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On Λ_r-compact and Λ_r-connected spaces

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ABSTRACT

The aim of this paper is to introduce and study two new classes of spaces, called Λ_r-compact and Λ_r-connected spaces. Some basic properties of these spaces are studied by utilizing Λ_r-open set.

Key words and phrases: *Λ_r-open set, Λ_r-compact space, Λ_r-connected space.*

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1. INTRODUCTION:

Λ_r-open sets is recently introduced by the authors [4] and studied Λ_r-T_0, Λ_r-T_1 and Λ_r-T_2 spaces, Λ_r-homeomorphisms and variants of continuity related to this concept in [4,5,6]. In this paper, we introduce two new classes of topological spaces called Λ_r-compact and Λ_r-connected spaces. To define and investigate these spaces, we use Λ_r-open sets.

Throughout the paper, (X, τ) (or simply X) will always denote a topological space. For a subset S of a topological space X, S is called regular-open if S = *Int cl* S. Then the complement S^c (= X-S) of a regular-open set S is called the regular-closed set. The family of all regular-open sets (resp. regular-closed sets) in (X, τ) will be denoted by $RO(X, \tau)$ (resp. $RC(X, \tau)$). A subset S of a topological space (X, τ) is called Λ_r-set [4] if $S = \Lambda_r(S)$ where $\Lambda_r(S) = \cap\{G \,/\, G \in RO(X, \tau) \text{ and } S \subseteq G\}$. The collection of all Λ_r-sets in (X, τ) is denoted by $\Lambda_r(X, \tau)$.

2. PRELIMINARIES:

Throught this paper, we adopt the notations and terminology of [4]. Let A be a subset of a space (X, τ). Then A is called a Λ_r-closed set if $A = S \cap C$ where S is a Λ_r-set and C is a closed set. The complement of a Λ_r-closed set is called Λ_r-open. The collection of all Λ_r-open (resp. Λ_r-closed) sets in (X, τ) is denoted by $\Lambda_r O(X, \tau)$ (resp. $\Lambda_r C(X, \tau)$). Also note that every open set is Λ_r-open; arbitrary union of Λ_r-open sets is Λ_r-open and arbitrary intersection of Λ_r-closed sets is Λ_r-closed. A point $x \in X$ is called a Λ_r-cluster point of A if for every Λ_r-open set U containing x, $A \cap U \neq \emptyset$. The set of all Λ_r-cluster points of A is called the Λ_r-closure of A and it is denoted by Λ_r-*cl*(A). Then Λ_r-*cl*(A) is the intersection of Λ_r-closed sets containing A and it is the smallest Λ_r-closed set containing A. Also A is Λ_r-closed if and only if A = Λ_r-*cl*(A). The union of Λ_r-open sets contained in A is called Λ_r-interior of A and it is denoted by Λ_r-*int*(A). Also A is Λ_r-open if and only if A = Λ_r-*int*(A) and Λ_r-*cl*(X-A) = X-(Λ_r-*int*(A)).

Before we entering into our work, we recall the following definitions.

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Definition: 2.1 A function f: X → Y is called

(i) Λ_r-continuous [5] if $f^{-1}(V)$ is a Λ_r-open set in X for each open set V in Y.
(ii) Λ_r-irresolute [5] if $f^{-1}(V)$ is a Λ_r-open set in X for each Λ_r-open set V in Y.
(iii) Λ_r-open [6] if the image of each open set in X is a Λ_r-open set in Y.
(iv) Λ_r^*-open [6] if the image of each Λ_r-open set in X is a Λ_r-open set in Y.
(v) Λ_r-closed [6] if the image of each closed set in X is a Λ_r-closed set in Y.
(vi) Λ_r^*-closed [6] if the image of each Λ_r-closed set in X is a Λ_r-closed set in Y.

3. Λ_r-COMPACT SPACES:

In this section, we introduce and study a class of compact space called Λ_r-compact space in topological spaces.

