ON $\Lambda_{Bg}$–CLOSER SETS IN TOPOLOGICAL SPACES

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ABSTRACT

We introduce new classes of sets called $\Lambda_{Bg}$-closed sets and $\Lambda_{Bg}$-open sets in topological spaces. We also investigate several properties of such sets. It turns out that $\Lambda_{Bg}$ - closed sets and $\Lambda_{Bg}$ - open sets are weaker forms of B-closed sets and B-open sets, respectively and stronger forms of Bg -closed sets and Bg-open sets, respectively.

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1. INTRODUCTION:

In 1986, Maki [9] introduced the notion of $\Lambda$-sets in topological spaces. A $\Lambda$-set is a set $A$ which is equal to its kernel (saturated set), i.e to the intersection of all open supersets of $A$. Arenas et al. [2] introduced and investigated the notion of $\lambda$-closed sets by involving $\Lambda$-sets and closed sets. Caldas et al. [4] introduced the notion of the $\lambda$-closure of a set by utilizing the notion of $\lambda$-open sets defined in [2]. Levine [7] introduced the notions of simply extended topological spaces. Abd El-Monsof et al. [1] introduced the notions of B-open sets and associated interior and closure operators on simply extended topological spaces.

In this paper, we introduce new classes of sets called $\Lambda_{Bg}$-closed sets and $\Lambda_{Bg}$-open sets in topological spaces. We also establish several properties of such sets. It turns out that $\Lambda_{Bg}$-closed sets and $\Lambda_{Bg}$-open sets are weaker forms of B-closed sets and B-open sets, respectively and stronger forms of Bg-closed sets and Bg-open sets, respectively.

2. PRELIMINARIES:

Throughout this paper, by $(X, \tau)$ and $(Y, \sigma)$ (or $X$ and $Y$) we always mean topological spaces. Let $A$ be a subset of $X$. We denote the interior, the closure and the complement of a set $A$ by $\text{int}(A)$, $\text{cl}(A)$ and $X \setminus A$ or $A^c$, respectively.

A subset $A$ of a space $(X, \tau)$ is called $\lambda$-closed [2] if $A = L \cap D$, where $L$ is a $\Lambda$-set and $D$ is a closed set. The complement of $\lambda$-closed set is called $\lambda$-open. A subset $A$ of a space $(X, \tau)$ is called semi-open [8] if $A \subseteq \text{cl}(\text{int}(A))$. The complement of semi-open set is called semi-closed. The intersection of all semi-closed subsets of $X$ containing $A$ is called the semi-closure [5] of $A$ and is denoted by $\text{scl}(A)$.

A subset $A$ of a space $(X, \tau)$ is called preopen [10] if $A \subseteq \text{int}(\text{cl}(A))$. The complement of preopen set is called preclosed. The intersection of all preclosed subsets of $X$ containing $A$ is called the preclosure of $A$ and is denoted by $\text{pcl}(A)$. The union of all preopen subsets of $X$ contained in $A$ is called the preinterior of $A$ and is denoted by $\text{pint}(A)$. A subset $A$ of a space $(X, \tau)$ is called regular open [13] if $A = \text{int}(\text{cl}(A))$. The complement of regular open set is called regular closed.

Let $X$ be a non empty set and Levine [7] defined $\tau(B) = \{ O \cup (O' \cap B) : O, O' \in \tau \}$ and called it simple extension of $\tau$ by $B$, where $B \notin \tau$. We recall the pair $(X, \tau(B))$ a simply extended topological spaces (briefly SETS). The elements of $\tau(B)$ are called B-open [1] sets and the complements are called B-closed sets [1]. The family of B-open sets of $X$ forms a topology. In other words, we can say, $A$ is closed set in $(X, \tau(B))$ or $A$ is a B-closed set in $(X, \tau)$. The B-closure of a subset $S$ of $X$, denoted by $Bcl(S)$ [1], is the intersection of B-closed sets of $X$ containing $S$ and the B-interior of $S$, denoted by $Bint(S)$, is the union of B-open sets contained in $S$. A subset $A$ of a space $(X, \tau)$ is called $\beta\lambda$-closed [12] if $A = L \cap D$, where $L$ is a $\Lambda$-set and $D$ is a B-closed. The complement of $\beta\lambda$-closed is called $\beta\lambda$-open.

