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AN ITERATIVE GOAL FOR SOLVING ROUGH BI-LEVEL LINEAR PROGRAMMING PROBLEM

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ABSTRACT

This paper proposes a bi- level linear programming problem with rough parameters in constraints, the objective functions are to be maximized with different goals, the approach in this paper was based on the goal programming method to develop the optimal solution of the two- level decision- maker, then we used the concepts of tolerance membership function to generate the optimal solution for this problem. An auxiliary problem is discussed as well as an example is presented.

***Key Words**: Bi –level programming, rough programming, goal programming.*

1. INTRODUCTION

Rough set theory [4] is an effective tool for data mining, and it has been the core problem in academic and application fields. Many scholars gave many discussions under different background and also obtained many important research results [3].

Bi-level programming problem is formulated for a problem in which two decision makers make decisions successively [6].

The majority of research on bi-level decision making has centered on the linear BLP. A set of approaches and algorithms of linear bi-level programming, such as the well-known Kuhn–Tucker approach, Kth-best approach, Branch-and-bound algorithm and genetic algorithm have been widely used [7].

The goal programming (GP) model is useful for decision makers to consider several objectives in finding a set of acceptable solutions simultaneously. Since only partial information can possibly be obtained, precisely determining the goal value of each objective might be difficult for decision makers; the main studies that incorporate uncertainty and imprecision into the GP are [5].

In [1] Emam proposed a bi-level integer non-linear programming problem with linear or non-linear constraints, and in which the non-linear objective function at each level are to maximized. It proposed a two planner integer model and a solution method for solving this problem. Emam proposed an interactive approach for solving bi-level integer multi-objective fractional programming problem [2].

In [8] Xu and Yao discussed a class of linear multi-objective programming problems with random rough coefficients and gave a crisp equivalent model.

In [6] Saraj and Sadeghi presented a fuzzy goal programming (FGP) technique for solving Quadratic Bi-Level Fractional Multi-Objective Programming (QBL-FMOP) Problem.

2. PROBLEM FORMULATION AND SOLUTION CONCEPT

Let $x_i \in R^{n_i}, (i = 1,2)$ be a vector variables indicating the first decision level's choice and the second decision level's choice,$n_i \geq 1, (i = 1,2)$.

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Let $H_i: R^{n_i} \to R^{N_i}, (i = 1,2)$ be the first level objective functions, and the second level objective functions,respectively. Let the FLDM and SLDM have N_1 and N_2 objective function, respectively.

Therefore, the bi- level multi-objective linear programming problem with definite goals contains rough parameters in constraints may be stated as follows:

[FLDM]

$$\max_{x_1} H_1(x_1, x_2) = \max_{x_1}(h_{11}, \dots, h_{1N_1}), \tag{1}$$

Where x_2 solves

[SLDM]

$$\max_{x_2} H_2(x_1, x_2) = \max_{x_2}(h_{21}, \dots, h_{2N_2}), \tag{2}$$

Subject to

$$G = \{(\xi x_1, \xi x_2) | g_i(\xi x_1, \xi x_2) \leq y, i = 1,2, \dots m, (x_1, x_2) \geq 0\}. \tag{3}$$

G is the bi-level linear constraint set contains rough parameters.$H_1$and $H_2$are linear functions with definite goals.

Now, we can write an associated goal programming for this problem with $(N_1 + N_2)$ goals as:

[FLDM]

$$\text{Achieve } (h_{11}(x), \dots, h_{1N_1}(x)) = (k_{11}, \dots\ k_{1N_1}), \tag{4}$$

where x_2 solves

[SLDM]

$$\text{Achieve}(h_{21}(x), \dots, h_{2N_2}(x)) = (k_{21}, \dots\ k_{2N_2}), \tag{5}$$

Subject to

$$x \in G.$$

where k_{1N_1}, k_{2N_2} are scalars and represent the aspiration levels associated with the objectives of the FLDM and SLDM, respectively.

3. THE TRANSFORMATION OF RANDOM ROUGH COEFFICIENT [8]

To convert the bi-level multi-objective linear programming problem with random rough coefficient in the constraints into the respective crisp equivalents for solving this trust probability constrains, this process is usually hard work for many cases but the transformation process is introduced in the following theorems.