Definition: 3.1 A collection $\{A_i: i\in I\}$ of Λ_r-open sets in a topological space (X,τ) is called a Λ_r-open cover of a subset B of X if $B \subseteq \cup \{A_i : i\in I\}$.

Definition: 3.2 A topological space (X,τ) is called Λ_r-compact if every Λ_r-open cover of X has a finite subcover.

Definition: 3.3 A subset B of a topological space (X,τ) is said to be Λ_r-compact relative to X if for every collection $\{A_i : i\in I\}$ of Λ_r-open subsets of X such that $B \subseteq \cup \{A_i : i\in I\}$, $\exists$ a finite subset I_0 of I such that $B \subseteq \cup \{A_i : i\in I_0\}$.

Definition: 3.4 A subset B of a topological space (X,τ) is said to be Λ_r-compact if B is Λ_r-compact as the subspace of X.

Theorem: 3.5 Every Λ_r-closed subset of a Λ_r-compact space X is Λ_r-compact relative to X.

Proof: Let A be a Λ_r-closed subset of a Λ_r-compact space X. Then A^c is Λ_r-open in X. Let $\{G_\alpha : \alpha \in I\}$ be any Λ_r-open cover of A. Then $\{G_\alpha : \alpha \in I\} \cup A^c$ is a Λ_r-open cover of X. Since X is Λ_r-compact, it has a finite subcover, say $\{G_1, G_2... G_n\}$. If this subcover contains A^c, we discard it. Otherwise leave the subcover as it is. Thus we have obtained a finite Λ_r-open subcover of A and so A is Λ_r-compact relative to X.

Theorem: 3.6 Let f be an Λ_r-irresolute surjective function from a topological space X onto a topological space Y. If X is Λ_r-compact, then Y is Λ_r-compact.

Proof: Let $\{A_i : i\in I\}$ be a Λ_r-open cover of Y. Then $\{f^{-1}(A_i) : i\in I\}$ is a Λ_r-open cover of X since f is Λ_r-irresolute. Since X is Λ_r-compact, X has a finite subcover, say $\{f^{-1}(A_1), f^{-1}(A_2)... f^{-1}(A_n)\}$. Since f is onto, $\{A_1, A_2... A_n\}$ is a finite subcover of Y and hence Y is Λ_r-compact.

Theorem: 3.7 Let f be an Λ_r-irresolute bijective function from a topological space X onto a topological space Y. If f is Λ_r^*-open and Y is Λ_r-compact, then X is Λ_r-compact.

Proof: Let $\{A_i: i\in I\}$ be a Λ_r-open cover of X. Since f is Λ_r^*-open, $\{f(A_i) : i\in I\}$ is a Λ_r-open cover of Y. Since Y is Λ_r-compact, Y has a finite subcover, say $\{f(A_1), f(A_2), ..., f(A_n)\}$. Since f is Λ_r-irresolute, $\{f^{-1}(f(A_1)), f^{-1}(f(A_2)), ..., f^{-1}(f(A_n))\}$ is a finite subcover of X. That is, $\{A_1, A_2, ..., A_n\}$ is a finite subcover of X. Hence X is Λ_r-compact.

Theorem: 3.8 A topological space X is Λ_r-compact if and only if for every family $\{F_\alpha : \alpha \in I\}$ of Λ_r-closed sets with finite intersection property, $\bigcap_{\alpha\in I} F_\alpha \neq \emptyset$.

Proof: Necessity: Let X be a Λ_r-compact space and suppose that $\{F_\alpha : \alpha \in I\}$ be a family of Λ_r-closed subsets of X with finite intersection property such that $\bigcap_{\alpha\in I} F_\alpha = \emptyset$.

Let us consider the Λ_r-open sets $U_\alpha = X - F_\alpha$.
Then $\cup \{U_\alpha : \alpha \in I\} = \cup \{X - F_\alpha : \alpha \in I\} = X - \cap \{F_\alpha : \alpha \in I\} = X - \emptyset = X$.

Hence $\{U_\alpha : \alpha \in I\}$ is a Λ_r-open cover of X.