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Theorem: 3.8

and $A = \{b\}$ is $B$-closed set but it is not closed.

Here $A = \{a, c, d\}$ is $B$-closed.

Example: 3.7

None of these implications is reversible as shown in the following example.

Remark: 3.4

Let $\{A_i : i \in I\}$ be a family of subsets of a space $X$. In general $\bigcap \{A_i : i \in I\} \not\subseteq Bcl(\bigcap \{A_i : i \in I\})$ and $A_i \neq Bcl(A_i)$.

Example: 3.5

Let $X = \{a, b, c\}, \tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $B = \{c\}$. Then $(B_i) = \{\emptyset, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}.

Let $A = \{a\}$ and $B = \{b\}$. Then $\Lambda_{Bg}(A \cap B) = \Lambda_{Bg}(\emptyset) = \emptyset$. Also, we have $\Lambda_{Bg}(A) = \{a, b\}$ and $\Lambda_{Bg}(B) = \{b\}$. Thus $\Lambda_{Bg}(A) \cap \Lambda_{Bg}(B) = \emptyset$.

Remark: 3.6

We have the following implications.

\[
\text{closed } \rightarrow \text{B-closed } \rightarrow \Lambda_{Bg} \text{-closed } \rightarrow \text{Bg - closed }
\]

None of these implications is reversible as shown in the following example.

Example: 3.7

Let $X = \{a, b, c, d\}, \tau = \{\emptyset, X, \{a, d\}\}$ and $B = \{c\}$. Then $\tau(B_i) = \{\emptyset, X, \{c\}, \{a, d\}, \{a, c, d\}\}.$

(1) Here $A = \{c\}$ is a B-closed set but it is not closed.

(2) Here $A = \{a, b, c\}$ is a $\Lambda_{Bg}$-closed set but it is not B-closed.

(3) Here $A = \{a, d\}$ is a $B_{g}$-closed set but it is not $\Lambda_{Bg}$-closed.

(4) Here $A = \{a, c, d\}$ is $B_{g}$-closed set but it is not B-closed.

(5) Here $A = \{a, b, d\}$ is $B_{g}$-closed set but it is not $\lambda$-closed.

(6) $\lambda$-closed sets and B-closed sets are independent of each other. Here $A = \{a, d\}$ is $\lambda$-closed set but it is not B-closed.

Theorem: 3.8

The union of two $\Lambda_{Bg}$-closed sets is $\Lambda_{Bg}$-closed.

Proof: Let $A \cup B \subseteq U$, then $A \subseteq U$ and $B \subseteq U$ where $U$ is $B_{g}$-open. As $A$ and $B$ are $\Lambda_{Bg}$-closed $Bcl(A \subseteq U$ and $Bcl(B) \subseteq U$. Hence $Bcl(A \cup B) = Bcl(A) \cup Bcl(B) \subseteq U$. 


\[\alpha, \beta \text{ are } \Lambda_{Bg} \text{-open.} \]

Let $\tau \leq \alpha \leq \beta$, then $\tau \leq \alpha \leq \beta$ is $\Lambda_{Bg}$-open.

Let $\tau \leq \alpha \leq \beta$, then $\tau \leq \alpha \leq \beta$ is $\Lambda_{Bg}$-open.