Theorem 1: Suppose that random variables $\tilde{\tilde{e}}_{rj}(\lambda)$ and $\tilde{\tilde{b}}_r(\lambda)$ are characterized by $\tilde{\tilde{e}}_{rj}(\lambda) \sim \mathcal{N}\left(e_{rj}(\lambda), V_r^e\right)$, $\tilde{\tilde{b}}_r(\lambda) \sim \mathcal{N}\left(b_r(\lambda), \left(\sigma_r^b\right)^2\right)$, where $e_{rj}(\lambda), b_r(\lambda)$ are rough variables, and $V_r^e, \left(\sigma_r^b\right)^2$are positive definite covariances. By Theorem 1, we have that $e_r(\lambda)^T x, b_r(\lambda)$are rough variables; then $e_r(\lambda)^T x - b_r(\lambda) = [(a, b), (c, d)](c \leq a \leq b \leq d)$ is also a rough variable. We assume that it is characterized by the following trust measure function:

$$\mathrm{Tr}\{e_r(\lambda)^T x - b_r(\lambda) \leq t\} = \begin{cases} 0 & \text{if } t \leq c, \\ \frac{t-c}{2(d-c)} & \text{if } c \leq t \leq a, \\ \frac{1}{2}\left(\frac{t-c}{d-c} + \frac{t-a}{b-a}\right) & \text{if } a \leq t \leq b, \\ \frac{1}{2}\left(\frac{t-c}{d-c} + 1\right) & \text{if } b \leq t \leq d, \\ 1 & \text{if } d \leq t. \end{cases}$$

where $\left(e_{rj}(\lambda)\right)_{n \times 1} = \left(e_{r1}(\lambda), e_{r2}(\lambda), \dots, e_{rn}(\lambda)\right)^T$. Then, we have that $\mathrm{Tr}\left\{\lambda \middle| \Pr\left\{\tilde{\tilde{e}}_r(\lambda)^T x \leq \tilde{\tilde{b}}_r(\lambda)\right\} \geq \theta_r\right\} \geq \eta_r$ if and only if

$$\begin{cases} a \geq M \geq c + 2(d-c)\eta_r & \text{if } c \leq M \leq a, \\ b \geq M \geq \frac{2\eta_r(d-c)(b-a) + C(b-a) + a(d-c)}{d-c+b-a} & \text{if } a \leq M \leq b, \\ d \geq M \geq (2\eta_r - 1)(d-c) + c & \text{if } b \leq M \leq d, \\ \quad M \geq d & \text{if } M \geq d. \end{cases}$$

To proof theorem 1 above, the reader is referred to [8].

3.1. The equivalent crisp problem of bi-level rough linear problem

The equivalent bi-level multi-objective linear programming problem equivalent to the bi- level multi-objective linear programming problem contains rough parameters in constraints with definite goals may be stated as follows:

[FLDM]

$$\max_{x_1} H_1(x_1, x_2) = \max_{x_1}(h_{11}, \dots, h_{1N_1}), \tag{6}$$

where x_2 solves

[SLDM]

$$\max_{x_2} H_2(x_1, x_2) = \max_{x_2}(h_{21}, \dots, h_{2N_2}), \tag{7}$$

Subject to

$$G = \{(x_1, x_2) | g_i(x_1, x_2) \le 0, i = 1,2, \dots m, (x_1, x_2) \ge 0\}. \tag{8}$$

where $h_1, h_2$are the objective functions of the FLDM, and SLDM.

Definition1: For any $x_1(x_1 \in G_1 = \{x_1 | (x_1, x_2) \in \mathrm{G}\})$ achieves the FLDM goals with underattainment or overattainment, if the decision-making variable $x_2(x_2 \in G_2 = \{x_2 | (x_1, x_2) \in G_1\})$ achieves the SLDM goals with underattainment or overattainment, then (x_1, x_2) is a feasible solution of the rough goal bi-Level multi-objective linear programming problem.

Definition 2: If (x_1^*, x_2^*) is a feasible solution of the rough goal bi-Level multi-objective linear programming problem, such that the FLDM achieves all goals; so (x_1^*, x_2^*) is the Pareto optimal solution of the rough goal bi-Level multi-objective linear programming problem.

4. A GOAL APPROACH FOR THE BI- LEVEL MULTI-OBJECTIVE LINEAR PROGRAMMING PROBLEM

To solve the bi- level multi-objective linear programming problem with definite goals, one first get the optimal solution of the FLDM with definite goals, and the SLDM should get his optimal solution with definite goals, as follows:-

4-1 The first level decision maker

First, the FLDM solves the following problem:

$$\text{Achieve } (h_{11}(x), \dots, h_{1N_1}(x)) = (k_{11}, \dots\ k_{1N_1}), \tag{9}$$

Subject to

$$x \in G.$$

where $k_{11}, \dots, k_{1N_1}$are scalars, and represent the aspiration levels associated with the objectives, $h_{11}, \dots, h_{1N_1}$, respectively.