Since X is Λ_r-compact, it has a finite subcover $\{U_{\alpha_i} : \alpha_i \in I_0\}$ where $I_0 = \{1,2,...,n\}$.

Then $X = \cup \{U_{\alpha_i} : \alpha_i \in I_0\} = \cup \{X - F_{\alpha_i} : \alpha_i \in I_0\} = X - \cap \{ F_{\alpha_i} : \alpha_i \in I_0\}$.

Hence $\cap \{F_{\alpha_i} : \alpha_i \in I_0\} = \emptyset$, which is a contradiction. Thus, if the family $\{F_\alpha : \alpha \in I\}$ of Λ_r-closed sets with finite intersection property, then $\bigcap_{\alpha\in I} F_\alpha \neq \emptyset$.

Sufficiency: Suppose that for every family $\{F_\alpha : \alpha \in I\}$ of Λr-closed sets with finite intersection property, $\bigcap_{\alpha\in I} F_\alpha \neq \emptyset$. Let $\{U_\alpha : \alpha \in I\}$ be a Λ_r-open cover of X.

Then $X = \cup \{U_\alpha : \alpha \in I\}$. Now, $\emptyset = X - \cup \{U_\alpha : \alpha \in I\} = \cap \{X - U_\alpha : \alpha \in I\}$.

Hence $\{X - U_\alpha : \alpha \in I\}$ is a family of Λ_r-closed sets with an empty intersection.

By the hypothesis, $\exists$ a finite subset $\{X - U_{\alpha_i} : \alpha_i \in I_0\}$ such that $\cap \{ X - U_{\alpha_i} : \alpha_i \in I_0\} = \emptyset$. Hence $X = X - \cap \{ X - U_{\alpha_i} : \alpha_i \in I_0\} = \cup \{ X - (X - U_{\alpha_i}) : \alpha_i \in I_0\} = \cup \{U_{\alpha_i} : \alpha_i \in I_0\}$.

This shows that $\{U_{\alpha_i} : \alpha_i \in I_0\}$ is a finite subcover of X and hence X is Λ_r-compact.

Theorem: 3.9 For any topological space X, the following properties are equivalent:

(i) X is Λ_r-compact
(ii) Every proper Λ_r-closed set is Λ_r-compact relative to X.

Proof: (i) → (ii) By Theorem 3.5, it is obvious.
(ii) → (i) Let $\{V_\alpha : \alpha \in I\}$ be a Λ_r-open cover of X. Then $X \subseteq \cup \{V_\alpha : \alpha \in I\}$.
We choose and fix one $\alpha_0 \in I$.
Then $X - V_{\alpha_0}$ is a proper Λ_r-closed subset of X and $X - V_{\alpha_0} \subseteq \cup \{V_\alpha : \alpha \in I - \{\alpha_0\}\}$.
Therefore $\{V_\alpha : \alpha \in I - \{\alpha_0\}\}$ is a Λ_r-open cover of $X - V_{\alpha_0}$.
By the hypothesis, $\exists$ a finite subset I_0 of $I - \{\alpha_0\}$ such that $X - V_{\alpha_0} \subseteq \{V_\alpha : \alpha \in I_0\}$.
Therefore $X \subseteq \cup \{V_\alpha : \alpha \in I_0 \cup \{\alpha_0\}\}$. Hence X is Λ_r-compact.

Theorem: 3.10 If a function $f : X \to Y$ is Λ_r-irresolute and a subset B of X is Λ_r-compact relative to X, then the image f(B) is Λ_r-compact relative to Y.

Proof: Let $\{A_i : i \in I\}$ be a family of Λ_r-open subsets of Y such that $f(B) \subseteq \cup \{A_i : i \in I\}$. Then $B \subseteq \cup \{f^{-1}(A_i) : i \in I\}$. By using assumption, $\exists$ a finite subset I_0 of I such that

$B \subseteq \cup \{f^{-1}(A_i) : i \in I_0\}$. Then we have $f(B) \subseteq \cup \{A_i : i \in I_0\}$.

This shows that f(B) is Λ_r-compact relative to Y.