Proof: Let $A \cup B \subseteq U$, then $A \subseteq U$ and $B \subseteq U$ where $U$ is $B_{g}$-open. As $A$ and $B$ are $\Lambda_{Bg}$-closed $Bcl(A \subseteq U$ and $Bcl(B) \subseteq U$. Hence $Bcl(A \cup B) = Bcl(A) \cup Bcl(B) \subseteq U$. 

\[\alpha, \beta \text{ are } \Lambda_{Bg} \text{-open.} \]
Proof: Suppose that $Bcl(A) \setminus A$ contains no nonempty $B$-closed subset of $X$. Thus $Bcl(A) \subseteq F$ or $F \subseteq [Bcl(A)]^c$. Then $F \subseteq [Bcl(A)]^c \cap (Bcl(A) \setminus A) \subseteq [Bcl(A)]^c \cap Bcl(A) = \emptyset$. This shows that $F = \emptyset$.

The converse of the above theorem is not true in general as it is shown in the following example.

Example: Let $A = \{d\}$ from Example 3.7. Then $Bcl(A) \setminus A = \{a, b\}$ does not contain nonempty $B$-closed set.

Definition: A topological space $(X, \tau)$ is called a $BT_1$-space if to each pair of distinct points $x, y$ of $(X, \tau)$ there exist a $B$-open set $U$ containing $x$ but not $y$ and a $B$-open set $V$ containing $y$ but not $x$.

Theorem: A topological space $(X, \tau)$ is a $BT_1$-space if and only if every subset of $X$ consisting of exactly one point is $B$-closed.

Proof: Let $(X, \tau)$ be a $BT_1$-space and $x$ be an arbitrary point of $X$. Then, we must show that $\{x\}$ is $B$-closed or equivalently that $((\{x\})^c)$ is $B$-open. If $((\{x\})^c) = \emptyset$, then it is clearly $B$-open. So, let $((\{x\})^c) \neq \emptyset$ and let $y \in ((\{x\})^c)$. Then $y \neq x$. But, $(X, \tau)$ being a $BT_1$-space there exist a $B$-open set $G$ containing $y$ but not $x$. Consequently, $y \in G \subseteq ((\{x\})^c)$. This shows that $((\{x\})^c)$ is neighbourhood of each of its points and therefore, $B$-open. Hence, $\{x\}$ is $B$-closed.

Conversely, let $(X, \tau)$ be a topological space such that every subset of $X$ consisting of exactly one point is $B$-closed. Let $x$ and $y$ be any two distinct points of $X$. Then, by hypothesis, $\{x\}$ as well as $\{y\}$ is $B$-closed. Consequently, $G = ((\{x\})^c) \cap H = ((\{y\})^c)$ are $B$-open sets such that $y \in G$ but $x \notin G$ and $x \in H$ but $y \notin H$. Hence $(X, \tau)$ is a $BT_1$-space.

Corollary: In a $BT_1$-space, every $\Lambda_{B\lambda}$-closed set is $B$-closed.

Proof: Let $A$ be a $\Lambda_{B\lambda}$-closed set in a $BT_1$-space $(X, \tau)$. Let $x \in Bcl(A) \setminus A$. Since $(X, \tau)$ is $BT_1$, $\{x\}$ is a $B$-closed set in $(X, \tau)$. By Theorem 3.11, there exists no nonempty $B$-closed set in $Bcl(A) \setminus A$ and so $Bcl(A) \setminus A = \emptyset$. Therefore $Bcl(A) = A$, i.e., $A$ is $B$-closed.

Theorem: A set $A$ is $\Lambda_{B\lambda}$-closed if and only if $Bcl(A) \setminus A$ contains no nonempty $B\lambda$-closed sets.

Proof: Necessity. Suppose that $A$ is $\Lambda_{B\lambda}$-closed. Let $S$ be a $B\lambda$-closed subset of $Bcl(A) \setminus A$. Then $A \subseteq S^c$. Since $A$ is $\Lambda_{B\lambda}$-closed, we have $Bcl(A) \subseteq S^c$. Consequently $S \subseteq [Bcl(A)]^c$. Hence $S \subseteq Bcl(A) \cap [Bcl(A)]^c = \emptyset$. Therefore $S$ is empty.

