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DIVISIBILITY RULES BY RD-ALGORITHM

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ABSTRACT

Let $z=a_n 10^{n-1}+a_{n-1}10^{n-2}+\cdots+a_1$ *and* $w=b_m 10^{m-1}+b_{m-1}10^{m-2}+\cdots+b_1$ *are dividend and odd divisor respectively. In this paper, we introduce many divisibility rules of special numbers with RD-Algorithm and we show that which there is a direct relationship between X and the speed of algorithm and calculations. X is n right digits of odd divisor.*

AMS Subject Classification: *13AXX, 13F15.*

Keywords: *Divisibility of Numbers, Prime Factors, RD-Algorithm.*

1. INTRODUCTION

In various section of number theory, different and difficult methods are considered for dividing special numbers. Also, some algorithms are introduced because of importance of this matter. A divisibility rule is a shorthand way of determining whether a given number is divisible by a fixed divisor without performing the division, usually by examining its digits. We introduced a new algorithm entitled "RD-Algorithm" in [1] for the first time in the world which we can represent the dividing of numbers to each other in a simple and rapid way. This algorithm could be useful in a mathematics competition such as mathcounts. In the other words, this algorithm reduced the number of digits of dividend. In this paper, in order to fasten dividing numbers, we fixed the 6 right digits of odd divisor, and use the extracted methods of the algorithm for the numbers which it's the 6 right digits of them are 000001, 857143, 666667, 888889. The more the X is higher, the more the speed of algorithm is high(X is the n right digits of odd divisor). A lot of study on divisibility of methods have been conducted for many years. In number theory, divisibility methods of whole numbers are very useful because they help us to quickly determine if a number can be divided by n. There are several different methods for divisibility of numbers with many variants and some of them can be found in [6, 7, 8, 9, 10, 11, 12, 13]. For example, in [14, 15] presented that numbers which are dividable to 11 should have the (sum of the odd numbered digits) - (sum of the even numbered digits) is divisible by 11. Similarly some studies are presented for special numbers such as 15, 17, 19, etc. In this paper, we suppose that $z=a_n 10^{n-1}+a_{n-1}10^{n-2}+\cdots+a_1=a_n a_{n-1}\ldots a_1$ and $w=b_m 10^{m-1}+b_{m-1}10^{m-2}+\cdots+b_1=b_m b_{m-1}\ldots b_1$ are dividend and odd divisor respectively. To test for divisibility by w, where w ends in 1, 3, 7, or 9, the following method can be used. (RD-Algorithm)

Theorem 1.1: If $z=a_n a_{n-1}\ldots a_1$ and $w=b_m b_{m-1}\ldots b_1$ are dividend and prime divisor respectively then:

1. If $w\,|\,|(b_m b_{m-1}\ldots b_2)a_1-(a_n a_{n-1}\ldots a_2)|$ and $b_1=1$, then $w|z$.
2. If $w\,|\,|((7w-1)/10)a_1+(a_n a_{n-1}\ldots a_2)|$ and $b_1=3$, then $w|z$.
3. If $w\,|\,|((3w-1)/10)a_1+(a_n a_{n-1}\ldots a_2)|$ and $b_1=7$, then $w|z$.
4. If $w\,|\,|((9w-1)/10)a_1+(a_n a_{n-1}\ldots a_2)|$ and $b_1=9$, then $w|z$.
5. If $w=5$ is prime divisor then the proof of $w|z$ is clear.
6. If $w=b_m b_{m-1}\ldots b_1$ is composite divisor, then with using of fundamental theorem of arithmetic, the proof of $w|z$ is obvious. ([1, 2])

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Corollary 1.1: (Divisibility by 11 with RD-Algorithm) If z=$a_n a_{n-1} \dots a_1$ is dividend and w=11 is odd divisor, then w|z if $w||(1)a_1 - (a_n a_{n-1} \dots a_2)|$.

