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EXESTENCE OF SEMI-PSEUDO RICCI SYMMETRIC MANIFOLD

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ABSTRACT

Conharmonic and concircular curvature tensors on Semi-Pseudo Ricci Symmetric Manifolds has been studied. Also Kaehlerian and Sasakian Semi-Pseudo Ricci Symmetric Manifolds, and anon trivial example of Semi-Pseudo Ricci Symmetric Manifold is obtained.

Keywords: *Riemannian manifold, Semi-Pseudo Ricci Symmetric manifold, Kaehlerian and Sasakian manifolds, Pseudo Ricci symmetric manifolds.*

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1. INTRODUCTION:

The notion of Semi-Pseudo Ricci Symmetric Manifolds was introduced by M. Tarafdar and the author in 1993 [1], which is defined as follows:

A non-flat n-dimensional Riemannian manifold (M_n, g) (n>3) is called Semi-Pseudo Ricci Symmetric Manifold if the Ricci tensor S satisfies:
(1.1) $(\nabla_X S)(Y,Z) = \pi(Y)S(X,Z) + \pi(Z)S(X,Y)$, where π is a non zero 1-form,

(1.2) $g(X,\rho) = \pi(X)$,

for every vector field X and ∇ denotes the operator of covariant differentiation with respect to the metric tensor g. Such a manifold is denoted by $(SPRS)_n$.

It is known [1] that on $(SPRS)_n$ the scalar curvature r = 0, and,

(1.3) $\pi(QX) = 0$,1.4) $S(X,\rho) = 0$,

where Q is the symmetric endomorphism of the tangent space at each point of (M_n, g) corresponding to the Ricci tensor.

The object of this paper is to further study Semi-Pseudo Ricci Symmetric Manifold. It is shown in section2 that $(SPRS)_n$ cannot be neither conharmonically (concircularly) flat nor conharmonically (concircularly) symmetric.

In section3&4 it is proved that $(SPRS)_n$ cannot be neither Kaehlerian nor Sasakian. Finally a non trivial example introduced in section5.

2. CONHARMONIC AND CONCIRCULAR CURVATURE TENSORS ON $(SPRS)_N$.

Conharmonic and concircular curvature tensors are given by [2],
(2.1) L(X,Y,Z) = R(X,Y,Z) - $\frac{1}{n-2}$[S(Y,Z)X – S(X,Z)Y – g(X,Z)QY + g(Y,Z)QX],

(2.2) Z(X,Y,Z) = R(X,Y,Z) - $\frac{r}{n)n-1)}$[g(Y,Z)X – g(X,Z)Y].

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Let the manifold be conharmonically flat, and then differentiating covariantly we get,

(2.3) $(\nabla_W R)(X,Y,Z) - \frac{1}{n-2}[(\nabla_W S)(Y,Z)X - (\nabla_W S)(X,Z)Y - g(X,Z)(\nabla_W Q)(Y) + g(Y,Z)(\nabla_W Q)(X)]$.

Contracting with respect to X, taking in account r = 0, we get,

(2.4) $\frac{1}{n-2}(\nabla_W S)(Y,Z) = 0$.

By virtue of (1.1) we have,

(2.5) $\pi(Y)S(X,Z) + \pi(Z)S(X,Y) = 0$.

Let Y = ρ, then by virtue of (1.4) we get $\pi(\rho)S(X,Z) = 0$. Since $\pi(\rho) \neq 0$, we have $S(X,Z) = 0$, which contradicts the hypothesis of $(SPRS)_n$. Thus we can state,

Theorem: 2.1 $(SPRS)_n$ cannot be conharmonically flat.

Further if the manifold is conharmonically symmetric then we have (2.3). Thus we can state,

Theorem: 2.2 $(SPRS)_n$ cannot be conharmonically symmetric.

Similarly we can prove,

Theorem: 2.3 $(SPRS)_n$ cannot be neither concircularly flat nor concircularly symmetric.

3. KAEHLERIAN $(SPRS)_n$.

An even dimensional manifold M_{2n} with a Riemannian metric g is a Kaehler manifold if there exist a mixed tensor F such that[2],

(3.1) $F^2 = I_n$,

(3.2) $g(\bar{X}, \bar{Y}) = g(X,Y)$, where $F(X) = \bar{X}$,

(3.3) $(\nabla_W F)Y = 0$,

Where ∇ as stated above. It is known [2] that on a Kaehler manifold,

(3.4) $S(\bar{X},Y) + S(X, \bar{Y}) = 0$.

If we define,

(3.5) $H'(X,Y) = -\frac{1}{2} C_3^1 \overline{R(X,Y)}$,
where C denotes the operator of contraction, then we have [2],

(3.6) $H'(X,Y) = S(X, \bar{Y})$,

(3.7) $dH' = 0$.