We consider the following bi-level multi-objective linear programming problem associated to the first goal as:

P_{11}: Minimize $D_{11} = d_{11}^- + d_{11}^+$, (10)

Subject to

$$h_{11}(x) + d_{11}^- - d_{11}^+ = k_{11},$$
$$x \in G,$$
$$d_{11}^-, d_{11}^+ \ge 0.$$

where d_{11}^- and d_{11}^+ are the underattainment and overattainment, respectively, of the first goal and $d_{11}^- \mathrm{X}\ d_{11}^+ = 0$.

Then the attainment problem associated with the second goal is equivalent to the optimization problemP_{12} , where:

P_{12}: Minimize$D_{12} = d_{12}^- + d_{12}^+$, (11)

Subject to

$$h_{12}(x) + d_{12}^- - d_{12}^+ = k_{12},$$
$$h_{11}(x) + d_{11}^- - d_{11}^+ = k_{11},$$
$$d_{11}^- + d_{11}^+ = D_{11}^*,$$
$$x \in G,$$
$$d_{1t}^-, d_{1t}^+ \ge 0, (t = 1,2).$$

The optimal solution of the linear goal programming model is given by $x^* = (x_1^F, x_1^F)$.

4-2. The second level decision maker

Second, in the same way, the SLDM independently solves:

$$\text{Achieve}(h_{21}(x), \dots, h_{2N_2}(x)) = (k_{21}, \dots\ k_{2N_2}), \tag{12}$$

Subject to

$$x \in G.$$

where $k_{21}, \dots, k_{2N_2}$ are scalars, and represent the aspiration levels associated with the objectives, $h_{21}, \dots, h_{2N_2}$, respectively.

The SLDM will do the same action as the FLDM till he obtain his optimal solution $x^* = (x_1^S, x_1^S)$.

5. FUZZY APPROACH OF BI-LEVEL LINEAR PROGRAMMING WITH ROUGH PARAMETERS PROBLEM

Now the solution of the FLDM and SLDM are disclosed. However, two solutions are usually different because of nature between two levels goals. The FLDM knows that using the optimal decisions x_1^F as a control factors for the SLDM, is not practical. It is more reasonable to have some tolerance that gives the SLDM an extent feasible region to search for his/her optimal solution, and reduce searching time or interactions. In this way, the range of decision variable x_1 should be around x_1^F with maximum tolerance t_1 and the following membership function specify x_1 as:

$$\mu(x_1) = \begin{cases} \frac{x_1 - (x_1^F - t_1)}{t_1} & x_1^F - t_1 \le x_1 \le x_1^F, \\ \frac{(x_1^F + t_1) - x_1}{t_1} & x_1^F \le x_1 \le x_1^F - t_1, \end{cases} \tag{13}$$

where x_1^F is the most preferred solution; the $(x_1^F - t_1)$ and $(x_1^F + t_1)$ are the worst acceptable decision; and that satisfaction is linearly increasing with the interval of $\lfloor x_1^F - t_1, x_1 \rfloor$ and linearly decreasing with $\lfloor x_1, x_1^F + t_1 \rfloor$, and other decision are not acceptable.

First, the FLDM goals may reasonably consider $h_1 \ge h_1^F$ is absolutely acceptable and $h_1 < \grave{h}_1$ is absolutely unacceptable, and that the preference with $\lfloor \grave{h}_1, h_1^F \rfloor$ is linearly increasing. This due to the fact that the SLDM obtained the optimum at (x_1^S, x_2^S), which in turn provides the FLDM the objective function values $\grave{h}_1$, makes any $h_1 < \grave{h}_1 = h_1(x_1^S, x_2^S)$ unattractive in practice.

The following membership functions of the FLDM can be stated as:

$$\mu[h_1(x)] = \begin{cases} 1 & \text{if } h_1(x) > h_1^F, \\ \frac{h_1(x) - \grave{h}_1}{h_1^F - \grave{h}_1} & \text{if } \grave{h}_1 \le h_1(x) \le h_1^F, \\ 0 & \text{if } \grave{h}_1 \ge h_1(x). \end{cases} \tag{14}$$

Second, the SLDM goals may reasonably consider the $h_2 \ge h_2^S$ is absolutely acceptable and $h_2 < \grave{h}_2 = h_2(x_1^F, x_2^F)$ is absolutely unacceptable, and that the preference with $\lfloor \grave{h}_2, h_2^S \rfloor$ is linearly increasing. In this way, the SLDM has the following membership functions for his/her goal:

$$\mu[h_2(x)] = \begin{cases} 1 & \text{if } h_2(x) > h_2^S, \\ \frac{h_2(x) - \grave{h}_2}{h_2^S - \grave{h}_2} & \text{if } \grave{h}_2 \le h_2(x) \le h_2^S, \\ 0 & \text{if } \grave{h}_2 \ge h_2(x). \end{cases} \tag{15}$$

Finally, in order to generate the satisfactory solution, which is also a Pareto optimal solution with overall satisfaction for all decision-makers, we can solve the following Tchebycheff problem.

$$\max \delta, \tag{16}$$

Subject to

$$\frac{(x_1^F + t_1) - x_1}{t_1} \ge \delta,$$
$$\frac{x_1 - (x_1^F - t_1)}{t_1} \ge \delta,$$
$$\mu[h_1(x)] \ge \delta,$$
$$\mu[h_2(x)] \ge \delta,$$

$x \in G,$
$t_i > 0\ , \delta \in [0,1].$

where δ is the over all satisfaction.

If the FLDM is satisfied with solution then satisfactory solution is reached. Otherwise, he/she should provide new membership function for the control variable and objectives to the SLDM, until a satisfactory solution is reached.

6. NUMERICAL EXAMPLE

To demonstrate the solution method for bi-level multi-objective linear programming problem under random rough coefficient in constraints can be written as:

[FLDM]
$\max_{x_1} \mathrm{H}_1(x_1, x_2) = \max_{x_1}[5x_1 + x_2, 2x_1 + x_2],$

where x_2 solves

[SLDM]
$\max_{x_2} \mathrm{H}_2(x_1, x_2) = \max[x_1\ +\ 2x_2, 2x_1\ +\ 2x_2],$
Subject to
$\xi_1 x_1\ +\ \xi_2 x_2 <=\ 45,$
$\xi_3 x_1 + \xi_4\ \ x_2 <=\ 30,$
$x_1, x_2 \geq 0.$

Assume that the rough parameters are defines as:
$\xi_1 \sim \mathcal{N}(\rho_1, 1)$, with $\rho_1 = ([2,3],[1,4])$, $\xi_2 \sim \mathcal{N}(\rho_2, 4)$, with $\rho_2 = ([1,2],[1,3])$,
$\xi_3 \sim \mathcal{N}(\rho_3, 1)$, with $\rho_3 = ([1,2],[0,3])$, $\xi_4 \sim \mathcal{N}(\rho_4, 2)$, with $\rho_4 = ([3,4],[2,5])$,

Let $\eta_r =\ 0.9.$

Now by using theorem 1, the equivalent crisp problem which equivalent to bi-Level multi-objective linear programming problem under rough parameters in constraints with definite goals, as follows:-

[FLDM]

Achieve $h_{11} = (5x_1 + x_2) = k_{11},$

Achieve $h_{12} = (2x_1 + x_2) = k_{12},$

Where x_2 solves

[SLDM]

Achieve $h_{21} = (x_1\ +\ 2x_2) = k_{21},$

Achieve $h_{22} = (2x_1\ +\ 2x_2) = k_{22}.$

Subject to
$x \in G = \{3.4\mathrm{x}_1 + 2.6\mathrm{x}_2 <=\ 45,$
$2.4\mathrm{x}_1 + 4.4\mathrm{x}_2 <=\ 30,$
$x_1 \geq 0, x_2 \geq 0 \geq 0\}.$

Then, calculating trust for every rough coefficients using trust measure function in theorem 1:
$\mathrm{Tr}\,\{\xi_1\} = 0.9, \mathrm{Tr}\,\{\xi_2\} = 0.9\ , \mathrm{Tr}\,\{\xi_3\} = 0.9, \mathrm{Tr}\,\{\xi_4\} = 0.9, \mathrm{Tr}\,\{\xi_5\} = 0.6, \mathrm{Tr}\,\{\xi_6\} = 0.9.$

So, with trust more than or equal $\propto$ is 0.6 the equivalent crisp problem which equivalent to bi-Level multi-objective linear programming problem under rough parameters in constraints.

Now, we can write an associated goal programming for this problem with goals as follows:

1- First, the FLDM solves his/her Problem as following:

Achieve $5x_1 + x_2 = k_{11}$,

Achieve $2x_1 + x_2 = k_{12,}$

Subject to
$x \in G$.

The aspiration levels of the goals are assumed to be $k_{11} = 50$, $k_{12,} = 20$, respectively. Then, the optimization problem associated with the first goal is formulated as follows:

P_{11}: $Minimize\ D_{11} = d_{11}^- + d_{11}^+$,
Subject to
$5x_1 + x_2 + d_{11}^- - d_{11}^+ = k_{11}$,
$x \in G$,
$d_{11}^-, d_{11}^+ \geq 0$.

The maximum degree of attainment of problem P_{11} is $D_{11}^* = 0.0002$ with the optimal solution$x_1 = (9.9998, 0.0011)$ and $d_{11}^- = 0, d_{11}^+ = 0.0002$.

The attainment problem for goal 2 of the FLDM is equivalent to problemP_{12}, where:

P_{12}: $Minimize\ D_{12} = d_{12}^- + d_{12}^+$
Subject to
$2x_1 + x_2 + d_{12}^- - d_{12}^+ = k_{12,}$
$5x_1 + x_2 + d_{11}^- - d_{11}^+ = 50$,
$d_{11}^- + d_{11}^+ = 0.0002$,
$x \in G$,
$d_{1t}^-, d_{1t}^+ \geq 0, (t = 1,2)$.

Therefore, the optimal solution of the model P_{12} is $x_2 = (9.9999, 0.0008), d_{11}^- = 0, d_{11}^+ = 0.0002, d_{12}^- = 0$, $d_{12}^+ = 0.0006$, so the optimal solution of the bi-level multi-objective linear goal programming model is given by x^* which will be the optimal solution of the FLDM $x^* = (x_1, x_2) = (9.9999, 0.0008)$.

2- Second, the SLDM solves his/her Problem as following:

Achieve$x_1 + 2x_2 = k_{21}$,

Achieve $2x_1 + 2x_2 = k_{22}$,
Subject to
$x \in G$.

The aspiration levels of the goals are assumed to be $k_{21} = 10, k_{22} = 18$ respectively. Then, the optimization problem associated with the first goal is formulated as follows:

P_{21}: $Minimize\ D_{21} = d_{21}^- + d_{21}^+$,
Subject to
$x_1 + 2x_2 + d_{21}^- - d_{21}^+ = k_{21}$,
$x \in G$,
$d_{21}^-, d_{21}^+ \geq 0$.

The maximum degree of attainment problem P_{21} is $D_{21}^* = 0$ with the optimal solution $x = (9.9987, 0.0006)$ and $d_{21}^- = 0, d_{21}^+ = 0$.

The attainment problem for goal 2 of the SLDM is equivalent to problemP_{21}, where:

P_{22}: $Minimize\ D_{22} = d_{22}^- + d_{22}^+$,
Subject to
$2x_1 + 2x_2 + d_{22}^- - d_{22}^+ = k_{22}$,
$x_1 + 2x_2 + d_{21}^- - d_{21}^+ = 10$,
$d_{21}^- + d_{21}^+ = 0$,
$x \in G$,
$d_{2t}^-, d_{2t}^+ \geq 0, (t = 1,2)$.

Therefore, the optimal solution of the model $P_{22}(x_1, x_2)$ is= $(7.9990{,}1.0005)$, $d_{21}^{-} = 0, d_{21}^{+} = 0, d_{22}^{-} = 0.0009$, $d_{22}^{+} = 0$, so the optimal solution of the bi-level multi-objective linear goal programming model is given byx^*which will be the optimal solution of the SLDM $x^* = (x_1, x_2) = (7.9990{,}1.0005)$.

3- Finally, we assume the FLDM control decision $x_1^F = 9.9999$ with the tolerance 1; the SLDM solves the following Tchebycheff problem as follows:

$Max\ \delta,$

Subject to

$$
\begin{aligned}
& x \in G, \\
& -x_1 - \delta \geq -10.9999, \\
& x_1 - \delta \geq 8.9999, \\
& (5x_1 + x_2) - 9.0048\,\delta \geq 40.9955, \\
& (2x_1 + x_2) - 3.00216\,\delta \geq 16.9985, \\
& (x_1 + 2x_2) + 0.0015\,\delta \geq 10.0015, \\
& (2x_1 + 2x_2) + 2.0024\,\delta \geq 20.0014, \\
& \delta \in [0, 1].
\end{aligned}
$$

Whose, optimal solution is: $(x_1, x_2) = (10.0001{,}0.9869), \delta = 0.9999$, $h_1 = (50.9874{,}20.9871)$, and $h_2 = (11.9739{,}21.974)$ Overall satisfaction for both decisions makers.

7. CONCLUSION

This paper proposes a bi- level linear programming problem with rough parameters in constraints, the linear objective functions are to be maximized with different goals, the suggested approach in this paper was mainly based on the goal programming method of Dauer and Krueger to develop the optimal solution of the two- level decision- maker, then we used the concepts of tolerance membership function together with the branch and bound technique to generate the optimal solution for this problem.

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