Proof: With using above theorem we have $w||(a_n a_{n-1} \dots a_2) - (1)a_1|=|(a_n a_{n-1} \dots a_3 \times 10 + (a_2 - a_1)|$. We should apply the theorem again. Therefore, we have $w||(a_n a_{n-1} \dots a_3) - (1)(a_2 - a_1)|$. With using above theorem we have $w||(a_n a_{n-1} \dots a_4) - (1)(a_3 - a_2 + a_1)|$. By resumption this algorithm, we have divisibility condition by 11.

Theorem 1.2: If z=$a_n a_{n-1} \dots a_1$ and w=$b_m b_{m-1} \dots b_1$ are dividend and odd divisor respectively then:

1. If z=$a_n a_{n-1} \dots a_1$ is dividend and w=$b_m b_{m-1} \dots b_1$ is odd divisor such that b_1=1, b_2=0, then w|z if $w||(b_m b_{m-1} \dots b_3)a_2 a_1 - a_n a_{n-1} \dots a_3|$.
2. If z=$a_n a_{n-1} \dots a_1$ is dividend and w=$b_m b_{m-1} \dots b_1$ is odd divisor such that b_1=3, b_2=4, then w|z if $w||((7w-1)/100)a_2 a_1 - (a_n a_{n-1} \dots a_3)|$.
3. If z=$a_n a_{n-1} \dots a_1$ is dividend and w=$b_m b_{m-1} \dots b_1$ is odd divisor such that b_1=7, b_2=6, then w|z if $w||((3w-1)/100)\, a_2 a_1 - (a_n a_{n-1} \dots a_3)|$.
4. If z=$a_n a_{n-1} \dots a_1$ is dividend and w=$b_m b_{m-1} \dots b_1$ is odd divisor such that b_1=9, b_2=8, then w|z if $w||((9w-1)/100)a_2 a_1 - (a_n a_{n-1} \dots a_3)|$. ([3])

Theorem 1.3: If z=$a_n \dots a_1$ and w=$b_m b_{m-1} \dots b_1$ are dividend and odd divisor respectively then:

1. If z=$a_n a_{n-1} \dots a_1$ is dividend and w=$b_m b_{m-1} \dots b_1$ is odd divisor such that b_1=1, b_2=0, b_3=0, then w|z if $w||(b_m b_{m-1} \dots b_4)a_3 a_2 a_1 - a_n a_{n-1} \dots a_4|$.
2. If z=$a_n a_{n-1} \dots a_1$ is dividend and w=$b_m b_{m-1} \dots b_1$ is odd divisor such that b_1=3, b_2=4, b_3=1, then w|z if $w||((7w-1)/1000)a_3 a_2 a_1 - (a_n a_{n-1} \dots a_4)|$.
3. If z=$a_n a_{n-1} \dots a_1$ is dividend and w=$b_m b_{m-1} \dots b_1$ is odd divisor such that b_1=7, b_2=6, b_3=6, then w|z if $w||((3w-1)/1000)a_3 a_2 a_1 - (a_n a_{n-1} \dots a_4)|$.
4. If z=$a_n a_{n-1} \dots a_1$ is dividend and w=$b_m b_{m-1} \dots b_1$ is odd divisor such that b_1=9, b_2=8, b_3=8, then w|z if $w||((9w-1)/1000)a_3 a_2 a_1 - (a_n a_{n-1} \dots a_4)|$. ([4])

Theorem 1.4: If z=$a_n a_{n-1} \dots a_1$ and w=$b_m b_{m-1} \dots b_1$ are dividend and odd divisor respectively then:

1. If z=$a_n a_{n-1} \dots a_1$ is dividend and w=$b_m b_{m-1} \dots b_1$ is odd divisor such that $b_4 b_3 b_2 b_1 = 0001$, then w|z if $w||(b_m b_{m-1} \dots b_5)a_4 a_3 a_2 a_1 - a_n a_{n-1} \dots a_5|$.
2. If z=$a_n a_{n-1} \dots a_1$ is dividend and w=$b_m b_{m-1} \dots b_1$ is odd divisor such that $b_4 b_3 b_2 b_1 = 7143$, then w|z if $w||((7w-1)/10000)a_4 a_3 a_2 a_1 - (a_n a_{n-1} \dots a_5)|$.
3. If z=$a_n a_{n-1} \dots a_1$ is dividend and w=$b_m b_{m-1} \dots b_1$ is odd divisor such that $b_4 b_3 b_2 b_1 = 6667$, then w|z if $w||((3w-1)/10000)a_4 a_3 a_2 a_1 - (a_n a_{n-1} \dots a_5)|$.
4. If z=$a_n a_{n-1} \dots a_1$ is dividend and w=$b_m b_{m-1} \dots b_1$ is odd divisor such that $b_4 b_3 b_2 b_1 = 8889$, then w|z if $w||((9w-1)/10000)\, a_4 a_3 a_2 a_1 - (a_n a_{n-1} \dots a_5)|$. ([5])

Theorem 1.5: If z=$a_n a_{n-1} \dots a_1$ and w=$b_m b_{m-1} \dots b_1$ are dividend and odd divisor respectively then:

1. If z=$a_n a_{n-1} \dots a_1$ is dividend and w=$b_m b_{m-1} \dots b_1$ is odd divisor such that $b_5 b_4 b_3 b_2 b_1 = 00001$, then w|z if $w||(b_m b_{m-1} \dots b_6)a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \dots a_6|$.
2. If z=$a_n a_{n-1} \dots a_1$ is dividend and w=$b_m b_{m-1} \dots b_1$ is odd divisor such that $b_5 b_4 b_3 b_2 b_1 = 57143$, then w|z if $w||((7w-1)/100000)a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \dots a_6|$.
3. If z=$a_n a_{n-1} \dots a_1$ is dividend and w=$b_m b_{m-1} \dots b_1$ is odd divisor such that $b_5 b_4 b_3 b_2 b_1 = 66667$, then w|z if $w||((3w-1)/100000)a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \dots a_6|$.
4. If z=$a_n a_{n-1} \dots a_1$ is dividend and w=$b_m b_{m-1} \dots b_1$ is odd divisor such that $b_5 b_4 b_3 b_2 b_1 = 88889$, then w|z if $w||((9w-1)/100000)a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \dots a_6|$.

Now, we introduce many divisibility rules of special numbers with RD-Algorithm and we show that which there is a direct relationship between X and the speed of algorithm. In this paper, X is the n right digits of odd divisor. In the other hand, we fixed the 6 right digits of odd divisor.

2. Divisibility Rules by RD-Algorithm

Theorem 2.1: If z=$a_n a_{n-1} \dots a_1$ is dividend and w=$b_m b_{m-1} \dots b_1$ is odd divisor such that $b_6 b_5 b_4 b_3 b_2 b_1 = 000001$, then w|z if $w||(b_m b_{m-1} \dots b_7)a_6 a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \dots a_7|$.

Proof: If $w||(b_m b_{m-1} \dots b_7)a_6 a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \dots a_7|$, then there exists an integer k such that $kw = (b_m b_{m-1} \dots b_7)a_6 a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \dots a_7$. Therefore, 1000000kw= $(b_m b_{m-1} \dots b_7)1000000a_6 a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \dots a_7$. Hence, we have $(1000000kw - a_6 a_5 a_4 a_3 a_2 a_1)w = -z$, so w|z.

Remark 2.2: In this paper, with using theorems for dividend and odd divisor, we can see the new numbers (as same as $\underline{0}$ in follow Example). If we don't know whether a new number is divisible by odd divisor, we should apply the theorems again.

Example 2.3: Is 2184000039 divisible by 56000001? By using above theorem | (56×39) −2184 | = $\underline{0}$. Therefore, 2184000039 is divisible by 56000001.

Theorem 2.4: If z=$a_n a_{n-1} \dots a_1$ is dividend and w=$b_m b_{m-1} \dots b_1$ is odd divisor such that $b_6 b_5 b_4 b_3 b_2 b_1 = 857143$, then w|z if $w \| ((7w-1)/1000000) a_6 a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \dots a_7|$.