Sufficiency. Suppose that $Bcl(A) \setminus A$ contains no nonempty $B\lambda$-closed sets. Let $A \subseteq G$ and $G$ be $B\lambda$-open. If $Bcl(A) \subseteq G$, then $Bcl(A) \cap G^c$ is a nonempty $B\lambda$-closed subset of $Bcl(A) \setminus A$. Therefore, $A$ is $\Lambda_{B\lambda}$-closed.

Theorem: If $A$ is a $\Lambda_{B\lambda}$-closed set of $(X, \tau)$ and $A \subseteq B \subseteq Bcl(A)$, then $B$ is a $\Lambda_{B\lambda}$-closed set of $(X, \tau)$.

Proof: Since $B \subseteq Bcl(A)$, we have $Bcl(B) \subseteq Bcl(A)$. Hence $(Bcl(B) \setminus B) \subseteq (Bcl(A) \setminus A)$. But by Theorem 3.16 $Bcl(A) \setminus A$ contains no nonempty $B\lambda$-closed subsets of $X$ and hence $Bcl(B) \setminus B$ does not contain $B\lambda$-closed sets. Again by Theorem 3.16, $B$ is $\Lambda_{B\lambda}$-closed.

Theorem: If $A$ is a $B\lambda$-open and $\Lambda_{B\lambda}$-closed set in $(X, \tau)$, then $A$ is $B$-closed in $(X, \tau)$.

Proof: Since $A$ is $B\lambda$-open and $\Lambda_{B\lambda}$-closed, $Bcl(A) \subseteq A$ and hence $A$ is $B$-closed in $(X, \tau)$.

Theorem: For each $x \in X$, either $\{x\}$ is $B\lambda$-closed or $\{x\}^c$ is $\Lambda_{B\lambda}$-closed in $(X, \tau)$.

Proof: Suppose $\{x\}$ is not $B\lambda$-closed in $(X, \tau)$. Then $\{x\}^c$ is not $B\lambda$-open and the only $B\lambda$-open set containing $\{x\}^c$ is the space $X$ itself. Therefore $Bcl(\{x\}^c) \subseteq X$ and so $\{x\}^c$ is $\Lambda_{B\lambda}$-closed in $(X, \tau)$.
Theorem: 3.20 Let $A$ be a $\Lambda_{Bg}$-closed set in $(X, \tau)$. Then

1. If $A$ is regular open, then $\text{pint}(A)$ and $\text{scl}(A)$ are also $\Lambda_{Bg}$-closed.
2. If $A$ is regular closed, then $\text{pcl}(A)$ is also $\Lambda_{Bg}$-closed.

Proof:

1. Since $A$ is regular open in $(X, \tau)$, we have $\text{scl}(A)=A \cup \text{int}(\text{cl}(A))=A$ and $\text{pint}(A)=A \cap \text{int}(\text{cl}(A))=A$. Thus $\text{scl}(A)$ and $\text{pint}(A)$ are $\Lambda_{Bg}$-closed in $(X, \tau)$.
2. Let $A$ be regular closed in $(X, \tau)$. Then $\text{pcl}(A)=A \cap \text{cl}(\text{int}(A))=A$. Thus $\text{pcl}(A)$ is $\Lambda_{Bg}$-closed in $(X, \tau)$.

Definition: 3.21 A space $X$ is said to be a $B$-normal space if for every pair of disjoint $B$-closed subsets $A$ and $B$ of $X$ there exist $B$-open sets $U, V$ such that $A \subseteq U$, $B \subseteq V$ and $U \cap V = \emptyset$.

Remark: 3.22 If $(X, \tau)$ is a $B$-normal space and suppose that $Y$ is a $Bg$-closed subset of $X$. Then $(Y, Y \cap \tau)$ is $B$-normal.

