International Journal of Mathematical Archive-5(7), 2014, 57-74
Available online through www.ijma.info ISSN 2229 – 5046

VARIABLE PREDICTION USING DATA ENVELOPMENT ANALYSIS

Ramesh Kumar N[1] and Vijaya Kumar K*[2]

*[1]Lecturer, Department of Statistics,
Sri Venkateswara Arts College, Tirupati, Andhra Pradesh, India.*

*[2]Lecturer, Department of Statistics,
Sri Govindarajaswamy Arts College, Tirupati, Andhra Pradesh, India.*

(Received On: 30-06-14; Revised & Accepted On: 28-07-14)

ABSTRACT

Simple formulae are derived to estimate potential revenue for a given target cost and potential inputs for given target revenue. Using these expression input losses are estimated for the total manufacturing sectors of 14 Indian states which account for 85 percent of country's total value added. The total manufacturing sector of all India is considered as a DMU to estimate the structural efficiency.

INTRODUCTION

In classical economic theory, if cost is given as a target the producer conditionally maximizes total revenue. However, if revenue is targeted the cost is minimized conditionally. These optimization problems are so solved that no concern is shown to the production units that are currently in competition. If one or more of the units in competition employ the best practice technique the producer of concern may turn out to be inefficient. Thus, the producer, either cost minimize or revenue maximize should model inefficiency into his optimization problems. The basic tool to model inefficiency is production function or its dual cost function.

The idea of production frontier and productive efficiency were first envisaged by Farrell (1957) and later had been well developed by Farrell and Fieldhouse (1962), Aigner and Chu (1968), Shephard (1970), Timmer (1971), Afriat (1972), Richmond (1974), Aigner, Lovell and Schmidt (1977) Fare (1978), Schmidt and Lovell (1979), Kopp (1981), Banker, Charness and Cooper (1984), Kalirajan (1985), Charness., Cooper and Thrall (1986), Greene (1990), Vijaya Kumar (2003), Ramesh Kumar (2003) and so on.

Objectives of the Study

The present study aims at predicting potential revenue against target cost and potential inputs against target revenue. It distinguishes inefficient decision making units (DMU's) from efficient units. Two parametric frontier production functions (i) the Cobb-Douglas and (ii) the variable returns to scale are considered as basic tools. Their dual factor minimal cost frontiers and expressions for potential input vectors for given target revenue and potential revenue for given target cost are derived.

The resultant measures are implemented for decision making units (DMU's) where each DMU is the total manufacturing sector of All India or a state of India. 14 states are considered as DMU's and these have also a DMU, which is included to estimate structural efficiency of the country. Potential revenues and potential inputs are predicted. For prediction, in particular, when parametric frontiers are used expressions of factor minimal cost and optimal revenue functions are required. These functional forms depend on certaing parameters which are estimated by postulating and solving certain programming problems. Since these estimates lack statistical properties. They are not amenable for any tests of significance.

Corresponding author: Vijaya Kumar K[2], [2]Lecturer, Department of Statistics,
Sri Govindarajaswamy Arts College, Tirupati, Andhra Pradesh, India.*

THE MODEL

For prediction Shephard's (1970) input and output sets are considered
Input Sets:

$$L\left(\frac{r}{R}\right) = \{x : R\ (x, r)\ \geq\ R\}$$

$$= \left\{x : R\left(x, \frac{r}{R}\right) \geq 1\right\}$$

where x: Input Vector
R(x, r): Optimal revenue
r: Output price vector
R: Target revenue

Output sets:

$$P\left(\frac{p}{c}\right) = \{u : Q\ (u, p)\ \leq c\}\ =\ \left\{u : Q\left(u, \frac{P}{c}\right) \leq 1\right\}$$

Where u: Output vector, P: Input Price Vector, C: Target Cost

Potential revenue is obtained by solving,

$$r\,\Gamma(p) = r\ \underset{u}{\mathrm{Max}}\ \{u : Q\ (u, p) \leq C\}$$

Potential inputs are obtained by solving

$$\mathrm{Min}\ \{\lambda : R\ (\lambda\ x, r) \geq R\}\ = \hat{\lambda}$$

Potential efficient inputs: $\hat{\lambda}\ x$

TARGET COST – POTENTIAL OUTPUT

Target cost is the cost specified by the entrepreneur whose desire is to predict potential revenue which not only depends on the target cost but also on other factors such as returns to scale.

If u is scalar valued potential output can be derived by solving the following optimization problem.

$$\Gamma(p) = \underset{u}{\mathrm{Min}}\ \{u : Q\ (u, p) \leq C\}$$

Where $\Gamma(p)$ is the Cost limited maximal output function

COBB-DOUGLAS PRODUCTION FONTIER–COST LIMITED MAXIMAL OUTPUT FUNCTION

Theorem: If the underlying production frontier is Cobb-Douglas of the form, $\hat{u} = A \prod_{i=1}^{n} x_i^{\alpha_i}$, then the cost limited maximal output function is $\Gamma(p) = \dfrac{C^{\theta}}{\left[B \prod p_i^{\alpha_{i/\theta}}\right]^{\theta}}$,

where C is the target cost

Proof: From the principle of duality we have

$$\frac{p}{Q(u, p)} = \nabla_x\ D(u, x)$$

Where p is the vector of prices, Q (u, p) is the factor minimal cost. The i[th] component of the vector $\nabla_x\ D(u, x)$ is, $\dfrac{\partial}{\partial x_i} D(u, x)$, D (u, x) being the input distance function.

For i[th] component we have $\dfrac{p_i}{Q(u, p)} = \dfrac{\partial}{\partial x_i}\left(\dfrac{\phi(x)}{u}\right)$

Since $\frac{\phi(x)}{u}$ is the input distance equation

$$\frac{p_i}{Q(u,p)} = \frac{1}{u}\frac{\partial \phi(x)}{\partial x_i} = \frac{1}{u}\frac{\alpha_i}{x_i}\phi(x)$$

$$p_i = Q\,(u,p)\,\frac{\phi(x)}{u}\,\frac{\alpha_i}{x_i}$$

$$\Rightarrow x_i = Q(u,p)\frac{\phi(x)}{u}\,\alpha_i\; p_i^{-1}$$

Substituting this in the frontier and by simplification we arrive at,

$$Q(u,p) = u^{\frac{1}{\theta}}\;[D(u,x)]^{\frac{1}{\theta}-1}\;B\left[\prod_i p_i^{\alpha_i/\theta}\right]$$

Since cost is minimized at x* which belongs to the isoquant of L (u),

$D\,(u, x^*) = 1$

Consequently, $Q(u,\,p) = u^{\frac{1}{\theta}}\;B\left[\prod_i p_i^{\alpha_i/\theta}\right]$

Where $B = A^{-\frac{1}{\theta}}\left[\prod_i \alpha_i^{-\alpha_i/\theta}\right]$

The cost limited maximal output is defined as,

$$\Gamma(p) = \underset{u}{\mathrm{Max}}\;\left\{u : Q\,(u,p) \le C\right\}$$

$$= \underset{u}{\mathrm{Max}}\left\{u : u^{\frac{1}{\theta}}\;B\prod_i p_i^{\alpha_i/\theta} \le C\right\}$$

Since Q (u, p) in this case is not only continuous in prices but also in output, we can find u^* such that

$$\Gamma\,(p) = \underset{u}{\mathrm{Max}}\left\{u : u^{\frac{1}{\theta}}\;B\prod_i p_i^{\alpha_i/\theta} \le C\right\} = u^*$$

$$\left(u^*\right)^{1/\theta} = \frac{C}{\left[B\prod_i p_i^{\alpha_i/\theta}\right]}$$

$$\Gamma(p) = u^* = \frac{C^\theta}{\left[B\prod_i p_i^{\alpha_i/\theta}\right]^\theta}$$

Corollary: If returns to scale are constant, $\theta = 1$

$$\Gamma(p) = \frac{C}{B\prod_i p_i^{\alpha_i}}$$

Γ (p) satisfies the following properties:

(i) $p = 0 \Rightarrow \Gamma(p) = +\infty$

(ii) $p \ge q \Rightarrow \Gamma(p) \le \Gamma(q)$

(iii) $\Gamma(p)$ is continuous function of input prices p_i (>0)

Theorem: If the underlying production frontier is homothetic, then

$$\Gamma(p) = F\left(\frac{C}{H(p)}\right)$$

Proof: If production frontier is homothetic its dual cost function can be expressed of the form,
Q (u, p) = f (u) H (p)

$$\Gamma(p) = \underset{u}{\text{Min}} \{u : Q(u,p) \le C\}$$

$$= \underset{u}{\text{Min}} \{u : f(u)\ H(p) \le C\}$$

If f (u) is continuous in u, we can find u^* such that
$f(u^*)\ H(p) = C$

$$f(u^*) = \frac{C}{H(p)};\ u^* = F\left(\frac{C}{H(p)}\right); \Gamma(p) = F\left(\frac{C}{H(p)}\right)$$

where C is targeted cost

VARIABLE RETURNS TO SCALE PRODUCTION FRONTIER–COST LIMITED MAXIMAL OUTPUT FUNCTION

Theorem: if the underlying production frontier is Zellner – Revenkar Variable returns to scale frontier, then

$$\Gamma(p) = u^{\alpha} e^{\theta u}\ B \prod_i p_i^{\alpha_i}$$

Proof: If the frontier production function is of the form,

$$u^{\alpha}\ e^{\theta}\, u = A \prod x_i^{\alpha_i},\ \sum_i \alpha_i = 1$$

then, the underlying factor minimal function makes the expression,

$$Q(u,p) = u^{\alpha}\ e^{\theta u}\ B \prod_i p_i^{\alpha_i}$$

Where p_i is unit price of i^{th} input

From Shephard's duality theorem

$$\frac{p_i}{Q(u,p)} = \frac{\partial}{\partial x_i}\left(\frac{\phi(x)}{f(u)}\right)$$

where $\phi(x) / f(u)$ is the input distance function Taking

$$\phi(x) = A \prod_i x_i^{\alpha_i}$$

$f(u) = u^{\alpha} e^{\theta u}$, we obtain,

$$\frac{p_i}{Q(u,p)} = \frac{\alpha_i}{x_i}\frac{\phi(x)}{f(u)} = \frac{\alpha_i}{x_i} D(u,x)$$

$$x_i = \alpha_i\ p_i^{-1}\ Q(u,p)\ D(u,x)$$

Replacing this expression in the place of x_i in $\phi(x)$,

$$\phi(x) = A\ D(u,x)\ Q(u,p)\ \left(\prod_i p_i^{-\alpha_i}\right)\left(\prod_i \alpha_i^{\alpha_i}\right)$$

$$f(u)\left[\frac{\phi(x)}{f(u)}\right] = A\ D(u,x)\ Q(u,p)\left(\prod_i p_i^{-\alpha_i}\right)\left(\prod_i \alpha_i^{\alpha_i}\right)$$

$$f(u)\ D(u,x) = A\, D(u,x)\ Q(u,p)\left(\prod_i p_i^{-\alpha_i}\right)\left(\prod_i \alpha_i^{\alpha_i}\right)$$

$$Q(u, p) = f(u)\ A^{-1}\left(\prod_i p_i^{\alpha_i}\right)\left(\prod_i \alpha_i^{-\alpha_i}\right)$$

$$Q(u,p) = f(u)\ B\left(\prod_i p_i^{\alpha_i}\right)$$

where $B = A^{-1}\left(\prod_i \alpha_i^{-\alpha_i}\right)$

The input sets L(u) of this frontier possess homothetic input structure since $\phi(x)$ is a homogeneous function. However, the output sets P(x) do not possess output homothetic structure since, $f(u) = u^{\alpha} e^{\theta u}$ is not a homogeneous function of u

Theorem: if the underlying production frontier is of the form
$u^{\alpha}\ e^{\theta u} = A \prod_i x_i^{\alpha_i}$, then the cost limited maximal output
$\Gamma(p)$ can be obtained as solution of the equation,

$$\left[\Gamma(p)\right]^{\alpha} e^{\theta\Gamma(p)} = \frac{C}{B}\prod_i p_i^{-\alpha_i}$$

Proof: $L(p) = \underset{u}{\text{Max}}\ \{u : Q(u, p) \le C\}$
where C is the target cost

$$\Gamma(p) = \underset{u}{\text{Max}}\left\{u : u^{\alpha} e^{\theta u}\ B\prod_i p_i^{\alpha_i} \le C\right\}$$

Let $\Gamma(p) = u^{*}$, for which $u^{*\alpha}\ e^{\theta u^{*}}\ B\prod_i p_i^{\alpha_i} = C$

$$\left(u^{*}\right)^{\alpha}\ e^{\theta u^{*}} = \frac{C}{B}\prod_i p_i^{-\alpha_i}$$

$$\left[\Gamma(p)\right]^{\alpha}\ e^{\theta\Gamma(p)} = \frac{C}{B}\prod_i p_i^{-\alpha_i}$$

TARGET REVENUE:

Some times the policy maker targets revenue and wishes to predict the necessary resources to achieve the targeted revenue. In such a case we use Shephard's output sets and compute maximum revenue, by solving the optimization problem,

$R(x, r) = \text{Max}\ \{u: u \in P(x)\}$

Also we use the sets

$$R\left(\frac{r}{R}\right) = \left\{x : R\left(x, \frac{r}{R}\right) \ge 1\right\} = \{x : R(x,r) \ge R\}$$

Theorem: In one input and multi-output case if R(x,r) is a continuous function of x then F(r) is obtained as solution of the equation,

$$R\,[F(r), r] = R$$

Proof: $F(r) = \underset{x}{\text{Max}}\ \{x : R(x,r) \ge R\}$
If R(x, r) is continuous function of x, then minimum input is attained such that
$R(x^{*}, r) = R$
When $F(r) = x^{*}$

$\Rightarrow R\,[F(r), r] = R$

where R is target revenue.

MAXIMAL REVENUE – HOMOTHETIC PRODUCTION STRUCTURE

The output set of a homothetic production structure is defined as,

$$P(x) = \{x : f(u) \le \phi(x)\}$$

If the production structure has homothetic output structure, then f(u) is linear homogeneous in u

$$P(x)=\left\{u : F\left(\frac{u}{\phi(x)}\right) \le 1\right\}$$

$$= \phi(x)\left\{\frac{u}{\phi(x)} : f\left(\frac{u}{\phi(x)}\right) \le 1\right\}$$

$$= \phi(x)\left\{\hat{u} : f(\hat{u}) \le 1\right\}$$

where $\hat{u} = \dfrac{u}{\phi(x)}$

Maximum revenue:

$$R(x,r) = \max_{x}\ \{r\ u : u \in P(x)\}$$

$$= \phi(x)\ \max_{\hat{u}}\ \{r\ \hat{u} : f(\hat{u}) \le 1\}$$

$$= \phi(x)\ B(r)$$

where $\hat{u} = \dfrac{u}{\phi(x)}$

$R(x, r) = \phi(x)\, B(r)$

The maximal revenue function splits into a product shown above stuject to the condition that f(u) is linear homogeneous in u

Theorem: If the underlying frontier production function is, $u^{\alpha}\ e^{\theta u} = A \prod_i x_i^{\alpha_i}$, the maximum revenue is obtained as the

Solution of the non-linear equation, $z^{\alpha}\ e^{\beta z} = r^{\alpha}\ \phi(x)$

where $z = R(x, r)$, $\beta = \theta/r$

Proof:

$$R(x, r) = \max_{u}\left\{r\,u : u \in P(x)\right\}$$

$$= \max_{u}\left\{r\,u : f(u) \le \phi(x)\right\}$$

$$= \max_{u}\left\{r\,u : u^{\alpha}\ e^{\theta u} \le A \prod_i x_i^{\alpha_i}\right\}$$

$$= \max_{u}\left\{r\,u : \frac{(ru)^{\alpha}}{r^{\alpha}}\ e^{\frac{\theta}{r}(ru)} \le A \prod_i x_i^{\alpha_i}\right\}$$

$$\Rightarrow \frac{[R(x,r)]^{\alpha}}{r^{\alpha}}\ e^{\beta R[(x,r)]} = \phi(x)$$

Let $R(x, r) = z$

$$\frac{z^{\alpha}}{r^{\alpha}}\ e^{\beta z} = \phi(x)$$

$$z^{\alpha}\ e^{\beta z} = r^{\alpha}\ \phi(x)$$

Theorem: If the production structure satisfies input homothetic structure, the predicted input vector that yields at least the targeted revenue R is,

$$\hat{x} = \frac{f\left(\frac{R}{r}\right)}{\phi(x_0)} x_0$$

Proof: consider the input set,

$$\{x : R(x, r) \geq R\} = \{x : r\ F[\phi(x)] \geq R\}$$

$$= \left\{x : F(\phi(x)) \geq \frac{R}{r}\right\}$$

$$= L_F\left(\frac{R}{r}\right)$$

Let x_0 be the input vector employed by the production unit whose efficiency is currently under evaluation.

Minimize λ such that $R(\lambda\ x_0, r) = R$

This optimization problem is equivalent to,

$$\text{Min } \lambda$$

Such that $\lambda\, x_0 \in L_F\left(\frac{R}{r}\right)$

Minimum λ occurs as and when

$$F(\phi(\lambda x)) = \frac{R}{r}$$

Since $\phi(x)$ is linear homogeneous in x due to the assumption that the production structure has input homothetic structure

$$F(\phi(\lambda\ x0)) = F(\lambda\ \ \phi(x_0)) = \frac{R}{r}$$

$$\Rightarrow\ \lambda\ \ \phi(x_0) = f\left(\frac{R}{r}\right)$$

$$\Rightarrow\ \lambda = \frac{f\left(\frac{R}{r}\right)}{\phi(x_0)}$$

Predicted input vector: $\hat{x} = \dfrac{f\left(\frac{R}{r}\right)}{\phi(x_0)} x_0$

If the underlying production frontier is VRS frontier, then the predicted input vector is

$$\hat{x} = \left\{\frac{(R/r)^{\alpha}\ e^{\theta(R/r)}}{A \prod_i x_{i0}{}^{\alpha}}\right\} x_0$$

ESTIMATION OF COBB-DOUGLAS PRODUCTION FRONTIER

C.P. Timmer (1971) proposed a linear programming problem whose solution yields the Cobb-Doublas frontier production function

$$\underset{(\ln A,\, \alpha_i)}{\text{Min}}\ \ln\ A + \sum_{i=1}^{m} \alpha_i\ \overline{\ln\ x_i}$$

Subject to $\ln A + \sum_i \alpha_i \ln x_{ij} \geq \ln u_j$

ln A is unrestricted for sign
$0 \leq \alpha_i \leq 1$, i = 1, 2, ……m

ESTIMATION OF VRS PRODUCTION FRONTIER:

$$\underset{(A,\,\alpha_j,\alpha\theta)}{\text{Min}} \ \ln A + \sum_i \alpha_i \overline{\ln x_i} - \alpha \overline{\ln u} - \theta \bar{u}$$

Subject to $\ln A + \sum_i \alpha_i \ln x_{ij} - \alpha \ln u_j - \theta u_j \geq 0$

$0 \leq \alpha_i \leq 1$, i = 1, 2, ……. M

$\alpha \geq 0, \theta \geq 0$

ln A is unrestricted for sign

RETURNS TO SCALE–HOMOGENEOUS PRODUCTION FUNCTIONS

Let ϕ (x) be homogeneous production frontier of degree $\in$

$\phi(\lambda x) = \lambda \phi(x)$ $\Rightarrow$ Returns to scale are constant
$= \lambda^{\in} \phi(x), \quad \lambda \geq 1$

$\in < 1$ $\Rightarrow$ Returns to scale are decreasing
$\in > 1$ $\Rightarrow$ Returns to scale are increasing

RETURNS TO SCALE CHARACTERIZATION – COBB-DOUGLAS FRONTIER

$$\hat{u} = A \prod_i x_i^{\alpha_i}$$

$\sum_i \alpha_i = 1 \Rightarrow$ Constant returns to scale

$\sum_i \alpha_i < 1 \Rightarrow$ Decreasing returns to scale

$\sum_i \alpha_i > 1 \Rightarrow$ Returns to scale are increasing

RETURNS TO SCALE CHARACTERIZATION – VARIABLE RETURNS TO SCALE FRONTIER:

$$u^{\alpha} e^{\theta u} = A \prod_i x_i^{\alpha_i}$$

$$\in(x,u) = \frac{\upsilon}{\alpha + \theta u}, \quad \upsilon = \sum_j \alpha_j$$

where $\in$ (x, u) is the elasticity of scale function

In the expression of elasticity of scale it can be noticed that returns to scale varies depending upon output u, but does not depend upon the input vector x.

APPLICATION

In variable prediction discussed and implemented in this work the chief tool is PRODUCTION FUNCTION, parametrically postulated. Two parametric specifications are considered, (a) the Cobb-Douglas frontier, which has a simple structure, but very widely used both in empirical and theoretical research and (b) the variable returns to scale production frontier for which the former frontier is a special case.

The decision making units (DMU's) are the total manufacturing sectors of different states of India and as well as that of India.

The policy maker or producer sometimes target cost and enquires for possible output augmentation or equivalently the potential revenue. We shall assume that in a production environment all units do not employ the same technique and/or posses the same managerial efficiency. The productive efficiency considered is only relative but not absolute. Efficiency co-efficient can be computed by comparing a typical unit's output and / or inputs with those of frontier units. If a unit better than the best emerges into the reference set the best units loose their efficiency by some extent.

Data: The data are taken from annual survey of industries (1999-2000). The variables chose for the study are 1. Value Added 2. No. of Persons Employed 3. Fixed Capital 4. Wages and Salaries including Employee's Contribution

Table - 1

STATES	TC/ P R
	C D FRONTIER
ALL INDIA	0.7399
A.P	0.5199
BIHAR	1.0000
GUJARAT	0.8033
HARYANA	0.9120
KARNATAKA	0.6147
KERALA	0.7474
M.P	0.7052
MAHARASHTRA	1.0000
ORISSA	0.6246
PUNJAB	0.8230
RAJASTHAN	0.6163
TAMILNADU	0.6545
UTTER PRADESH	0.5811
WEST BENGAL	0.4098

T C: Target Cost, P R: Potential Revenue, CRTS: Constant Returns to Scale

Table (1) presents target cost to potential revenue ratios for total manufacturing sectors of 14 Indian states which account for 85 per cent of the Country's total value added. The total manufacturing sector of all India is augmented to these states to that we have 15 DMU's. The total manufacturing sector of all India adjusts scale inefficiency towards constant returns and if it becomes cost efficient, 74 per cent of potential revenue is accounted for by target cost. And 26 percent, addition over target cost remains to be profit. The conditional profit estimated is the profit foregone due to productive inefficiency.

The inclusion of the total manufacturing sector of all India helps to estimate structural efficiency.
The TC/PR ratio exhibited wide variation over 14 total Manufacturing sectors of India. For these states we obtain,

$$0.4098 \leq \frac{\text{TC}}{\text{PR}} \leq 1.000$$

TARGET REVENUE–POTENTIAL INPUTS TOTAL MANUFACTURING SECTOR OF ALL INDIA

The total manufacturing sector of ALL INDIA is augmented to those of 14 states as a DMU, since the inclusion helps to study structural change in manufacturing sector of All India.

Table – 2: C D PRODUCTION FRONTIER – CRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.7311	0.8042	0.8773	0.9504	1.0235

E I D C: Efficiency input Determining Coefficient
T R C : Target Revenue Co efficient

If current revenue is target revenue the total manufacturing sector of All India attains the target revenue mixing inputs as usual and by adjusting its inefficiency both technical and scale, utilizing only 73 percent of its current inputs.

If target revenue is raised by 10 percent more than the current revenue, the target is achieved, employing 80 percent of the current inputs. A 10% increase in target revenue can be achieved, increasing efficient inputs by exactly 10 percent.

This is not a surprise, since the frontier production function that serves as a reference technology admits constant returns to scale.

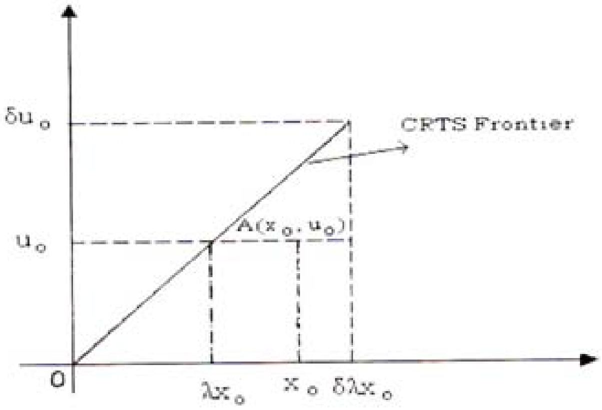

Figure -1

The ray that emanates from the origin is constant returns to scale frontier, for which $\hat{u} = a\ x$, where is $\hat{u}$ frontier output.

The DMU operating at A is inefficient. With input x_0 it produces u_0. For this DMU efficient input is, $\hat{x} = \lambda\, x_0,\ 0 \le \lambda \le 1$

Suppose current revenue is target revenue.

Current revenue: r u_0

Efficient input: $\lambda\ x_0$

Let θ be the rate by which target revenue is increased.

New target revenue: $(1+\theta)\ r\ u_o = r\ (1+\theta)\, u_0 = r\ \ \delta\, u_0$

qhere $\delta = (1+\theta),\ \ 0 \le \theta \le 1$

$$u_0 = a\,(\lambda\, x_0)$$

$$\delta u_0 = a\,(\delta\,\lambda\, x_0)$$

Thus, if current revenue is increased by θ percent the efficient inputs (λx_0) also have increased by θ percent. This result holds good for any linear homogeneous production function. The Cobb-Douglas frontier production function is homogeneous of degree $\sum_i \alpha_i$. if this sum equal to one the frontier becomes linear homogeneous, consequently admits constant returns to scale. If returns to scale are constant a one percent increase in target revenue requires a one percent rise in efficient inputs and production cost.

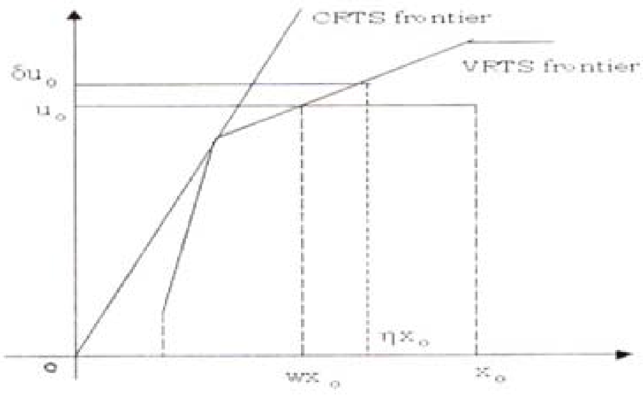

Figure -2

The units that operates at A is inefficient. It admits decreasing returns to scale. The efficient input required to produce u_0 is wx_0. Thus wx_0 is efficient input that generates the revenue ru_0. Let the next target revenue be, δ ru_0. The efficient input requirement to attain the target is, ηx_0. Additional revenue augmentation is r $(\delta-1)u_0$, achieved by an increase of the inputs $(\eta - w)x_0$.

Consider homogeneous production frontier $\phi(x)$ that admits decreasing returns scale. If u_0 is current output, producible by an input vector x_0, then $u_0 \leq \phi(x_0)$. Let $\lambda > 0$ be such that $u_0 = \phi(\lambda x_0)$

The efficient input that can produce u_0, if returns to scale are decreasing is λx_0, if δ ru_0 is next target, we have. $\delta ru_0 = \delta\phi(\lambda x_0) = \phi\left(\delta^{\in^{-1}}\lambda x_0\right)$

where $\in > 0$ is the degree of homogeneity of the frontier production function, for which returns to scale are decreasing.

Consequently, $\in < 1 \Rightarrow \in^{-1} > 1$

The efficient inputs have to be increased by more than one percent to bring about a one percent increase in target revenue.

Here, $\delta > 1,\ 0 < \lambda \leq 1, \in < 1$

$$\delta < \delta^{\in^{-1}}$$

The Cobb-Douglas frontier implies decreasing returns to scale to the total manufacturing sector of all India. The estimated returns to scale are,

$$\in = \alpha_1 + \alpha_2 = 0.8611 \Rightarrow \in^{-1} = 1.1613$$

The DMU is pure technical efficient since $u_o = \phi(x_o)$

Table -3: CD Frontier - DRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	1.0000	1.1170	1.2358	1.3562	1.4781

If TRC is multiplied with current revenue target revenue can be obtained. If current inputs are multiplied by EIDC potential inputs are obtained. The governing returns to scale are decreasing.

Total Manufacturing sector of Andhra Pradesh

The total manufacturing sector of Andhra Pradesh shares six percent of the country's total value added. Its fixed capital and total persons employed are respectively 6.7 and 11 percent in all India's total fixed capital and number of person employed.

Table -4: CD Production Frontier – CRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.5111	0.5622	0.6133	0.6644	0.7155

Assuming returns to scale as constant against each target revenue potential efficient inputs are estimated for CD frontier. The degree of returns to scale implied by the parametric frontier are estimated for the total manufacturing sector of Andhra Pradesh as,

$\in = \alpha_1 + \alpha_2 = 0.8857$

$\in^{-1} = 1.1291$

One percent increase in target revenue can be achieved by more than one percent increase in efficient inputs.

Table -5: CD Frontier – DRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.6131	0.6828	0.7532	0.8245	0.8964

$\phi(\lambda x_0) = u_0$

$\lambda^{\in} \phi(x_0) = u_0$

$$\lambda^{\in} = \frac{u_0}{\phi(x_o)} \quad \Rightarrow \quad \lambda = \left[\frac{u_0}{\phi(x_o)}\right]^{\in^{-1}}$$

W = λ = 0.6131

If the DMU eliminates its pure technical efficiency utilizing 90 percent of its current inputs 40 percent more revenue than its current revenue can be generated.

TOTAL MANUFACTURING SECTOR OF BIHAR

Only one percent of country's total value added is accounted by the total manufacturing sector of Bihar.

Table – 6: C D Production Frontier – CRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	1.0000	1.1000	1.2000	1.3000	1.4000

Potential input requirements are tabulated above against target revenues using CD frontier production functions, which admit CRTS.

The Cobb-Douglas production frontier of this DMU is,

$$\hat{u} = 1.00 \;\; x_1^{0.0223} \, x_2^{0.9777}$$

The elasticity of frontier output with respect to labour is marginally different from zero. The linear programming estimate of elasticity of frontier output with respect to fixed capital is 0.9777 implying that frontier output is more sensitive to changes in fixed capital than changes in labour in labour input. The estimated returns to scale are constant, since,

$\in = \alpha_1 + \alpha_2 = 1.00$

TOTAL MANUFACTURING SECTOR OF GUJARAT

12.4 percent of the country's total valued added is accounted for by the total manufacturing sector of Gujarat. Gujarat is one of the most industrialized states in India. 16.7 percent of India's fixed capital is accounted by this DMU. The share of it in India's Industrial employment is 10 percent.

Table – 7: CD Production Frontier – CRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.6828	0.7511	0.8194	0.8877	0.9560

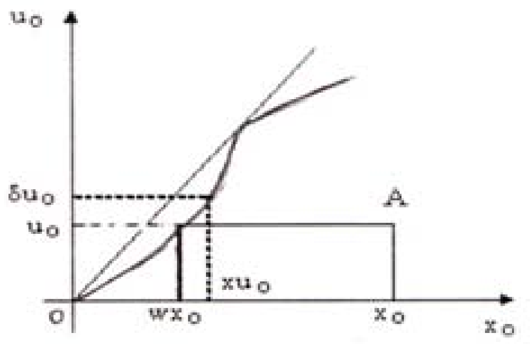

Figure -3

The production unit that operates at A is inefficient. It can attain technical efficiency by reducing its inputs to λx_0. It enjoys increasing return to scale. Let $\delta > 1$ be the target revenue co-efficient, so that target revenue is $r\delta u_0$. A one percent increase in target revenue can be attained increasing efficient inputs by less than one percent. The target revenue can be generated employing the input vector ηx_0.

Additional revenue augmentation : $r(\delta - 1)u_0$

Additional efficient input augmentation : $(\eta - w)x_0$

Consider a homogeneous function whose degree of homogeneity is greater than one, so that it admits increasing returns to scale.

$u_0 \leq \phi(x_0)$

$\Rightarrow \quad u_0 = \phi(\lambda\, x_0),\ 0 < \lambda \leq 1$

$\Rightarrow \quad u_0$ is produced is $\lambda\, x_0$

where $\lambda\, x_0$ (= wx_0) is efficient input

It target revenue is $r\delta u_0$

$$\delta\, u_0 = \delta\, \phi\, (\lambda\, x_0)$$
$$= \phi\left(\lambda\ \delta^{\in^{-1}}\ x_0\right)$$

$\in\ > 1 \Rightarrow \in^{-1} < 1$, where $\delta\ >\ 1$

Thus, a one percent increase in target revenue can be accomplished by less than one percent increase in efficient input.

$\in = \alpha 1 + \alpha 2 = 1.1378$

$\Rightarrow \in\text{-}1 = 0.8789$

Table – 8: CD Frontier – IRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.8684	0.9445	1.0196	1.0936	1.1675

If the DMU eliminates it pure technical inefficiency it can generate 40 percent more revenue that it is currently receiving by employing 17 percent more input than it is currently combining

TOTAL MANUFACTURING SECTOR OF HARYANA

4 Percent of Country's total value added is accounted for by the total manufacturing sector of Haryana. Although the state is small on the Country's geographical map it is an important industrialized state. It accounts for 3 percent of country's value added and provides 3 percent employment in the country's total industrial employment.

Table – 9: C D Production Frontier – CRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.8925	0.9818	1.0710	1.1603	1.2195

The CD frontier estimates reveal that returns to scale are increasing the estimated frontier is,

$$\hat{u} = 0.2464\, x_1^{1.0594}\ x_2^{0.1023}$$

Return to Scale: $\in = \alpha_1 + \alpha_2 = 1.1617$

Elasticity of frontier output with respect to labour input is 1.0594 where as the frontier output elasticity with respect to capital is only 0.1023, implying that frontier output is sensitive more to the changes in labour input than capital input.

$\in = 1.1617$

$\Rightarrow \in^{-1} = 0.8789$

The pure technical efficiency implied by CD frontier is 0.9892.

Table – 10: CD Production Frontier – IRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.8608	1.0738	1.1573	1.2398	1.3215

If the current revenue is the target revenue, it can be achieved employing 86 percent of current inputs, if pure technical inefficiency is eliminated, from the last row of the above table it follows that 40 percent more than the current revenue can be generated by employing 32 percent more inputs than currently mixed.

TOTAL MANUFACTURING SECTOR OF KARNATAKA

5.2 Percent of country's total net value added is accounted for by the total manufacturing sector of Karnataka. The DMU shares 6.7 percent of India's total fixed capital. It provides 6 percent of country's total industrial employment.

Table – 11: C D Production Frontier – CRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.6144	0.6758	0.7373	0.7987	0.8602

If the DMU eliminates technical and scale inefficiency 40 percent more of the current revenue can be achieved mixing 86 percent of its current inputs.

$\in = \alpha_1 + \alpha_2 = 1.1617$
$\Rightarrow \in^{-1} = 0.8608$

The pure technically efficiency measured by the CD production frontier is, 0.6952. since returns to scale are increasing a one percent increase in target revenue can be attained by less than one percent increase in efficient inputs.

Table – 12: C D Frontier – IRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.6952	0.7546	0.8133	0.8713	0.9287

If the DMU eliminates pure technical inefficiency it can generate 40 percent more revenue that what it is currently earning.

TOTAL MANUFACTURING SECTOR OF KERALA

2.3 Percent of Country's total value added is accounted for by the total manufacturing sector of Kerala. Its share in industrial employment is 2.3 percent of Country's total industrial employment. 1.5 percent of India's fixed capital is accounted for by the DMU.

Table – 13: C D Production Frontier – CRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.7475	0.8223	0.8970	0.9718	1.0465

Potential inputs are tabulated above against the respective target revenues by increasing target revenue 10 percent in each step, up to 40 percent.

If the total manufacturing sector of Kerala eliminates pure technical and scale efficiencies, additional 40 percent revenue can be earned by increasing current inputs by only 4.7 percent.

Returns to scale implied by the CD frontier are also decreasing.

$$\in = \alpha_1 + \alpha_2 = 0.8857$$

The pure technical efficiency estimated is,

$$W = 0.8857$$

Since returns to scale are decreasing a one percent increase in target revenue can be accomplished by more than one percent increase in efficient inputs.

Table – 14: C D Production Frontier – DRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.8885	0.9894	1.0916	1.1948	1.2991

11 percent of input losses are accounted for by pure technical inefficiency. If the DMU eliminates its pure technical inefficiency, 40 percent additional revenue can be earned by increasing current inputs by 30 percent.

TOTAL MANUFACTURING SECTOR OF MADHYA PRADESH

It is geographically a vide state of India. Its industrial development is not in tune of its size. 3.6 percent of the Country's total value added is accounted for by the total manufacturing sector of Madhya Pradesh. It accounts for 4.2 percent of India's fixed capital and shares 3.2 percent of Industrial employment.

Table – 15: C D Production Frontier – CRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.7096	0.7806	0.8516	0.9225	0.9935

If the total manufacturing sector of Madhya Pradesh eliminates its pure and scale inefficiency 40 percent additional revenue can be generated with the inputs it is currently applying.

$\in = \alpha 1 + \alpha 2 = 1.1617$
$\Rightarrow \in^{-1} = 0.8608$

Input losses due to pure technical inefficiency are about 6 percent.

Table – 16: CD Production Frontier – IRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.9393	1.0196	1.0989	1.1773	1.2548

If the DMU eliminates pure technical inefficiency an additional revenue of 40 percent can be generated with an additional input of 25percent.

TOTAL MANUFACTURING SECTOR OF MAHARASHTRA

It is highly industrialized state. 22 percent of Country's total valued added is accounted for by the total manufacturing sector of Maharashtra. Its share in fixed capital and employment are respectively 17.5 and 15 percent.

Table – 17: C D Production Frontier – CRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	1.0000	1.1000	1.2000	1.3000	1.4000

A one percent increase in target revenue requires a one percent increase in its inputs, since returns to scale are constant.

TOTAL MANUFACTURING SECTOR OF ORISSA

.7 percent of India's total value added is accounted for by the total manufacturing sector of Orissa. Its contribution to the country's fixed capital and industrial employment are respectively 2.4 and 1.6 percent.

Table – 18: C D Production Frontier – CRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.6283	0.6911	0.7539	0.8167	0.8796

The CD frontier estimates reveal that a 40 percent increase in the target revenue is possible with 88 percent of the current inputs if the total manufacturing sector of Orissa eliminates its pure technical and scale inefficiencies.

Since returns to scale are increasing a one percent increase in target revenue can be realized by less than one percent increase in efficient inputs. The parametric frontier estimates pure technical efficiency as, W = 0.97, 3 percent of input losses are attributed to pure technical inefficiency.

Table – 19: CD Production Frontier – IRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.9700	1.0410	1.1104	1.1783	1.2113

$\in = \alpha_1 + \alpha_2 = 1.3484$
$\Rightarrow \in^{-1} = 0.7416$

To attain 40 percent of additional revenue the inputs should be increased by 25 percent, if the DMU eliminates its pure technical inefficiency which is only marginal.

TOTAL MANUFACTURING SECTOR OF PUNJAB

3.6 percent of India's valued added is accounted for by the total manufacturing sector of Punjab. It shares 2.5 percent of fixed capital and 4.1 percent of industrial employment of the country

Table – 20: C D Production Frontier – CRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.8443	0.9287	1.0131	1.0975	1.1820

If the DMU eliminates its scale inefficiency additional 40 percent revenue can be earned employing 18 percent more of its currently consumed inputs.

A one percent increase in target revenue requires increasing efficient inputs by more than one percent.

Returns to scale estimated using CD frontier are decreasing

$\in = \alpha_1 + \alpha_2 = 0.8857$

$\Rightarrow \in^{-1} = 1.1291$,

additional 40 percent revenue can be generated employing 46 percent of additional inputs.

Table -21: C D Production Frontier – DRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	1.0000	1.1136	1.2286	1.3448	1.4622

TOTAL MANUFACTURING SECTOR OF RAJASTHAN

3.4 Percent of total value added, 5 percent of fixed capital and 2.9 percent of industrial employment respectively of India are accounted by the total manufacturing sector of Rajasthan.

Table -22: C D Production Frontier – CRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.6423	0.7065	0.7708	0.8350	0.8992

Returns to scale predicted by the CD frontier production function are decreasing.

$\in = \alpha_1 + \alpha_2 = 1.1945$

$\Rightarrow \in^{-1} = 0.8371$

The estimated pure technical efficiency is, W = 0.9546, which implies that five percent of input losses are attributed to pure technical inefficiency.

Table -23: C D Production Frontier – IRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.9546	1.0336	1.1120	1.1891	1.2652

Since returns to scale are increasing a one percent increase in target revenue requires less than one percent increase in efficient inputs. 40 percent increase in target revenue requires 27 percent more input than that currently applied, if only the unit reduces its pure technical efficiency to zero level.

TOTAL MANUFACTURING SECTOR OF TAMILNADU:

Tamil Nadu is one of the major industrialized states of India. 9.5 percent of the value added of India is accounted for by Tamil Nadu. 13.5 percent of all India Industrial employees work in the total manufacturing sector of Tamil Nadu. It shares 9.3 percent of the country's fixed capital

Table -24: C D Production Frontier – CRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.6358	0.6994	0.7630	0.8266	0.8902

What follows from the CD frontier (CRTS) is that if the total manufacturing sector of Tamil Nadu eliminates both pure and scale inefficiencies, 40 percent additional revenue can be earned employing 90 percent of the current inputs.

Pure technical efficiency assessed by CD frontier production function is, W = 0.7473

Estimated returns to scale by CD frontier are decreasing, implying that a one percent augmentation of additional revenue requires increasing efficient inputs by more than one percent.

Table -25: C D Production Frontier – DRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.7473	0.8322	0.9181	1.0050	1.0927

CD frontier estimate of returns to scale:

$\in = \alpha_1 + \alpha_2 = 0.8857$

$\Rightarrow \in^{-1} = 1.1291$

If the total manufacturing sector of Tamil Nadu eliminates pure technical inefficiency, additional revenue of 40 percent can be earned by increasing its current inputs by only 9 percent.

TOTAL MANUFACTURING SECOR OF UTTAR PRADESH

6.6 Percent of the Country's value added is accounted for the total manufacturing sector of Uttar Pradesh. It is the largest state of India in area and population. Its share in fixed capital and industrial employment are respectively 9.3 and 6.9 percent in India's total fixed capital and industrial employment.

Table – 26: C D Production Frontier – CRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.5848	0.6433	0.7018	0.7603	0.8188

If the total manufacturing sector of Uttar Pradesh can eliminate its pure technical and scale inefficiencies, 40 percent more revenue can be earned by employing 82 percent of the current inputs, a prediction based on CD frontier production function.

The degree of returns to scale predicted by CD frontier is,
$\in = \alpha_1 + \alpha_2 = 1.1617$,
$\Rightarrow \in^{-1} = 0.8608$

A one percent increase in target revenue can be attained by less than one percent increase in efficient inputs. pure technical efficiency predicted by the parametric frontier is, W = 0.6998

Table – 27: C D Production Frontier – IRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.6998	0.7596	0.8187	0.8771	0.9319

If this DMU eliminates it pure technical inefficiency it can earn 40 percent more of its current inputs. In the absence of pure technical inefficiency, employing current inputs it can increase its revenue to 51 percent more than that of currently it generates.

TOTAL MANUFACTURING SECTOR OF WEST BENGAL

3.7 percent of India's value added is accounted for by the total manufacturing sector of West Bengal. Its share in country's fixed capital and industrial employment are respective 4.3 and 7.2 percent.

Table – 28: C D Production Frontier – CRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.4997	0.5496	0.5996	0.6496	0.6995

The CD frontier estimate of pure technical efficiency is, W = 0.5722
43 percent of input losses are due to pure technical inefficiency.
$\in = \alpha_1 + \alpha_2 = 0.8857, \Rightarrow \in^{-1} = 1.1291$

Table – 29: C D Production Frontier – DRTS

TRC	1.0	1.1	1.2	1.3	1.4
EIDC	0.5722	0.6372	0.7013	0.7695	0.8366

If this DMU eliminates its pure technical inefficiency 40 percent more revenue can be generated by employing 84 percent of the current inputs.

In the absence of pure technical inefficiency, with current inputs additional revenue of 64 percent can be attained.

CONCLUSION

Simple formulae are derived to estimate potential revenue for a given target cost and potential inputs for given target revenue. Using these expression input losses and revenue losses are estimated for the total manufacturing sectors of 14 Indian states which account for 85 percent of country's total valued added. The total manufacturing sector of all India is considered as a DMU to estimate the structural efficiency.

REFERENCES

1. Afriat, S.N. (1972), "Efficiency Estimation of Production Functions", International Economic Review, 13, 568-598.
2. Aigner, D.J., and Chu, S.F., (1968), "On Estimating the Industry Production Function", American Economic Review, 58, 826-839.
3. Aigner, D.J., C.A.K. Lovell and Schmidt, P., (1977), "Formulation and Estimation of Stochastic Frontier Prodcution Function Models", Journal of Econometrics, 23, 21-37.
4. Banker, R.D., A Charnes, and Cooper W.W., (1981), "Models for the Estimation of Technical and Scale Inefficiency in a Data Envelopment Analysis", Management Science, 30, 1078-1092
5. Charnes, A., W.W, Cooper and Thrall, R.M. (1986), "Classifying and haracterization Efficiencies and Inefficiencies in a Data Envelopment Analysis", Operations Research Letters, 5, 105-110.
6. Fare, R.C. (1978), "Measuring the Technical Efficiency of Production", Journal of Economic Theory, 19, 150-162.
7. Farell M.J., (1957), "The Measurement of Productive Efficiency", Journal of Royal Statistical Society, Series – A, 120, 235-290.
8. Farell, M.J. and Fiedlhouse, M. (1962), "Estimating Efficiency in Production function under increasing Returns to Scale", Journal of Roryal Statistical Society, Series-A, 125, 252-267.
9. Ramesh Kumar, N. (2003) "Variable Prediction Using Data Envelopment Analysis", Ph. D. Thesis, S. V. University, Tirupati, Andhra Pradesh, India.
10. Vijaya kumar, K. (2003), "Nonnormal Disturbances: Testing normality in Linear Statistical Models", Ph. D. Thesis, S. V. University, Tirupati, Andhra Pradesh, India.

Source of support: Nil, Conflict of interest: None Declared