DIVISIBILITY RULES BY RD-ALGORITHM

H. Khosravi* and P. Jafari

Department of Mathematics, Faculty of Science, Mashhad Branch, Islamic Azad University, Mashhad, P. Box 91735-413, Iran.

(Received On: 24-06-14; Revised & Accepted On: 24-07-14)

Abstract

Let \(z=a_n 10^{n-1} + a_{n-1} 10^{n-2} + \cdots + a_1\) and \(w=b_m 10^{m-1} + b_{m-1} 10^{m-2} + \cdots + b_1\) are dividend and odd divisor respectively. In this paper, we introduce many divisibility rules of special numbers with RD-Algorithm and we show that which there is a direct relationship between \(X\) and the speed of algorithm and calculations. \(X\) is \(n\) right digits of odd divisor.

AMS Subject Classification: 13AXX, 13F15.

Keywords: Divisibility of Numbers, Prime Factors, RD-Algorithm.

1. INTRODUCTION

In various section of number theory, different and difficult methods are considered for dividing special numbers. Also, some algorithms are introduced because of importance of this matter. A divisibility rule is a shorthand way of determining whether a given number is divisible by a fixed divisor without performing the division, usually by examining its digits. We introduced a new algorithm entitled “RD-Algorithm” in [1] for the first time in the world which we can represent the dividing of numbers to each other in a simple and rapid way. This algorithm could be useful in a mathematics competition such as mathcounts. In the other words, this algorithm reduced the number of digits of dividend. In this paper, in order to fasten dividing numbers, we fixed the 6 right digits of odd divisor, and use the extracted methods of the algorithm for the numbers which it’s the 6 right digits of them are 000 001, 857143, 666667, 888889. The more the \(X\) is higher, the more the speed of algorithm is high (\(X\) is the \(n\) right digits of odd divisor). A lot of study on divisibility of methods have been conducted for many years. In number theory, divisibility methods of whole numbers are very useful because they help us to quickly determine if a number can be divided by \(n\). There are several different methods for divisibility of numbers with many variants and some of them can be found in [6, 7, 8, 9, 10, 11, 12, 13]. For example, in [14, 15] presented that numbers which are dividable to 11 should have the \((\text{sum of the odd numbered digits}) - (\text{sum of the even numbered digits})\) is divisible by 11. Similarly some studies are presented for special numbers such as 15, 17, 19, etc. In this paper, we suppose that \(z=a_n 10^{n-1} + a_{n-1} 10^{n-2} + \cdots + a_1 = a_n a_{n-1} \ldots a_1\) and \(w=b_m 10^{m-1} + b_{m-1} 10^{m-2} + \cdots + b_1 = b_m b_{m-1} \ldots b_1\) are dividend and odd divisor respectively. To test for divisibility by \(w\), where \(w\) ends in 1, 3, 7, or 9, the following method can be used. (RD-Algorithm) Theorem 1.1: If \(z=a_n a_{n-1} \ldots a_1\) and \(w=b_m b_{m-1} \ldots b_1\) are dividend and prime divisor respectively then:

1. If \(w|(b_m b_{m-1} \ldots b_2)a_1 - (a_n a_{n-1} \ldots a_2)\) and \(b_1 = 1\), then \(w|z\).
2. If \(w|((7w - 1)/10)a_1 + (a_n a_{n-1} \ldots a_2)\) and \(b_1 = 3\), then \(w|z\).
3. If \(w|((3w - 1)/10)a_1 + (a_n a_{n-1} \ldots a_2)\) and \(b_1 = 7\), then \(w|z\).
4. If \(w|((9w - 1)/10)a_1 + (a_n a_{n-1} \ldots a_2)\) and \(b_1 = 9\), then \(w|z\).
5. If \(w = 5\) is prime divisor then the proof of \(w|z\) is clear.
6. If \(w=b_m b_{m-1} \ldots b_1\) is composite divisor, then with using of fundamental theorem of arithmetic, the proof of \(w|z\) is obvious. ([1, 2])

Corresponding author: H. Khosravi*

Department of Mathematics, Faculty of Science, Mashhad Branch, Islamic Azad University, Mashhad, P. Box 91735-413, Iran.
Corollary 1.1: (Divisibility by 11 with RD-Algorithm) If \( z = a_na_{n-1} \ldots a_1 \) is dividend and \( w = 11 \) is odd divisor, then \( w \mid z \) if \( w \mid (1)a_1 - (a_na_{n-1} \ldots a_2) \).

Proof: With using above theorem we have \( w \mid (a_na_{n-1} \ldots a_2) - 1 \mid (a_2 - a_1) \). We should apply the theorem again. Therefore, we have \( w \mid (a_na_{n-1} \ldots a_3) - (a_3 - a_2) \). With using above theorem we have \( w \mid (a_na_{n-1} \ldots a_4) - (a_4 - a_3) \). By resumption this algorithm, we have divisibility condition by 11.

Theorem 1.2: If \( z = a_na_{n-1} \ldots a_1 \) and \( w = b_nb_{n-1} \ldots b_1 \) are dividend and odd divisor respectively then:
1. If \( z = a_na_{n-1} \ldots a_1 \) is dividend and \( w = b_nb_{n-1} \ldots b_1 \) is odd divisor such that \( b_1 = b, b_2 = 0, \) then \( w \mid z \) if \( w \mid (b_1a_2a_1 - a_2a_1 \ldots a_3) \).
2. If \( z = a_na_{n-1} \ldots a_1 \) is dividend and \( w = b_nb_{n-1} \ldots b_1 \) is odd divisor such that \( b_1 = 3, b_2 = 4, \) then \( w \mid z \) if \( w \mid ((7w - 1)/100)a_2a_1 - (a_2a_1 \ldots a_3) \).
3. If \( z = a_na_{n-1} \ldots a_1 \) is dividend and \( w = b_nb_{n-1} \ldots b_1 \) is odd divisor such that \( b_1 = 7, b_2 = 6, \) then \( w \mid z \) if \( w \mid ((3w - 1)/100)a_2a_1 - (a_2a_1 \ldots a_3) \).
4. If \( z = a_na_{n-1} \ldots a_1 \) is dividend and \( w = b_nb_{n-1} \ldots b_1 \) is odd divisor such that \( b_1 = 9, b_2 = 8, \) then \( w \mid z \) if \( w \mid ((9w - 1)/100)a_2a_1 - (a_2a_1 \ldots a_3) \).

Corollary 1.2: Theorem 1.2 gives the new divisibility rules in case of even divisor and the other condition is also fulfilled and can be easy applied to the odd dividend.

Remark 2.1: In this paper, we introduce the divisibility rules of special numbers with RD-Algorithm and show that there is a direct relationship between X and the speed of algorithm. In this paper, X is the n right digits of odd divisor. In the other hand, we fixed the 6 right digits of odd divisor.

2. Divisibility Rules by RD-Algorithm

Theorem 2.1: If \( z = a_na_{n-1} \ldots a_1 \) is dividend and \( w = b_nb_{n-1} \ldots b_1 \) is odd divisor such that \( b_2b_3b_2b_1 = 00001 \), then \( w \mid z \) if \( w \mid (b_2b_3b_2b_1 \ldots b_1a_2a_1a_2a_1 - a_2a_1 \ldots a_3) \).

Proof: If \( w \mid (b_2b_3b_2b_1 \ldots b_1a_2a_1a_2a_1 - a_2a_1 \ldots a_3) \), then there exists an integer k such that \( kw = (b_2b_3b_2b_1 \ldots b_1a_2a_1a_2a_1 - a_2a_1 \ldots a_3) \). Therefore, \( 1000000kw = (b_2b_3b_2b_1 \ldots b_1a_2a_1a_2a_1 - a_2a_1 \ldots a_3) \). Hence, we have \( 1000000kw = (a_2a_1a_2a_1 - a_2a_1 \ldots a_3)w = -z \), so \( w \mid z \).

Remark 2.2: In this paper, with using theorems for dividend and odd divisor, we can see the new numbers (as same as \( \theta \) in follow Example). If we don't know whether a new number is divisible by odd divisor, we should apply the theorems again.
Example 2.3: Is 2184000039 divisible by 56000001? By using above theorem | (56×39) − 2184 | = 0. Therefore, 2184000039 is divisible by 56000001.

Theorem 2.4: If \( z = a_n a_{n-1} \ldots a_1 \) is dividend and \( w = b_m b_{m-1} \ldots b_1 \) is odd divisor such that \( b_m b_{m-1} b_3 b_2 b_1 = 857143 \), then \( \exists \ w/z \) if \( w/\left|((7w-1)/1000000)a_6 a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \ldots a_7\right| \).

Proof: If \( w/\left|((7w-1)/1000000)a_6 a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \ldots a_7\right| \), then there exists an integer \( k \) such that \( kw/\left|((7w-1)/1000000)a_6 a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \ldots a_7\right| \)w = −z, so \( w/z \).

Example 2.5: Is 5066701483285 divisible by 17857143? By using above theorem \( (125×483285) = 5066701483285 \). But the divisibility 55344554 by 17857143 is not clear. Therefore, by using above theorem for 55344554 to 17857143, we have \( |(125×44554)−55|=43069195 \). But the divisibility 43069195 by 17857143 is not clear. Therefore, by using above theorem for 43069195 to 17857143, we have \( |(125×69195)−43|=8649332<17857143 \). Therefore, 506671483285 is not divisible by 17857143.

Theorem 2.6: If \( z = a_n a_{n-1} \ldots a_1 \) is dividend and \( w = b_m b_{m-1} \ldots b_1 \) is odd divisor such that \( b_6 b_5 b_4 b_3 b_2 b_1 = 666667 \), then \( w/\exists \ w/z \) if \( w/\left|((3w-1)/1000000)a_6 a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \ldots a_7\right| \).

Proof: If \( w/\left|((3w-1)/1000000)a_6 a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \ldots a_7\right| \), then there exists an integer \( k \) such that \( kw/\left|((3w-1)/1000000)a_6 a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \ldots a_7\right| \)w = −z, so \( w/z \).

Example 2.7: Is 19630333443 divisible by 59666667? By using above theorem \( |(179×333443)−19630| = 59666667 \). Therefore, 19630333443 is divisible by 5966667.

Remark 2.8: \( 3×66\ldots 67−1 \ldots n \equiv 0 \) \( (4) \).

Corollary 2.9: If \( z = a_n a_{n-1} \ldots a_1 \) is dividend and \( w = b_m b_{m-1} \ldots b_1 \) is odd divisor such that \( b_1 = \ldots = 6 \), then \( w/\exists \ w/z \) if \( w/\left|((3w−1)/1000000)a_6 a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \ldots a_7\right| \).

Theorem 2.10: If \( z = a_n a_{n-1} \ldots a_1 \) is dividend and \( w = b_m b_{m-1} \ldots b_1 \) is odd divisor such that \( b_6 b_5 b_4 b_3 b_2 b_1 = 888889 \), then \( w/\exists \ w/z \) if \( w/\left|((9w−1)/1000000)a_6 a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \ldots a_7\right| \).

Proof: If \( w/\left|((9w−1)/1000000)a_6 a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \ldots a_7\right| \), then there exists an integer \( k \) such that \( kw/\left|((9w−1)/1000000)a_6 a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \ldots a_7\right| \)w = −z, so \( w/z \).

Example 2.11: Is 3157321157978264 divisible by 2888889? By using above theorem \( |(26×978264)−3157321157| = 3131886293 \). But the divisibility 3131886293 by 2888889 is not clear. Therefore, by using above theorem for 3131886293 to 2888889, we have \( |(26×886293)−313|=23040487 \). But the divisibility 23040487 by 2888889 is not clear. Therefore, by using above theorem for 23040487 to 2888889, we have \( |(26×40487)−23|=1052639<2888889 \). Therefore, 3157321157978264 is not divisible by 2888889.

Remark 2.12: \( 9×88\ldots 89−1 \ldots n \equiv 0 \) \( (5) \).

Corollary 2.13: If \( z = a_n a_{n-1} \ldots a_1 \) is dividend and \( w = b_m b_{m-1} \ldots b_1 \) is odd divisor such that \( b_1 = 9 \), then \( w/\exists \ w/z \) if \( w/\left|((9w−1)/1000000)a_6 a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \ldots a_7\right| \).

3. ACKNOWLEDGEMENTS

The authors thank the research council of Mashhad Branch (Islamic Azad University). Also, we would like to thank the referee for his/her many helpful suggestions.

REFERENCES


Source of support: Research Council of Mashhad Branch (Islamic Azad University), Iran.
Conflict of interest: None Declared

[Copy right © 2014. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]