Proof: Let $E$ and $F$ be $B$-closed in $X$ and suppose that $(Y \cap E) \cap (Y \cap F) = \emptyset$. Then $Y \subseteq (E \cap F )^c \in \tau$ and hence $\text{Bcl}(Y) \subseteq (E \cap F )^c$. Thus $(\text{Bcl}(Y) \cap E) \cap (\text{Bcl}(Y) \cap F) = \emptyset$. Since $(X, \tau)$ is $B$-normal, there exists disjoint $B$-open sets $U$ and $V$ such that $\text{Bcl}(Y) \cap E \subseteq U$ and $\text{Bcl}(Y) \cap F \subseteq V$. It follows then that $Y \cap E \subseteq U \cap Y$ and $Y \cap F \subseteq V \cap Y$.

Remark: 3.23 By Remark 3.6, every $\Lambda_{Bg}$-closed set of a $B$-normal space is $B$-normal.

Definition: 3.24 A subset $S$ of $X$ is said to be locally-B-closed if $S = U \cap F$, where $U$ is $B$-open and $F$ is $B$-closed in $(X, \tau)$.

Theorem: 3.25. For a subset $S$ of $(X, \tau)$, the following are equivalent:

1. $S$ is locally-B-closed.
2. $S = P \cap \text{Bcl}(S)$ for some $B$-open set $P$.
3. $\text{Bcl}(S) - S$ is $B$-closed.
4. $S \cup (X - \text{Bcl}(S))$ is $B$-open.
5. $S \subseteq \text{Bint}(S) \cup (X - \text{Bcl}(S))$.

Proof:

1. $(\rightarrow)$ If $S = P \cap \text{Bcl}(S)$ for some $B$-open set $P$, then $S \subseteq \text{Bcl}(S) \subseteq Q$, where $P \subseteq \text{Bcl}(S)$. So $S = P \cap Q$ implies $\text{Bcl}(S) \cup Q$. Since $S \subseteq P$ implies $S \subseteq \text{Bcl}(S)$, hence $S = P \cap \text{Bcl}(S)$.
2. $(\rightarrow)$ Since $S \cup (X - \text{Bcl}(S)) = X$, $\text{Bcl}(S) - S$ is $B$-closed.
3. $(\rightarrow)$ Since $S \cup (X - \text{Bcl}(S)) = X$, $\text{Bcl}(S) = S$. Hence $S \subseteq \text{Bint}(S) \cup (X - \text{Bcl}(S))$.
4. $(\rightarrow)$ $S \subseteq \text{Bint}(S) \cup (X - \text{Bcl}(S))$ implies $S = \text{Bint}(S) \cup (X - \text{Bcl}(S)) \cap \text{Bcl}(S)$.

Theorem: 3.26 Let $A$ be locally-B-closed subset of $(X, \tau)$. For the set $A$, the following properties are equivalent:

1. $A$ is $B$-closed;
2. $A$ is $\Lambda_{Bg}$-closed;
3. $A$ is $Bg$-closed.

Proof: By Remark 3.6, it suffices to prove that (3) implies (1). By Theorem 3.25 $A \cup (\text{Bcl}(A))^c$ is $B$-open in $(X, \tau)$ since $A$ is locally-B-closed. Now $A \cup (\text{Bcl}(A))^c$ is an $B$-open set of $(X, \tau)$ such that $A \subseteq A \cup (\text{Bcl}(A))^c$. Since $A$ is $Bg$-closed, then $\text{Bcl}(A) \subseteq A \cup (\text{Bcl}(A))^c$. But $\text{Bcl}(A) \cap (\text{Bcl}(A))^c = \emptyset$. Thus we have $\text{Bcl}(A) \subseteq A$ and hence $A$ is $B$-closed.

Definition: 3.27 A subset $A$ in $(X, \tau)$ is said to be $\Lambda_{Bg}$-open in $(X, \tau)$ if and only if $A^c$ is $\Lambda_{Bg}$-closed in $(X, \tau)$.