Let us assume that the manifold be Kaehlerian. Then from (3.6) and (3.7) by virtue of (3.4) we get,

(3.8) $(\nabla_X S)(Y, \bar{Z}) + (\nabla_Y S)(\bar{Z},X) + (\nabla_Z S)(X, \bar{Y}) = 0$.

By virtue of (1.10) and (3.4) we have,

(3.9) $\pi(Y)S(X,\bar{Z}) + \pi(\bar{Y})S(X,Z) = 0$.

Now let ρ = Y, we get,

(3.10) $S(X, \bar{Z})\rho - S(X,Z)\,\bar{\rho} = 0$.

But ρ and $\bar{\rho}$ are linearly independent, and therefore we have $S(X,Z) = 0$, which contradicts the hypothesis of $(SPRS)_n$. Thus we can state,

Theorem: 3.1 $(SPRS)_n$ cannot be Kaehler manifold.

4. SASAKIAN $(SPRS)_n$.

Let (M^n, g) be n-dimensional (n=2m+1, m>1) differentiable manifold with contact form η, associated vector field ξ, and (1-1) tensor field Φ. Then M_n is called Sasakian manifold [3] if ξ is a killing vector field, and,

(4.1) $(\nabla_X\Phi)Y = g(X,Y)\xi + \eta(Y)X$,

where ∇ as stated above. On Sasakian manifold we have [3],

(4.2) $\Phi\xi - 0$,

(4.3) $\eta(\xi) = 1$,

(4.4) $S(X,\xi) = (n-1)\eta(X)$,

(4.5) $(\nabla_X S)(Y,\xi) = (n-1)\, g(\Phi X,Y) + S(Y,\Phi X)$.

Assume the manifold be Sasakian, and let Z = ξ in (1.1) we get,

(4.6) $(\nabla_X S)(Y,\xi) = \pi(Y)S(X,\xi) + \pi(\xi)S(X,Y)$.

By virtue of (4.4) and (4.5) we get,

(4.7) $(n-1)\, g(\Phi X,Y) + S(Y,\Phi X) = (n-1)\,\eta(X)\,\pi(Y) + \pi(\xi)S(X,Y)$.

Now if X = ξ, we can have by virtue of (4.2) and (4.3),

(4.8) $(n-1)\,\pi(Y) + (n-1)\,\eta(Y)\pi(\xi) = 0$.

Next if Y = ξ we get $\pi(\xi) = 0$, which contradicts the hypothesis of $(SPRS)_n$. Thus we can state,

Theorem4.1) $(SPRS)_n$ cannot be Sasakian manifold.

5- $(SPRS)_4$:

Let us consider $M^4 = \{(x^1, x^2, x^3, x^4) \in R^4\}$ be an open subsets of R^4 endowed with the metric,

(5.1) $d^2 = g_{ij}dx^i dx^j = f(x^1,x^3)d(x^1)^2 + 2dx^1dx^2 + d(x^3)^2 d(x^4)^2$,
where i , j = 1,2,3,4, and f is continuously differentiable function of x^1 and x^3 such that $f_{.33} \neq 0$, and $f_{.331} \neq 0$, where (.) denote the partial differentiation with respect to coordinates. Therefore the only non vanishing Ricci tensors and their covariant derivatives[4] are,

(5.2) $S_{11} = \frac{1}{2}f_{.33}$; $S_{11,1} = \frac{1}{2}f_{.331}$, $S_{11,3} = \frac{1}{2}f_{.333}$,

where (,) denotes the covariant differentiation with respect to the metric tensor g. Hence under consideration is a Riemannian manifold which is neither Ricci symmetric non Ricci recurrent and it is of vanishing scalar curvature tensor.

If we now define the 1-form,

(5.3) $\pi_i(x) = \begin{cases} \frac{f_{.331}}{2f_{.33}}, & i = 1 \\ 0, & otherwise \end{cases}$

for any point x ϵ M. Then (1.1) reduces to the form,

(5.4) $S_{11,1} = \pi_1 S_{11} + \pi_1 S_{11}$

and all other cases vanishes identically. Therefore (5.4) holds because we have,

$\frac{1}{2}f_{.331} = S_{11,1} = 2\pi_1 S_{11} = \frac{1}{2}f_{.331}$

Thus we can state,

Theorem3.1) A Riemannian manifold (M^4,g) endowed with the metric (5.1) is a semi-pseudo Ricci symmetric manifold with vanishing scalar curvature tensor which is neither Ricci symmetric nor Ricci recurrent.

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