (2ik + M^2) = 0$$
$$m = \pm\sqrt{2ik + M^2}$$

Then C.f. = $Ae^{\ell\xi} + Be^{-\ell\xi}$ where A, B are constants and $\ell = \sqrt{2ik + M^2}$ & P.I. is

$$\frac{1}{D^2-(2ik+\mathrm{M}^2)}\{-(2ik+\mathrm{M}^2)e^{0\xi}\} = 1$$

$$r_0(\xi) = \mathrm{A}e^{\ell\xi} + \mathrm{B}e^{-\ell\xi} + 1 \qquad (24)$$

Using boundary conditions (23) in equation (24), we obtain

$$0 = \mathrm{A} + \mathrm{B} + 1$$
$$1 = 1 + \mathrm{A}e^{\ell} + \mathrm{B}e^{-\ell}$$

Solving these equtns we get

$$\mathrm{B} = -\frac{e^{\ell}}{(e^{\ell}-e^{-\ell})} = -\frac{e^{\ell}}{2\sinh\ell}$$
$$\mathrm{A} = \frac{e^{-\ell}}{(e^{\ell}-e^{-\ell})} = \frac{e^{-\ell}}{2\sinh\ell}$$

Making use of equation (24) we finally get:

$$r_0(\xi) = 1 + \frac{e^{-\ell}}{2\sinh\ell}e^{\ell\xi} - \frac{e^{\ell}}{2\sinh\ell}e^{-\ell\xi}$$

$$= 1 + \frac{1}{2\sinh\ell}(e^{-\ell}e^{\ell\xi} - e^{\ell}e^{-\ell\xi})$$

$$\Rightarrow r_0(\xi) = 1 - \frac{\sinh\ell(1-\xi)}{\sinh\ell} \qquad (25)$$

Now from equtn (21):

Auxiliary equation –

$$m^2 - (1+\lambda\beta)^{-1}[2ik + \mathrm{M}^2 + i\lambda] = 0$$

$$m = \pm\left[\frac{2ik + \mathrm{M}^2 + i\lambda}{1+\lambda\beta}\right]^{1/2} \quad \text{and}$$

$$\Rightarrow r_1(\xi) = 1 + \mathrm{A}e^{m\xi} + \mathrm{B}e^{-m\xi}$$

Also from equtn (22):

$$r_2(\xi) = 1 + \mathrm{A}e^{n\xi} + \mathrm{B}e^{-n\xi} \quad \text{Where} \quad n = \pm\left[\frac{2ik + \mathrm{M}^2 - i\lambda}{1+\lambda\beta}\right]^{1/2}$$

Employing the same procedure as for r_0, the solution of equation (21) & equation (22) subject to boundary conditions (23) are given by:

$$r_1(\xi) = 1 - \frac{\sinh m(1-\xi)}{\sinh m} \qquad (26)$$

And $$r_2(\xi) = 1 - \frac{\sinh n(1-\xi)}{\sinh n} \tag{27}$$

The solution (25) corresponds to the steady part which gives u_{10} & u_{20} as the primary and secondary Velocity components respectively . From equation (25) we have for large k :

$$u_{10} \approx 1 - e^{-\ell_R \xi} \cos \ell_I \xi \tag{28}$$

$$u_{20} \approx e^{-\ell_{Rr}\xi} \sin \ell_I \xi \tag{29}$$

Where $$\ell_R = \frac{1}{\sqrt{2}}[M^2 + \sqrt{M^4 + 4k^2}]^{1/2} \, and$$
$$\ell_I = \frac{1}{\sqrt{2}}[-M^2 + \sqrt{M^4 + 4k^2}]^{1/2}$$

And R & I in the subscripts indicate the real & imaginary parts. The solution (26) & (27) together give the unsteady part of the flow. These solutions depend on β, for large k the primary and secondary velocity components u_1 & u_2 respectively. For the fluctuating flow are given by:

$$u_{11}(\xi, t) \approx 2\cos \omega t - e^{-m_R \xi} \cos(m_I \xi - \omega t) - e^{-n_R \xi} \cos(n_I \xi + \omega t) \tag{30}$$

$$u_{21}(\xi, t) \approx e^{-m_R \xi} \sin(m_I \xi - \omega t) + e^{-n_R \xi} \sin(n_I \xi + \omega t) \tag{31}$$

In which
$$m_R = (C_1)^{-1}[\sqrt{\sqrt{A_1^2 + B_1^2} + A_1}]$$
$$m_I = (C_1)^{-1}[\sqrt{\sqrt{A_1^2 + B_1^2} - A_1}]$$
$$n_R = (C_1)^{-1}[\sqrt{\sqrt{A_2^2 + B_2^2} + A_2}]$$
$$n_I = (C_1)^{-1}[\sqrt{\sqrt{A_2^2 + B_2^2} - A_2}]$$
$$A_1 = M^2 + (2k + \lambda)\lambda\beta$$
$$A_2 = M^2 + (2k - \lambda)\lambda\beta$$
$$B_1 = (2k + \lambda) - M^2\lambda\beta$$
$$B_2 = (2k - \lambda) - M^2\lambda\beta$$
$$C_1 = \sqrt{2(1 + \lambda^2\beta^2)}$$

We note that steady solution (25) is independent on β. It means that primary & secondary velocity components u_{10} & u_{20} respectively for present steady flow do not depend upon nature of the fluid.

The amplitudes and phase differences in terms of u_{10} & u_{20} are given by:

$$R_0 = \sqrt{u_{10}^2 + u_{20}^2}, \qquad \theta_0 = \tan^{-1}\frac{u_{20}}{u_{10}}, \tag{32}$$

And for unsteady flow

$$R_1 = \sqrt{u_{11}^2 + u_{21}^2},$$
$$\theta_1 = \tan^{-1}\frac{u_{21}}{u_{11}}, \tag{33}$$

RESULT AND DISCUSSION:

The investigation of the velocity, magnetic field ,frequency & non-newtonian effects on the flow of an incompressible conducting fluid bounded between two rigid non-conducting parallel plates have been carried out in the preceding paragraphs. The solutions are obtained for steady and unsteady velocity field from equation (25) to (27).Numerical computions are discussed in the following points:

(1) The primary velocity u_{10}, secondary velocity u_{20}, resultant velocity R_0 and phase angle θ_0 are shown for various values of k & M. It is observed that u_{10} increases with increase o M for Small k ,however, for large rotation parameter k, u_{10} decreases with the increases of M and is approximately one for large k in the upper half of the channel width. It shows that u_{20} increases in the lower half of the channel for small k and becomes approximately zero in the upper half of the channel width. These observations can also be expected from equation (28) and (29).

These equations show the existence of a thin boundary layer of order $\mathrm{O}(\ell_R^{-1})$ in the vicinity of the lower plate which decreases with the increase in Hartmann number M or the rotation parameter k. The behavior of R_0 is almost the same as that of u_{10}, and θ_0 decreases with increasing M for any value of rotation large to small. It is also evident that θ_0 increases with small rotation whereas it decreases with large rotation and is approximately zero in the upper half of the channel.

(2) The expressions (30) & (31) represent the shear oscillations are ω/m_1 & ω/n_1 and the amplitude of these these oscillations decay exponentially with ξ.These expression also show the emergence of a boundary layer of thickness of order $\mathrm{O}(m_R^{-1})$ superimposed with a boundary layer of thickness of order $\mathrm{O}(n_R^{-1})$. These boundary layers which are a direct consequence of the cyclonic and anticyclonic components of the imposed harmonic oscillations decrease with increase in M, β & k. It may be noted that in second grade fluid the boundary layer thickness increases. Also, the present analysis exhibits a striking difference between the structure of hydrodynamic & the hydromagnetic boundary layers.

(3) In case of resonance $(2\Omega - \omega = 0 or 2k - \lambda = 0)$, the solution of equation which represent the value of m & n in equation (26) & (27) is :

$$r_2(\xi) = 1 - \frac{\sinh \tilde{\mathrm{M}}(1-\xi)}{\sinh \tilde{\mathrm{M}}} \tag{34}$$

Where $$\tilde{\mathrm{M}}^2 = \mathrm{M}^2(1 + i\lambda\beta)^{-1} \tag{35}$$

We note that when $\mathrm{M} = 0$ then $r_2(\xi)$ for Newtonian and second grade fluids is the same and is given by

$$r_2(\xi) = \xi.$$

CONCLUSION:

The effect of rotation & magnetic field on unsteady couette flow of a viscous incompressible electrically conducting fluid between two horizontal parallel porous plates in a rotating medium is investigated. It is found that magnetic field has tendency to retard the fluid flow in both the primary & secondary flow directions. Rotation retards primary flow whereas it accelerates secondary flow. Also there exists incipient flow reversal near the stationary plate in primary flow Direction on increasing rotation parameter. Suction accelerates primary flow whereas it retards secondary flow.

Injection retards both the primary & secondary flows, fluid flow in both the primary & secondary flow directions increases on increasing time t and the solution for small values of time t, obtained by Laplace transforms technique converges more rapidly than that of general solution.

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