Every $B$-open set in $(X, \tau)$ is $\Lambda_{Bg}$-open in $(X, \tau)$ but not conversely. It can be verified from the following example.

Example: 3.28 Let $A = \{a\}$ from Example 3.7. Then $A$ is $\Lambda_{Bg}$-open set but it is not $B$-open in $(X, \tau)$.

Theorem: 3.29 The intersection of two $\Lambda_{Bg}$-open sets is $\Lambda_{Bg}$-open.

Proof: This is obvious by Theorem 3.8.

Theorem: 3.30 A set $A$ is $\Lambda_{Bg}$-open in $(X, \tau)$ if and only if $F \subseteq \text{Bint}(A)$ whenever $F$ is $B\lambda$-closed in $(X, \tau)$ and $F \not\subseteq A$.
Proof: Suppose that $F \subseteq Bcl(A)$ whenever $F$ is $\beta\lambda$-closed and $F \subseteq A$. Let $A' \subseteq G$, where $G$ is $\beta\lambda$-open. Hence $G^c \subseteq A$. By assumption $G^c \subseteq Bcl(A)$ which implies that $(Bcl(A))^c \subseteq G$, so $Bcl(A^c) \subseteq G$. Hence $A'$ is $\Lambda_{BG}$-closed, i.e., $A$ is $\Lambda_{BG}$-open.

Conversely, let $A$ be $\Lambda_{BG}$-open. Then $A^c$ is $\Lambda_{BG}$-closed. Also let $F$ be a $\beta\lambda$-closed set contained in $A$. Then $F^c$ is $\beta\lambda$-open. Therefore whenever $A' \subseteq F^c$, $Bcl(A') \subseteq F^c$. This implies that $F \subseteq (Bcl(A))^c = Bcl(A)$. Thus $F \subseteq Bcl(A)$.

Theorem: 3.31 A set $A$ is $\Lambda_{BG}$-open in $(X, \tau)$ if and only if $G = X$ whenever $G$ is $\beta\lambda$-open and $Bcl(A) \cup A \subseteq G$.

Proof: Let $A$ be $\Lambda_{BG}$-open, $G$ be $\beta\lambda$-open and $Bcl(A) \cup A \subseteq G$. This gives $G^c \subseteq (Bcl(A))^c \cap (A')^c = (Bcl(A))^c \cap A^c = Bcl(A^c) \setminus A^c$. Since $A' \subseteq \Lambda_{BG}$-closed and $G^c$ is $\beta\lambda$-closed, by Theorem 3.16 it follows that $G^c = \emptyset$. Therefore $X = G$. Conversely, suppose that $F$ is $\beta\lambda$-closed and $F \subseteq A$. Then $Bcl(A) \cup A' \subseteq Bcl(A) \cup F^c$. It follows that $Bcl(A) \cup F^c = X$ and hence $F \subseteq Bcl(A)$. Therefore $A$ is $\Lambda_{BG}$-open.

Theorem: 3.32 If $Bcl(A) \subseteq B \subseteq A$ and $A$ is $\Lambda_{BG}$-open in $(X, \tau)$, then $B$ is $\Lambda_{BG}$-open in $(X, \tau)$.

Proof: Suppose $Bcl(A) \subseteq B \subseteq A$ and $A$ is $\Lambda_{BG}$-open in $(X, \tau)$. Then $A' \subseteq B \subseteq Bcl(A)$ and $A^c \subseteq Bcl(A^c)$. By Theorem 3.17, $B$ is $\Lambda_{BG}$-open in $(X, \tau)$.

Theorem: 3.33. A set $A$ is $\Lambda_{BG}$-closed in $(X, \tau)$ if and only if $Bcl(A) \setminus A$ is $\Lambda_{BG}$-open in $(X, \tau)$.

