MANPOWER SYSTEM WITH ERLANG DEPARTURE AND ONE BY ONE RECRUITMENT

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ABSTRACT

A System in which employees leave the organization during busy period and new employees are recruited one by one after the busy period is treated. After the busy period, recruitment starts. In this Model busy period has general distribution and the inter departure time of employees has Erlang Phase two distribution and Joint Laplace stieltjes Transforms of busy period and recruitment time and their means are obtained.

Key Words: Manpower system, Loss of Manpower and Recruitment, Joint Transform, Erlang Phase two.

INTRODUCTION

It is very common in business world to face the availability and shortage of Manpower periods. These two periods are liable to be dependent on each other due to various reasons such as larger number of employees leaving the organization, unavailability of suitable persons in the job market, stringent recruitment policies and such similar reasons. There are many reasons for an employee to leave the organization. It may be for higher salary or to join family or for higher education etc… If more persons leave, then the business would be severely affected during busy period. As loss and shortcomings are inevitable and fund management are to be done during busy period and in the recruitment period, one may have to speed up recruitments using different strategies in order to start business early. The duration of busy period and the duration to recruit employees are random and they occur alternately in a business organization. When a busy period is long, one may like to speed up recruitment so as to start the next busy period early. A systematic approach to manpower system has been made as early as 1947 by Vajda [8] and others. For clear understanding of manpower planning one can refer to Bartholomew [1], Grinold and Marshall [2] and Vajda [9]. Lesson [3] has given methods to compute wastages (Resignation, dismissal and death) and promotion intensities which produce the proportions corresponding to some desired planning proposals. Stochastic models are designed for wastage and promotion in manpower system by Vassiliou [10]. Subramanian V [6] has made an attempt to provide optimal policy for recruitment training, promotion, and wastages in manpower planning models with special provisions such as time bound promotions, cost of training and voluntary retirement scheme. For other manpower models one may refer Setlhare K [5]. For three characteristics system in manpower models one may refer to Mohan C and Ramanarayanan R [4]. Manpower system with Recruitment Period Depending on Attrition during the busy period by K. Usha and R.Ramanarayanan [7]. In this paper we consider the busy period of operation of an organization has general distribution and the inter–departure time of employees has Erlang distribution phase 2. After the busy period, recruitments of employees start to fill up the vacancies one by one due to employees leaving the organization during the busy period. Two distinct distributions for the recruitment times depending on the length of busy period is within or exceeding a random threshold length is considered. The joint transform is obtained to derive the mean number of employees left and recruitment time. Numerical examples are also presented with illustration.

Model A: General Busy Period and Erlang departure

The main assumptions of the model are given below.
1. The busy period T of an organization is a random variable with cumulative distribution function (Cdf) F(x) and probability density function (pdf) f(x)

2. The inter-departure times of employees are independent random variables with Erlang distribution with rate λ. Let N be the number of employees left during the operation time T phase 2 and the recruitment time S has general distribution with Cdf $G_1(y)$ and pdf $g_1(y)$ when the operation time T is less than the threshold time U and it has Cdf $G_2(y)$ and pdf $g_2(x)$ when T is more than the threshold U to speed up recruitment.

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3. Threshold U has exponential distribution with parameter μ.

We may derive the joint distribution of T, N, and S as follows.

The joint probability density function of T and S and probability function of N is

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}P(T \le x,\ N = n,\ S \le y)\right) = f(x, n, y)$$

$$\left\{ e^{-\lambda x}\left[\frac{(\lambda x)^{2n}}{(2n)!} + \frac{(\lambda x)^{2n+1}}{(2n+1)!}\right]e^{-\mu x}g_{1(n)}(y) + e^{-\lambda x}\left[\frac{(\lambda x)^{2n}}{(2n)!} + \frac{(\lambda x)^{2n+1}}{(2n+1)!}\right]\left(1 - e^{-\mu x}\right)g_{2(n)}(y)\right\} \quad (1)$$

$$n = 1, 2, 3, \dots$$

Here $g_{1(n)}(y)$ is the n-fold convolution of pdf $g_1(y)$ and $g_{2(n)}(y)$ is the n-fold convolution of pdf $g_2(y)$

The first term of equation (1) is the part of the pdf that the busy period is x, the recruitment one by one time is y, the number of employees left is n and the busy period duration is within the threshold time. The second term of equation (1) is the part of the pdf that the busy period is x, the recruitment time is y, the number of employees left is n and the busy period duration exceeds the threshold time. Since the inter departure times of employees Erlang distribution with phase2 and rate λ exactly $2n$ or $2n+1$ exponential times are completed during the busy period $(0,x)$ is considered for writing equation(1).

Let us define the joint Laplace transform cum generating function as follows.

$$E\left[e^{-tT}e^{-sS}\theta^N\right] = \int_0^\infty \int_0^\infty \sum_{n=0}^\infty f(x, n, y)e^{-tx}.e^{-sy}\theta^n\, dx\, dy$$

$$= \int_0^\infty \int_0^\infty \sum_{n=0}^\infty e^{-tx}e^{-sy}\left[\begin{array}{c} f(x)e^{-\lambda x}\left[\dfrac{(\lambda x)^{2n}}{(2n)!} + \dfrac{(\lambda x)^{2n+1}}{(2n+1)!}\right]e^{-\mu x}g_{1(n)}(y) + \\[4mm] f(x)e^{-\lambda x}\left[\dfrac{(\lambda x)^{2n}}{(2n)!} + \dfrac{(\lambda x)^{2n+1}}{(2n+1)!}\right]\left(1 - e^{-\mu x}\right)g_{2(n)}(y)\end{array}\right]\theta^n\, dx dy,$$

$$0 \le \theta \le 1, \text{and } T,\ S \ge 0$$

On simplification, we obtain

$$= \frac{1}{2}\left[\left(\frac{\sqrt{\theta}+1}{\sqrt{\theta}}\right)f^*\left(t+\mu+\lambda-\lambda\theta^{1/2}g_1^*(s)\right) + \left(\frac{\sqrt{\theta}-1}{\sqrt{\theta}}\right)f^*\left(t+\mu+\lambda+\lambda\theta^{1/2}g_1^*(s)\right)\right]$$

$$+ \frac{1}{2}\left[\left(\frac{\sqrt{\theta}+1}{\sqrt{\theta}}\right)f^*\left(t+\lambda-\lambda\theta^{1/2}g_2^*(s)\right) + \left(\frac{\sqrt{\theta}-1}{\sqrt{\theta}}\right)f^*\left(t+\lambda+\lambda\theta^{1/2}g_2^*(s)\right)\right]$$

$$- \frac{1}{2}\left[\left(\frac{\sqrt{\theta}+1}{\sqrt{\theta}}\right)f^*\left(t+\mu+\lambda-\lambda\theta^{1/2}g_2^*(s)\right) + \left(\frac{\sqrt{\theta}-1}{\sqrt{\theta}}\right)f^*\left(t+\mu+\lambda+\lambda\theta^{1/2}g_2^*(s)\right)\right]$$

$$(2)$$

Here $*$ indicates Laplace transform.

Put $\theta = 1$ in equation (2), we get

$$E\left[e^{-sS}e^{-tT}\right] = f*(t+\mu+\lambda-\lambda g_1^*(s)) + f*(t+\lambda-\lambda g_2^*(s)) - f*(t+\mu+\lambda-\lambda g_2^*(s)) \quad (3)$$

Using differentiation of equation (3), we get

$$E(T) = -\left[f*'(0)\right]$$

Assume that f(.) is exponential density with parameter δ.

$$E(T) = \frac{1}{\delta}$$

$$(4)$$

Using differentiation of equation (3), we get

$$E(S) = \left[E(S_1) - E(S_2) \right] \left(\frac{\lambda \delta}{(\delta + \mu)^2} \right) + \frac{\lambda}{\delta} E(S_2)$$

(5)

Put s= 0 and t=0 in equation (2), we get

$$E\left[\theta^N\right] = \frac{1}{2}\left(\frac{\sqrt{\theta}+1}{\sqrt{\theta}}\right) f^*\left(\lambda - \lambda\theta^{1/2}\right) + \frac{1}{2}\left(\frac{\sqrt{\theta}-1}{\sqrt{\theta}}\right) f^*(\lambda + \lambda\theta^{1/2})$$

(6)

Using differentiation of equation (6), we get

$$E[N] = \left(\frac{\lambda}{2\delta}\right)$$

$$= \frac{\lambda}{2} E[T]$$

(7)

NUMERICAL ILLUSTRATION

We consider the case of busy period, the inter-departure time of employees, one by one recruitment time of S_1 and S2 have Erlang distribution with rate parameter values respectively δ, λ, E $[S_1]$ and E $[S_2]$.

We fix the value of λ=2, E $[S_1]$ =2 and E $[S_2]$ =1

We vary the busy period parameter δ= 5,10,15,20.25.30 and the threshold rate parameter μ=10,20,30,40,50,60 in the equations (4),(5) and (7), we get

Table of E[T]

$\dfrac{\delta \rightarrow}{\mu}$	5	10	15	20	25	30
10	.2000	.1000	.0667	.0500	.0400	.0333
20	.2000	.1000	'0667	'0500	.0400	.0333
30	.2000	.1000	.0667	.0500	.0400	.0333
40	.2000	.1000	.0667	.0500	.0400	.0333
50	.2000	.1000	.0667	.0500	.0400	.0333
60	.2000	.1000	.0667	.0500	.0400	.0333
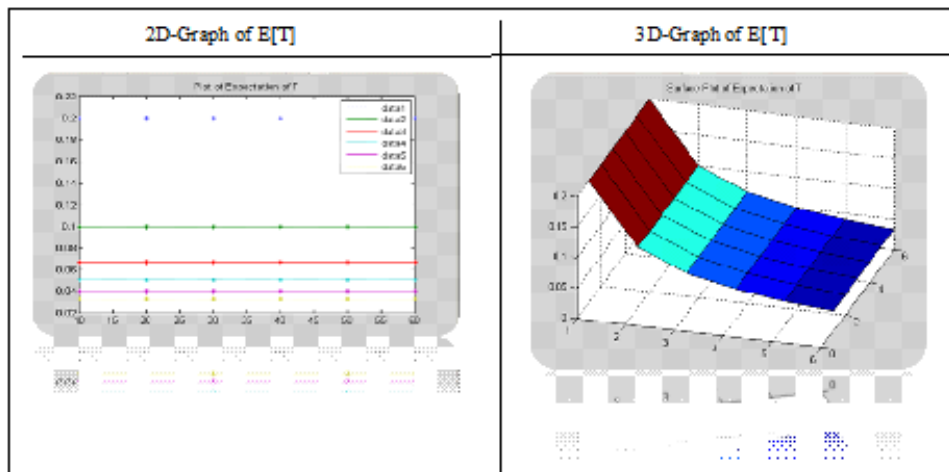
From the above table, as the value of δ increases E [T] decreases and when the value of μ increases, E [T] remains the same.

Table of E[S]

$\dfrac{\delta \rightarrow}{\mu}$	5	10	15	20	25	30
10	1.2444	.6500	.4480	.3444	.2808	.2375
20	1.2160	.6222	.4245	.3250	.2647	.2240
30	1.2082	.6125	.4148	.3160	.2565	.2167
40	1.2049	.6080	.4099	.3111	.2518	.2122
50	1.2033	.6056	.4071	.3082	.2489	.2094
60	1.2024	.6041	.4053	.3062	.2469	.2074
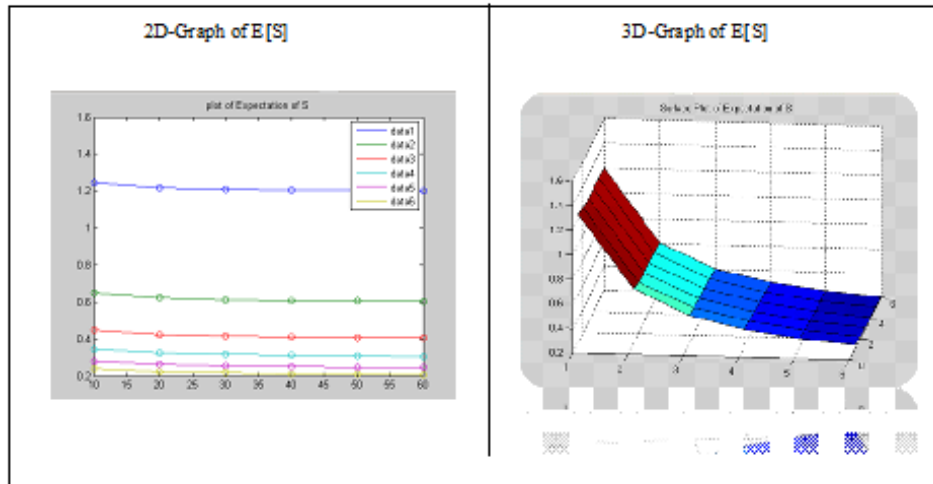
From the above table, as the value of δ increases, E[S] decreases and when the value of μ increases, E[S] decreases.

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