Theorem: 3.11 Every Λ_r-continuous image of a Λ_r-compact space is compact.

Proof: Let $f: X \to Y$ be a Λ_r-continuous function from a Λ_r-compact space X onto a topological space Y. Let $\{A_i : i \in I\}$ be an open cover of Y. Since f is Λ_r-continuous, $\{f^{-1}(A_i): i \in I\}$ is a Λ_r-open cover of X. Since X is Λ_r-compact, it has a finite subcover, say $\{f^{-1}(A_1), f^{-1}(A_2)... f^{-1}(A_n)\}$. Since f is onto, $\{A_1, A_2... A_n\}$ is an open cover of Y and hence Y is compact.

Theorem: 3.12 If $f: X \to Y$ is a Λ_r^*-closed surjective function such that for each $y \in Y$, $f^{-1}(y)\}$ is Λ_r-compact relative to X and Y is Λ_r-compact, then X is Λ_r-compact.

Proof: Let $\{U_\alpha : \alpha \in I\}$ be a Λ_r-open cover of X. Then $X = \bigcup_{\alpha\in I} U_\alpha$.
For each $y \in Y$, $\{f^{-1}(y)\}$ is Λ_r-compact relative to X.
Then $\exists$ a finite subset I_0 of I such that $\{f^{-1}(y)\} \subseteq \cup\{U_\alpha : \alpha \in I_0\}$.

Take $U_y = \cup\{U_\alpha : \alpha \in I_0\}$ and $V_y = Y - f(X-U_y)$.
Then U_y is Λ_r-open and so $X-U_y$ is Λ_r-closed. Since f is Λ_r^*-closed, $f(X-U_y)$ is Λ_r-closed in Y.
Hence $Y - f(X-U_y)$ is Λ_r-open and hence V_y is Λ_r-open in Y. Now, $f^{-1}(y) \in U_y$.
That is, $f^{-1}(y) \notin X-U_y$. That implies $f(f^{-1}(y)) \notin f(X-U_y)$. Since f is onto, $y \notin f(X-U_y)$ and hence $y \in Y - f(X-U_y) = V_y$. Also $V_y = Y - f(X-U_y)$.

That implies $f^{-1}(V_y) = f^{-1}(Y- f(X-U_y)) = f^{-1}(Y) - f^{-1}(f(X-U_y)) = X - f^{-1}(f(X-U_y))$ (since f is onto) $\subseteq X-(X-U_y) = U_y$. Thus V_y is a Λ_r-open set in Y containing y such that $f^{-1}(V_y) \subseteq U_y$. Hence $\{V_y : y\in Y\}$ is a Λ_r-open cover of Y. Since Y is Λ_r-compact, it has a finite subcover, say $\{V_{y_1}, V_{y_2}, ..., V_{y_n}\}$. That is, $Y = \cup\{V_{y_i} : i = 1,2,...,n\}$. Therefore $X = f^{-1}(Y) = f^{-1}(\cup\{V_{y_i} : i = 1,2,...,n\}) = \cup\{f^{-1}(V_{y_i}) : i = 1,2,...,n\} \subseteq \cup\{U_{y_i} : i = 1,2,...,n\} = \cup\{U_\alpha : \alpha \in I_i, i = 1,2,...,n\}$. This shows that X is Λ_r-compact.

4. Λ_r-CONNECTED SPACES:

In this section, we introduce a new class of connected space called Λ_r-connected space and prove some of its properties.

Definition: 4.1 A topological space X is said to be Λ_r-connected if there does not exist a pair A, B of non-empty disjoint Λ_r-open subsets of X such that $X = A \cup B$, otherwise X is called Λ_r-disconnected. In this case, the pair (A, B) is called a Λ_r-disconnection of X. A subset A of a space X is Λ_r-connected if it is Λ_r-connected as a subspace.

Theorem: 4.2 For a topological space X, the following are equivalent:

(i) X is Λ_r-connected
(ii) The only subsets of X which are both Λ_r-open and Λ_r-closed are the sets X and Ø
(iii) Each Λ_r-continuous function of X into a discrete space Y with atleast two points is a constant function

Proof: (1) → (2) Let U be a both Λ_r-open and Λ_r-closed subset of X. Then X-U is both Λ_r-open and Λ_r-closed. Since X is Λ_r-connected and X is the disjoint union of Λ_r-open sets U and X-U, one of these must be empty. Hence either U = Ø or U = X. (2) → (1) Suppose that X is Λ_r-disconnected. Then $X = A \cup B$ where A and B are nonempty Λ_r-open sets such that $A \cap B = Ø$. Since B = X-A is Λ_r-open, A is both Λ_r-open and Λ_r-closed. Hence by (2), A = Ø or X. That is, either A = Ø or B = Ø, which is a contradiction. Therefore X is Λ_r-connected. (2)→ (3) Let f: X → Y be a Λ_r-continuous function from a topological space X into a discrete topological space Y. For each $y \in Y$, {y} is both open and closed in Y since Y is a discrete topological space. Since f is Λ_r-continuous, $f^{-1}(y)$ is both Λ_r-open and Λ_r-closed in X. Hence X is covered by Λ_r-open and Λ_r-closed covering $\{f^{-1}(y): y\in Y\}$. By (2), $f^{-1}(y)$ = Ø or X for each $y \in Y$. If $f^{-1}(y)$ = Ø for each $y \in Y$, then f fails to be a map. Hence $\exists$ only one point $y \in Y$ such that $f^{-1}(y) = X$ which shows that f is a constant function. (3) → (2) Let U be both Λ_r-open and Λ_r-closed in X. Suppose U ≠ Ø. Let f: X → Y be a Λ_r-continuous function from a topological space X into a discrete topological space Y defined by f (U) = {y} and f(X-U) = {w} where $y,w \in Y$ and $y \neq w$. By (3), f is constant. Hence U = X.

Theorem: 4.3 Every Λ_r-connected space is connected.

Proof: Let X be a Λ_r-connected space. If possible, let X be not connected. Then X can be written as $X = A \cup B$ where A and B are disjoint nonempty open sets in X. Since every open set is Λ_r-open, $X = A \cup B$ where A and B are disjoint nonempty Λ_r-open sets in X. Therefore X is Λ_r-disconnected. This is a contradiction. Hence X is connected.

Remark: 4.4 The following example shows that the converse of the above theorem need not be true.

Example: 4.5 Let X = {a,b,c,d} and τ = {X,Ø,{a},{b,c},{a,b,c}}. Then $\Lambda_rO(X,\tau)$ = {X,Ø, {a},{a,d},{b,c},{a,b,c},{b,c,d}}. Here (X,τ) is connected but Λ_r-disconnected since X = {a,d} ∪ {b,c} where {a,d} and {b,c} are two disjoint nonempty Λ_r-open sets in X.

Definition: 4.6 Let (X,τ) be a topological space and A be a subset of X. Then we define the Λ_r-frontier of A by Λ_r-$Fr(A) = \Lambda_r$-$cl(A) \setminus \Lambda_r$-$int(A) = \Lambda_r$-$cl(A) \cap \Lambda_r$-$cl(X-A)$.

Next, we characterize Λ_r-connectedness interms of Λ_r-frontier.

Theorem: 4.7 A space X is Λ_r-connected if and only if every non-empty proper subset of X has a non-empty Λ_r-frontier.

Proof: Suppose that X is Λ_r-connected and A be a proper non-empty subset of X. If possible, let Λ_r-Fr(A) = Ø. Then Λ_r-cl(A) $\cap$ Λ_r-cl(X-A) = Ø.
This implies that Λ_r-cl(A) $\subseteq$ X-(Λ_r-cl(X-A)) = X-(X-Λ_r-int(A)) = Λ_r-int(A). This shows that Λ_r-int(A) = A = Λ_r-cl(A) and hence A is both Λ_r-open and Λ_r-closed. Then by Theorem 4.2, X is Λ_r-disconnected. This contradiction proves that A has a non-empty Λ_r-frontier.

Conversely, suppose that X is Λ_r-disconnected. Then X can be written as X = A $\cup$ B where A and B are two non-empty disjoint Λ_r-open subsets of X. Since B = X-A is Λ_r-open, A is both Λ_r-open and Λ_r-closed. Hence A = Λ_r-int(A) and A = Λ_r-cl(A). Now we have

Λ_r-Fr(A) = Λ_r-cl(A) $\cap$ Λ_r-cl(X-A) = A $\cap$ (X-Λ_r-int(A)) = A $\cap$ (X-A) = Ø. So A has empty Λ_r-frontier. By this contradiction, X is Λ_r-connected.

Definition: 4.8 Let (X, τ) be a topological space and A be a subspace of X.Then the class of Λ_r-open sets in A is defined in a natural way as: $\Lambda_r O(A,\tau_A) = \{ A \cap O : O \in \Lambda_r O(X, \tau) \}$ where $\Lambda_r O(X, \tau)$ is the class of Λ_r-open sets in X. That is, G is Λ_r-open in A if and only if G = A $\cap$ O, where O is Λ_r-open in X.

Theorem: 4.9 Let (A, B) be a Λ_r-disconnection of a space X and C be a Λ_r-connected subspace of X. Then C is contained in A or in B.

Proof: Suppose that C is neither contained in A nor in B. Since (A, B) is a Λ_r-disconnection of a space X, X = A $\cup$ B where A and B are non-empty disjoint Λ_r-open subsets of X. Then C $\cap$ A and C $\cap$ B are both non-empty Λ_r-open sets of C such that (C $\cap$ A) $\cap$ (C $\cap$ B) = Ø and (C $\cap$ A) $\cup$ (C $\cap$ B) = C. This gives that (C $\cap$ A, C $\cap$ B) is a Λ_r-disconnection of C. Hence C is Λ_r-disconnected. This contradiction proves the theorem.

Theorem: 4.10 Let $X = \bigcup_{\alpha\in I} \{X_\alpha\}$, where each X_α is Λ_r-connected and $\bigcap_{\alpha\in I} \{X_\alpha\} \neq Ø$. Then X is Λ_r-connected.

Proof: Suppose that X is Λ_r-disconnected and (A, B) is a Λ_r-disconnection of X. Since each X_α is Λ_r-connected, by Theorem 4.9, $X_\alpha \subseteq$ A or $X_\alpha \subseteq$ B. Since $\bigcap_{\alpha\in I} \{X_\alpha\} \neq Ø$, all X_α are contained in A or in B. This gives that X $\subseteq$ A or X $\subseteq$ B. If X $\subseteq$ A, then B = Ø or if X $\subseteq$ B, then A = Ø. This contradiction proves that X is Λ_r-connected.

Theorem: 4.11 A space X is Λ_r-connected if and only if for every pair of points x, y in X, there is a Λ_r-connected subset of X which contains both x and y.

Proof: Necessity: Since the Λ_r-connected space X itself contain these two points, it is obvious. Sufficiency: Suppose that for any two points x and y of X, there is a Λ_r-connected subspace $C_{x,\,y}$ of X such that x, y$\in C_{x,\,y}$. Let a$\in$ X be a fixed point and $\{C_{a,\,x} : x\in X\}$ be a class of all Λ_r-connected subspace of X which contain the points a and x. Then X = $\bigcup_{x\in X} \{C_{a,\,x}\}$ and $\bigcap_{x\in X} \{C_{a,\,x}\} \neq Ø$. Therefore by Theorem 4.10, X is Λ_r-connected.

Theorem: 4.12 If f : X → Y is a Λ_r-continuous surjective function and X is Λ_r-connected, then Y is connected.

Proof: Suppose that Y is not connected. Then Y can be written as Y = A $\cup$ B where A and B are disjoint nonempty open sets in Y. Since f is Λ_r-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are Λ_r-open in X. Since f is onto, X= $f^{-1}(Y)=f^{-1}(A \cup B)=f^{-1}(A) \cup f^{-1}(B)$. Thus, X = $f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are nonempty Λ_r-open sets in X such that $f^{-1}(A) \cap f^{-1}(B) = \phi$. Therefore X
is Λ_r-disconnected, which is a contradiction. Hence Y is connected.

Theorem: 4.13 If f : X → Y is a Λ_r-irresolute surjective function and X is Λ_r-connected, then Y is Λ_r-connected.

Proof: Suppose that Y is Λ_r-disconnected. Then Y can be written as Y = A $\cup$ B where A and B are disjoint nonempty Λ_r-open sets in Y. Since f is Λ_r-irresolute, $f^{-1}(A)$ and $f^{-1}(B)$ are Λ_r-open in X. Since f is onto, $f^{-1}(A)$ and $f^{-1}(B)$ are nonempty subsets of X and X = $f^{-1}(Y)$. Thus, X = $f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nonempty Λ_r-open sets in X. Hence X is Λ_r-disconnected, which is a contradiction. Therefore Y is Λ_r-connected.

Theorem: 4.14 If f : X → Y is a Λ_r^*-open bijective function and Y is Λ_r-connected, then X is Λ_r-connected.

Proof: Suppose that X is Λ_r-disconnected. Then X can be written as $X = A \cup B$ where A and B are disjoint nonempty Λ_r-open subsets of X. Since f is Λ_r^*-open, f (A) and f (B) are Λ_r-open in Y. Also f (A) and f (B) are nonempty subsets of Y. Since f is injective,

$f(A) \cap f(B) = f(A \cap B) = f(\phi) = \phi$. Since f is onto, f(X) = Y. Thus, $Y = f(X) = f(A \cup B) = f(A) \cup f(B)$

where f(A) and f(B) are disjoint nonempty Λ_r-open sets in Y and so Y is Λ_r-disconnected. This is a contradiction.

Hence X is Λ_r-connected.

Theorem: 4.15 If $f : X \to Y$ is a Λ_r-open bijective function and Y is Λ_r-connected, then X is connected.

Proof: Suppose that X is not connected. Then $X = A \cup B$ where A and B are disjoint nonempty open sets in X. Since f is Λ_r-open, f (A) and f (B) are Λ_r-open in Y. Since f is bijective, $f(A) \cap f(B) = \phi$ and f(X) = Y. Therefore $Y = f(A) \cup f(B)$ where f(A) and f(B) are disjoint nonempty Λ_r-open sets in Y. Hence Y is Λ_r-disconnected, which is a contradiction.

Hence X is connected.

REFERENCES:

[1] Ahmad Al-Omari and Mohd. Salmi Md. Noorani – New Characterization of compact spaces – Proceedings of the 5th Asian Mathematical Conference, Malaysia 2009.

[2] K. Balachandran, P. Sundaram and H. Maki – On generalized continuous maps in topological spaces – *Mem. Fac. Sci. Kochi. Univ. (Math)*12 (1991), 5-13.

[3] S. Eswaran and A. Pushpalatha - τ^* -generalized compact spaces and τ^*-generalized connected spaces in topological spaces – *International Journal of Engineering Science and Technology* – Vol. 2(5), 2010, 2466-2469.

[4] M. J. Jeyanthi, Adem Kilicman, S. Pious Missier and P. Thangavelu, Λ_r-sets and Separation Axioms, *Malaysian Journal of Mathematical Sciences,* 5 (1) 2011 : 45-60

[5] Jeyanthi.M. J, Pious Missier.S and Thangavelu. P, Λ_r-homeomorphisms and Λ_r^*-homeomorphisms, *Journal of Mathematical Sciences and Computer Applications*. (To appear).

[6] M. J. Jeyanthi, S. Pious Missier and P. Thangavelu, On Λ_r-open sets and functions. (Submitted)

[7] Miguel Galdas Cueva – Semi-generalized continuous maps in topological spaces – *Portugaliae Mathematica* – Vol. 52 Fasc.4 – 1995.

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