Proof: Necessity. Suppose that $A$ is $\Lambda_{BG}$-closed in $(X, \tau)$. Let $F \subseteq Bcl(A) \setminus A$, where $F$ is $\beta\lambda$-closed. By Theorem 3.16, $F \neq \emptyset$. Therefore $F \subseteq Bcl(Bcl(A) \setminus (A))$ and by Theorem 3.30, $Bcl(A) \setminus A$ is $\Lambda_{BG}$-open in $(X, \tau)$.

Sufficiency. Let $A \subseteq G$ where $G$ is $\beta\lambda$-open. Then $Bcl(A) \cap G^c \subseteq Bcl(A) \cap A^c = Bcl(A) \setminus A$. Since $Bcl(A) \cap G^c$ is $\beta\lambda$-closed and $Bcl(A) \setminus A$ is $\Lambda_{BG}$-open, by Theorem 3.30, we have $Bcl(A) \cap G^c \subseteq Bcl(Bcl(A) \setminus A) = \emptyset$. Hence $A$ is $\Lambda_{BG}$-closed in $(X, \tau)$.

Theorem: 3.34 For a subset $A \subseteq X$, the following properties are equivalent.

(1) $A$ is $\Lambda_{BG}$-closed.
(2) $Bcl(A) \setminus A$ contains no nonempty $\beta\lambda$-closed set.
(3) $Bcl(A) \setminus A$ is $\Lambda_{BG}$-open.

Proof: This follows from Theorems 3.16 and 3.33.

Theorem: 3.35 A subset $A$ in $(X, \tau)$ is $\Lambda_{BG}$-closed if and only if $\text{cl}_{\beta\lambda}({\{x\}}) \cap A \neq \emptyset$ for every $x \in Bcl(A)$.

Proof: Necessity. Suppose that $\text{cl}_{\beta\lambda}({\{x\}}) \cap A = \emptyset$ for some $x \in Bcl(A)$. Then $X \setminus \text{cl}_{\beta\lambda}({\{x\}})$ is a $\beta\lambda$-open set containing $A$. Furthermore, $x \in Bcl(A) \setminus (X \setminus \text{cl}_{\beta\lambda}({\{x\}}))$ and hence $Bcl(A) \cap X \setminus \text{cl}_{\beta\lambda}({\{x\}})$. This shows that $A$ is not $\Lambda_{BG}$-closed.

Sufficiency. Suppose that $A$ is not $\Lambda_{BG}$-closed. There exists a $\beta\lambda$-open set $U$ containing $A$ such that $Bcl(A) \setminus U = \emptyset$. There exists $x \in Bcl(A)$ such that $x \notin U$; hence $\text{cl}_{\beta\lambda}({\{x\}}) \cap U = \emptyset$. Therefore, $\text{cl}_{\beta\lambda}({\{x\}}) \cap A = \emptyset$ for some $x \in Bcl(A)$.

4. FUNCTIONS:

Definition: 4.1 A function $f : (X, \tau) \to (Y, \sigma)$ is said to be

(1) $\beta\lambda$-irresolute if $f^{-1}(V)$ is $\beta\lambda$-open in $X$ for every $\beta\lambda$-open set $V$ of $Y$,
(2) $\beta\lambda$-closed if $f(F)$ is $\beta\lambda$-closed in $Y$ for every $\beta\lambda$-closed set $F$ of $X$,
(3) $\beta$-continuous if $f^{-1}(V)$ is $\beta$-closed in $X$ for every $\beta$-closed set $V$ of $Y$.

Definition: 4.2 A map $f : (X, \tau) \to (Y, \sigma)$ is said to be $\beta$-closed if the image of every $\beta$-closed set in $(X, \tau)$ is $\beta$-closed set in $(Y, \sigma)$.

Theorem: 4.3 Let $f : (X, \tau) \to (Y, \sigma)$ be $\beta\lambda$-irresolute $\beta$-closed function. If $A$ is $\Lambda_{BG}$-closed in $X$, then $f(A)$ is $\Lambda_{BG}$-closed in $Y$.