Proof: If $w \| ((7w-1)/1000000) a_6 a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \dots a_7|$, then there exists an integer k such that kw=$((7w-1)/1000000) a_6 a_5 a_4 a_3 a_2 a_1 - (a_n a_{n-1} \dots a_7)$. Therefore, $(1000000k - 7a_6 a_5 a_4 a_3 a_2 a_1)w = -z$, so w|z.

Example2.5: Is 5066071483285 divisible by 17857143? By using above theorem |(125×483285) −5066071|=55344554. But the divisibility 55344554 by 17857143 is not clear. Therefore, by using above theorem for 55344554 to 17857143, we have |(125×344554) −55|=43069195. But the divisibility 43069195 by 17857143 is not clear. Therefore, by using above theorem for 43069195 to 17857143, we have |(125×69195) −43|=8649332<17857143. Therefore, 5066071483285 is not divisible by 17857143.

Theorem2.6: If z=$a_n a_{n-1} \dots a_1$ is dividend and w=$b_m b_{m-1} \dots b_1$ is odd divisor such that $b_6 b_5 b_4 b_3 b_2 b_1 = 666667$, then w|z if $w \| ((3w-1)/1000000) a_6 a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \dots a_7|$.

Proof: If $w \| ((3w-1)/1000000) a_6 a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \dots a_7|$, then there exists an integer k such that kw=$((3w-1)/1000000)\, a_6 a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \dots a_7$. Therefore, $(1000000k - 3a_6 a_5 a_4 a_3 a_2 a_1)w = -z$, so w|z.

Example 2.7: Is 19630333443 divisible by 59666667? By using above theorem |(179×333443) −19630| = 59666667. Therefore, 19630333443 is divisible by 59666667.

Remark 2.8: $3 \times \underbrace{66 \dots 6}_{n-th} 7 - 1 \overset{10^{n+1}}{\equiv} 0$. ([4])

Corollary 2.9: If z=$a_n a_{n-1} \dots a_1$ is dividend and w=$b_m b_{m-1} \dots b_1$ is odd divisor such that b_1=7, $b_2 = b_3 = \dots = b_j$ =6, then w|z if $w \| ((3w-1)/10^j) a_j \dots a_2 a_1 - (a_n a_{n-1} \dots a_{j+1})| : 2 \le j \le m$. ([4])

Theorem 2.10: If z=$a_n a_{n-1} \dots a_1$ is dividend and w=$b_m b_{m-1} \dots b_1$ is odd divisor such that $b_6 b_5 b_4 b_3 b_2 b_1 = 888889$, then w|z if $w \| ((9w-1)/1000000) a_6 a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \dots a_7|$.

Proof: If $w \| ((9w-1)/1000000) a_6 a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \dots a_7|$, then there exists an integer k such that kw=$((9w-1)/1000000) a_6 a_5 a_4 a_3 a_2 a_1 - (a_n a_{n-1} \dots a_7)$. Therefore, $(1000000k - 9a_6 a_5 a_4 a_3 a_2 a_1)w = -z$, so w|z.

Example 2.11: Is 3157321157978264 divisible by 2888889? By using above theorem | (26×978264) −3157321157| = 3131886293. But the divisibility 3131886293 by 2888889 is not clear. Therefore, by using above theorem for 3131886293 to 2888889, we have | (26×886293) −3131|=23040487. But the divisibility 23040487 by 2888889 is not clear. Therefore, by using above theorem for 23040487 to 2888889, we have |(26×40487) −23|=1052639<2888889. Therefore, 3157321157978264 is not divisible by 2888889.

Remark 2.12: $9 \times \underbrace{88 \dots 8}_{n-th} 9 - 1 \overset{10^{n+1}}{\equiv} 0$. ([5])

Corollary 2.13: If z=$a_n a_{n-1} \dots a_1$ is dividend and w=$b_m b_{m-1} \dots b_1$ is odd divisor such that b_1=9, $b_2 = b_3 = \dots = b_j$ =8, then w|z if $w \| ((9w-1)/10^j) a_j \dots a_2 a_1 - (a_n a_{n-1} \dots a_{j+1})|$. ([5])

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