Proof: Let $A$ be a $\Lambda_{BG}$-closed set of $X$ and $V$ a $\beta\lambda$-open set of $Y$ containing $f(A)$. Since $f$ is $\beta\lambda$-irresolute, $f^{-1}(V)$ is $\beta\lambda$-open in $X$ and $A \subseteq f^{-1}(V)$. Since $A$ is $\Lambda_{BG}$-closed, $Bcl(A) \subseteq f^{-1}(V)$ and $f(A) \subseteq f(Bcl(A)) \subseteq V$. Since $f$ is $\beta$-closed, we obtain $Bcl(f(A)) \subseteq V$. This shows that $f(A)$ is $\Lambda_{BG}$-closed in $Y$.
Lemma: 4.4 A function \( f : (X, \tau) \to (Y, \sigma) \) is \( \lambda_\theta \)-closed if and only if for each subset \( B \) of \( Y \) and each \( U \in B\sigma(X, \tau) \) containing \( f^{-1}(B) \), there exists \( V \in B\sigma(Y, \sigma) \) such that \( B \subseteq V \) and \( f^{-1}(V) \subseteq U \).

Proof: Necessity. Suppose that \( f \) is \( \lambda_\theta \)-closed. Let \( B \subseteq Y \) and \( U \in B\sigma(X, \tau) \) containing \( f^{-1}(B) \). Put \( V = Y - f(X - U) \). Then we obtain \( \forall \in B\sigma(Y, \sigma) \), \( B \subseteq V \) and \( f^{-1}(V) \subseteq U \).

Sufficiency. Let \( F \) be any \( \lambda_\theta \)-closed set of \( (X, \tau) \). Set \( f(F) = B \), then \( F \subseteq f^{-1}(B) \) and \( f^{-1}(Y - B) \subseteq X - F \in B\sigma(X, \tau) \). By hypothesis, there exists \( V \in B\sigma(Y, \sigma) \) such that \( Y - B \subseteq V \) and \( f^{-1}(V) \subseteq X - F \). Therefore we obtain \( Y - V \subseteq B = f(F) \subseteq Y - V \). Hence \( f(F) = Y - V \) and \( f(F) \) is \( \lambda_\theta \)-closed in \( (Y, \sigma) \). Therefore, \( f \) is \( \lambda_\theta \)-closed.

Theorem: 4.5 Let \( f : (X, \tau) \to (Y, \sigma) \) be a \( \lambda \)-continuous \( \lambda_\theta \)-closed function. If \( B \) is a \( \lambda_{BG} \)-closed set of \( (Y, \sigma) \), then \( f^{-1}(B) \) is \( \lambda_{BG} \)-closed in \( (X, \tau) \).

Proof: Let \( B \) be a \( \lambda_{BG} \)-closed in \( (Y, \sigma) \) and \( U \) a \( \lambda \)-open set of \( (X, \tau) \) containing \( f^{-1}(B) \). Since \( f \) is \( \lambda \)-closed, by Lemma 4.4 there exists a \( \lambda \)-open set \( V \) of \( (Y, \sigma) \) such that \( B \subseteq V \) and \( f^{-1}(V) \subseteq U \). Since \( B \) is \( \lambda_{BG} \)-closed in \( (Y, \sigma) \), \( \text{Cl}(B) \subseteq V \) and hence \( f^{-1}(B) \subseteq f^{-1}(\text{Cl}(B)) \subseteq f^{-1}(V) \subseteq U \). Since \( f \) is \( \lambda \)-continuous, \( f^{-1}(\text{Cl}(B)) \) is \( \lambda \)-closed and hence \( \text{Cl}(f^{-1}(B)) \subseteq U \). This shows that \( f^{-1}(B) \) is \( \lambda_{BG} \)-closed in \( (X, \tau) \).

REFERENCES:


