CHARACTERISTICS OF SOME GENERALIZED OPEN SETS

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ABSTRACT

We discuss the properties of δ– preopen sets and the relations of δ– preclosure and δ– preinterior operators with the closure and interior operators of some well known generalized topologies. Also, we characterize regular open sets, δ– open sets, semiopen sets, preopen sets, b– open sets and β– open sets.

Key words and Phrases: δ– preopen, δ– open, semiopen, preopen, b– open and β– open sets, δ– preclosure and δ– preinterior.

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1. INTRODUCTION AND PRELIMINARIES:

In 1993, Raychaudhuri and Mukherjee [11] introduced and studied δ– preopen sets in topological spaces. In 1997, Á.Császár [4] introduced and studied generalized open subsets of a set X defined in terms of monotonic functions from ϕ(X) to ϕ(X). δ– preopen set is a particular kind of generalized open set introduced by Á.Császár [4] but different from the well known generalized open sets, namely semiopen set, preopen set, b– open set and β– open set. In this paper, we further study these sets and discuss the relation between the interior and closure operators of these generalized open sets.

Let X be any nonempty set. Denote by Γ, the collection of all mappings γ: ϕ(X)→ϕ(X) such that A ⊆ B implies γ(A) ⊆ γ(B). As defined in [4], we mention here the following sub collections of Γ.

\[ \Gamma_0 = \{ \gamma \in \Gamma \mid \gamma(\emptyset) = \emptyset \}, \]
\[ \Gamma_1 = \{ \gamma \in \Gamma \mid \gamma(X) = X \}, \]
\[ \Gamma_2 = \{ \gamma \in \Gamma \mid \gamma(A) = \gamma(A) \text{ for every subset } A \text{ of } X \}, \]
\[ \Gamma_3 = \{ \gamma \in \Gamma \mid \gamma(A) \subseteq A \text{ for every subset } A \text{ of } X \} \]
\[ \Gamma_4 = \{ \gamma \in \Gamma \mid \gamma(A) \subseteq A \text{ for every subset } A \text{ of } X \} \]

Let γ ∈ Γ. A subset A of X is said to be γ – open [4] if A ⊆ γ(A). B is said to be γ – closed [4] if its complement is γ – open. The smallest γ – closed set containing A is called the γ – closure of A [4] and is denoted by clγ(A). The largest γ – open set contained in A is called the γ – interior of A [4] and is denoted by intγ(A). If γ1, γ2 ∈ Γ, then we will denote γ1° γ2 by γ1γ2. For γ ∈ Γ, define γ∗: ϕ(X)→ϕ(X) by γ∗(A) = X – γ(A – X) [4] for every subset A of X. Also, in Proposition 1.7 of [4], it is established that (γ∗)∗ = γ, (iγ)∗ = ciγ and (cγγ)∗ = iγ. We say that ι γ is a dual of k γ if i γ = k γ and clearly, if i is a dual of ι, then k is a dual of i. If I is a collection of some of the symbols 0, 2, –, + and 1, then ΓI = { γ ∈ Γ \ γ ∈ I, for every i ∈ I}. For example, γ ∈ Γ01 means that γ ∈ Γ0, γ ∈ Γ1 and γ ∈ Γ2. A subfamily I of ϕ(X) is called a generalized topology [6] if Φ ∈ I and I is closed under arbitrary union. By a space (X, τ), we will always mean the topological space (X, τ). If (X, τ) is a space, then \( \Gamma_1(\tau) = \{ \gamma \in \Gamma \mid \gamma(G \cap \tau) \subseteq \gamma(G \cap \tau) \text{ for every } G \in G \text{ and } A \subseteq X \} \). A subset A of a space (X, τ) is said to be regular open if A = intcl(A)). where int and cl are the interior and closure operators in (X, τ). The family of all regular open sets is a base for a topology τo, coarser than τ, which is called the semiregularization of the topology τ. int and cl are the interior and closure operators in (X, τ). A subset A of X is said to be a – open [9] (resp. semiopen [7], preopen [8], b – open [2], β – open[3]) if A ⊆ intcl(A)). (resp. A ⊆ cl(int(A)), A ⊆ intcl(A)), A ⊆ intcl(int(A)), A ⊆ cl(intcl(A))). A subset A of X is said to be a – closed (resp. semiclosed, preclosed, b – closed, β – closed) if X – A is a – open (resp. semiopen, preopen, b – open, β – open). The family of all a – open sets (resp. semiopen, preopen, b – open, β – open) sets of a space will be denoted by (A) (resp. (S), (P), (X), (B), (β)). These families are generalized topologies whose closure and interior operators are denoted by cτo, cτ, cτ, cτ, cτ, cτ, cτ and cτ, respectively. We use the following lemmas without mentioning it explicitly.

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**Lemma: 1.1** Let \((X, \tau)\) be a space, \(A \subset X\) and \(\mathcal{d} = \{\text{int cl int, cl int, int cl }\}.\) If \(x \in \mathcal{d},\) then the following hold [1, 2].

(a) \(i_d(A) = A \cap \mathcal{d}(A).\)
(b) \(c_d(A) = A \cup \mathcal{d}(A).\)
(c) \(i_d(A) = \mathcal{d}_i(A) \cup \mathcal{d}_c(A).\)
(d) \(c_d(A) = \mathcal{d}_c(A) \cap \mathcal{d}_d(A).\)

**Lemma: 1.2** Let \((X, \tau)\) be a space and \(A \subset X.\) Then the following hold.

(a) If \(A\) is open, then \(\text{cl}(A) = \delta \text{cl}(A)\) [12].
(b) If \(A\) is closed, then \(\text{int}(A) = \delta \text{int}(A)\) [12].
(c) \(A\) is regular open if and only if \(A = \delta \text{int} \circ \text{cl}(A).\)

2. \(\delta \) – Preopen Sets:

Let \(\lambda = \text{int} \circ \delta \text{cl},\) the composition of the operators \(\text{int}\) and \(\delta \text{cl}\). Since \(\text{int} \in \Gamma_{012}^*\) and \(\delta \text{cl} \in \Gamma_{012}^*\), by Theorem 1.11 of [4], \(\lambda \in \Gamma_{01}\) and by Corollary 1.14 of [4], \(\lambda \in \Gamma_2.\) If \(\mu = \{A \subset \lambda(A)\},\) then \(\mu\) is the family of all \(\lambda\) – open sets which is nothing but the family of all \(\delta\) – preopen sets [11] and so \(\lambda\) – closed sets are nothing but \(\delta\) – preclosed sets. Clearly, \(\emptyset \in \mu,\) \(X \in \mu\) and by Proposition 2 of [11], arbitrary union of elements of \(\mu\) is in \(\mu.\) Therefore, \(\mu\) is a generalized topology with \(X \in \mu.\) Moreover, one can easily prove that RO(X) \(\subset \tau_3 \subset \tau \subset \tau^* \subset \pi(X) \subset \mu,\) where RO(X) (resp. \(\tau^*\)) is the family of all regular open (resp. \(\tau^*\)-open) sets in \((X, \tau).\) The following Theorem 2.1 gives some properties of \(\delta\) – preopen sets.

**Theorem: 2.1** Let \((X, \tau)\) be a space and \(\lambda = \text{int} \circ \delta \text{cl}.\) Then the following hold.

(a) \(\lambda \in \Gamma_3(t_3),\) or equivalently, \(G \cap \text{int}(\delta \text{cl}(A)) \subset \text{int}(\delta \text{cl}(G \cap A))\) for every \(\delta\) – open set \(G\) and every subset \(A\) of \(X.\)
(b) \(A \in \mu\) if and only if \(A \cap V \in \mu\) for every \(\delta\) – open (resp. regular open) set \(V\) [11, Theorem 3].
(c) \(\lambda^* \in \Gamma_{012}^*\) and \(\lambda^* = \delta \text{cl} \circ \text{int}\).

**Proof:** (a) In any space \((X, \tau),\) \(\text{cl}\) and \(\text{int} \in \Gamma_3(t_3).\) Therefore, \(\text{int} \circ \delta \text{cl} \in \Gamma_3(t_3).\) Since \(\text{int} \in \Gamma_3(t_3),\) \(\text{int} \in \Gamma_3(t_3)\) and so by Proposition 2.1 of [4], \(\lambda \in \Gamma_3(t_3).\)

(b) If \(A \in \mu,\) then \(A \subset \lambda(A)\) and so \(V \cap A \subset V \cap \lambda(A) \subset \lambda(V \cap A),\) by (a), for every \(\delta\) – open set \(V.\) Therefore, \(V \cap A \in \mu.\) Converse follows from the fact that \(X\) is both regular open and \(\delta\) – open.

(c) Since \(\lambda \in \Gamma_{012}^*\), by Proposition 1.7 of [4], \(\lambda^* \in \Gamma_{012}^*\). Since \(\text{int} \circ \text{cl} (\text{resp. } \delta \text{cl} \circ \text{int})\) are dual to each other, \(\lambda^* = (\text{int} \circ \delta \text{cl})^* = \text{cl} \circ \text{int}.\)

**Theorem: 2.2** Let \((X, \tau)\) be a space. Then a subset \(A\) of \(X\) is a \(\delta\) – preclosed set if and only if \(\lambda^*(A) = \delta \text{cl}(\delta \text{int}(A)) \subset A.\)

**Proof:** \(A\) is \(\delta\) – preclosed if and only if \(A\) is \(\lambda\) – closed if and only if \(X - A \subset \lambda(X - A)\) if and only if \(\lambda^*(A) = \delta \text{cl}(\delta \text{int}(A)) \subset A.\)

**Theorem: 2.3** Let \((X, \tau)\) be a space. Then \(\tau_3 = \{V \mid A \cap V \in \mu \text{ for every } A \in \mu\}\) is a topology and \(\tau_3 \subset \tau \subset \mu.\)

**Proof:** Since \(\lambda \in \Gamma_3,\) \(\tau_3\) is a topology by Theorem 2.16 of [4]. \(\tau_3 \subset \tau_3\) follows from Theorem 2.1(b). That \(\tau_3 \subset \mu\) is clear, since \(X \in \mu.\)

For the generalized topology \(\mu = \{A \subset X \mid A \subset \lambda(A)\}\) and a subset \(A\) of \(X,\) define \(i_d(A) = U \{U \in \mu \mid U \subset A\}\) and \(c_d(A) = \cap \{X - U \mid U \in \mu, A \subset X - U\}.\) Then \(i_d = \delta \text{Preint} [10]\) and \(c_d = \delta \text{Precl} [10].\) Also, note that Theorem 2.1(b) can be deduced from Theorem 2.4(c) as well as from Corollary 2.5(c).

**Theorem: 2.4** Let \((X, \tau)\) be a space. Then the following hold.

(a) \(c_d \in \Gamma_{012}^*\) and \(i_d \in \Gamma_{012}^*\)
(b) \(c_d \in \Gamma_3(t_3)\) [11, Lemma 1(a)]
(c) \(i_d \in \Gamma_3(t_3)\)
(d) \(i_d(A) = A \cap \text{int}(\delta \text{cl}(A))\) for every subset \(A\) of \(X.\)
(e) \(c_d(A) = A \cup \text{cl}(\delta \text{int}(A))\) for every subset \(A\) of \(X.\)
(f) If \(A \in \tau_3,\) then \(c_d(A) = \text{cl}(A)\) [11, Theorem 4].
(g) If \(A\) is \(\tau_3\) – closed, then \(i_d(A) = \text{int}(A).\)
Proof: (a) The proof follows from the definition.

(b) By Theorem 2.1(a), \( \lambda \in \Gamma (\tau) \) and so by Corollary 2.6 of \([4]\), \( c \lambda \in \Gamma (\tau) \).

(c) By Theorem 2.1(a), \( \lambda \in \Gamma (\tau) \) and so by Proposition 2.4 of \([4]\), \( i \lambda \in \Gamma (\tau) \).

(d) Since \( \text{int} \) is a decreasing idempotent monotonc operator, \( \delta \text{cl} \) is an increasing monotonc operator and \( \mu = \text{int} \delta \text{cl} \), by Theorem 1.3 of \([5]\), \( i \lambda (A) = \mathcal{A} \cap \text{int}(\delta \text{cl}(A)) \) for every subset \( A \) of \( X \).

The proof of (e) follows from (d). The proof of (f) follows from (e). The proof of (g) follows from (d).

**Corollary 2.5** Let \( (X, \tau) \) be a space. Then the following hold.

(a) \( c \lambda (G \cap c \lambda (A)) = c \lambda (G \cap A) \) for every \( \delta \)-open (resp. regular open) set \( G \) and every subset \( A \) of \( X \).

(b) \( c \lambda (G) = c \lambda (G \cap A) \) for every \( \delta \)-open (resp. regular open) set \( G \) and every subset \( A \) of \( X \) such that \( c \lambda (A) = X \).

(c) \( G \cap i \lambda (A) \subset i \lambda (G \cap A) \) for every \( \delta \)-open set \( G \) and every subset \( A \) of \( X \).

(d) If \( A \in \mu \) and \( G \) is \( \delta \)-open, then \( G \cap A \in \mu \).

Proof: (a) Let \( G \) be \( \delta \)-open and \( A \subset X \). By Theorem 2.4(b), \( G \cap c \lambda (A) \subset c \lambda (G \cap A) \) and so \( c \lambda (G \cap c \lambda (A)) \subset c \lambda (G \cap A) \). Clearly, \( c \lambda (G \cap A) \subset c \lambda (G \cap c \lambda (A)) \) and so the proof follows.

(b) The proof follows from (a).

(c) Since \( i \lambda \in \Gamma (\tau) \), by Theorem 2.4(c), \( G \cap i \lambda (A) \subset i \lambda (G \cap A) \) for every \( \delta \)-open set \( G \) and every subset \( A \) of \( X \).

(d) If \( A \in \mu \), by (c), \( G \cap A \subset i \lambda (G \cap A) \) for every \( \delta \)-open set \( G \) and every subset \( A \) of \( X \) and so \( G \cap A \in \mu \).

Therefore, \( G \cap A \in \mu \).

3. IDENTITIES INVOLVING \( i \lambda \) AND \( c \lambda \):

In the rest of the section, we discuss the relation between the operators \( \delta \text{int}, \delta \text{cl}, c \lambda, i \lambda, c \mu, i \mu, c \nu, i \nu \), \( c \sigma, i \sigma, c \tau, i \tau, c \beta \) and \( i \beta \).

Throughout this section, the dual of an identity is denoted by the corresponding alphabet with the suffix “1” and written almost in the same line.

**Theorem 3.1** Let \( (X, \tau) \) be a space and \( A \subset X \). Then the following hold.

(a) \( \delta \text{int}(c \lambda (A)) = \delta \text{int}(\text{cl}(A)) \). (a1) \( \delta \text{cl}(i \lambda (A)) = \delta \text{cl}(\text{int}(A)) \).

(b) \( \delta \text{cl}(c \lambda (A)) = \delta \text{cl}(c \mu (A)) \). (b1) \( \delta \text{cl}(i \lambda (A)) = \delta \text{cl}(\text{int}(A)) \).

(c) \( c \lambda (\delta \text{int}(A)) = \text{int}(\delta \text{cl}(i \lambda (A))) \). (c1) \( c \lambda (\delta \text{cl}(A)) = \delta \text{cl}(\text{int}(A)) \).

(d) \( c \lambda (\delta \text{int}(A)) = \delta \text{cl}(\text{int}(A)) \). (d1) \( c \lambda (\delta \text{cl}(A)) = \delta \text{cl}(A) \).

Proof: (a) \( \delta \text{int}(c \lambda (A)) = \delta \text{int}(\text{cl}(A)) \), by Lemma 1.1(b) and so \( \delta \text{int}(c \lambda (A)) \supset \delta \text{int}(\text{cl}(A)) \) = \( \delta \text{int}(\text{cl}(A)) \). Also, \( \delta \text{int}(c \lambda (A)) = \delta \text{int}(\text{cl}(A)) \supset \delta \text{int}(\text{cl}(A)) \). Hence \( \delta \text{int}(c \lambda (A)) \supset \delta \text{int}(\text{cl}(A)) \).

(b) \( \delta \text{cl}(c \lambda (A)) = \delta \text{cl}(\text{cl}(A)) \), by Lemma 1.1(b) and so \( \delta \text{cl}(c \lambda (A)) = \delta \text{cl}(A) \supset \delta \text{cl}(\text{cl}(A)) \) = \( \delta \text{cl}(A) \). Again, \( \delta \text{cl}(A) \supset \delta \text{cl}(c \lambda (A)) \) = \( \delta \text{cl}(A) \). Therefore, \( \delta \text{cl}(c \lambda (A)) = \delta \text{cl}(A) \).

(c) \( c \lambda (\delta \text{int}(A)) = \text{int}(\delta \text{cl}(A)) \), by Lemma 1.1(b) and so \( c \lambda (\delta \text{int}(A)) = \text{int}(\delta \text{cl}(A)) \).

(d) \( c \lambda (\delta \text{int}(A)) = \text{int}(\delta \text{cl}(A)) \), by Lemma 1.1(b) and so \( c \lambda (\delta \text{int}(A)) = \text{cl}(\delta \text{int}(A)) \).

**Theorem 3.2** Let \( (X, \tau) \) be a space and \( A \subset X \). Then the following hold.

(a) \( c \lambda (\text{int}(c \lambda (A))) = c \lambda (\text{int}(c \lambda (A))) \). (a1) \( i \lambda (\text{cl}(i \lambda (A))) = c \lambda (\text{cl}(i \lambda (A))) \).

(b) \( i \lambda (\text{cl}(i \lambda (A))) = i \lambda (\text{cl}(i \lambda (A))) \). (b1) \( c \lambda (\text{cl}(i \lambda (A))) = c \lambda (\text{cl}(i \lambda (A))) \).

(c) \( c \lambda (\text{cl}(i \lambda (A))) = c \lambda (\text{cl}(i \lambda (A))) \). (c1) \( i \lambda (\text{cl}(i \lambda (A))) = i \lambda (\text{cl}(i \lambda (A))) \).

(d) \( c \lambda (\text{cl}(i \lambda (A))) = c \lambda (\text{cl}(i \lambda (A))) \). (d1) \( i \lambda (\text{cl}(i \lambda (A))) = i \lambda (\text{cl}(i \lambda (A))) \).
Proof:
(a) \( cl(int(c(A))) = cl(int(A \cup cl(\partial int(A)))) \supset cl(int(A)) \). Also, \( cl(int(c(A))) = cl(int(A) \cup cl(\partial int(A))) \supset cl(int(A)) \). Therefore, \( cl(int(c(A))) = cl(int(A)) \). Also, \( cl(int(c(A))) = cl(int(A) \cup cl(\partial int(A))) = cl(int(A)) \). Then the following are equivalent
\[ c(A) \subseteq cl(int(c(A))) \] by Lemma 1.1(d) and so \( c(A) \subseteq cl(int(c(A))) \).

(b) \( \partial int(i_1(A)) = \partial int(cl(i_1(A))) \), by Theorem 3.2(a) and so the above equality just proved and so \( i_1(\partial cl(A)) \).

(c) \( c(d) \subseteq c(A) \). Hence \( c(d) \subseteq c(A) \).

(d) \( c(d) \subseteq c(A) \). Hence \( c(d) \subseteq c(A) \).

4. CHARACTERIZATIONS OF GENERALIZED OPEN SETS:

In this section, we characterize regular open sets, \( \delta \)-open sets, \( \alpha \)-open sets, semiopen sets, preopen sets, \( \beta \)-open sets, \( \delta \)-preopen sets and \( \delta \)-semiopen sets in terms of the compositions of generalized interior and closure operators.

Theorem 4.1 Let \( (X, t) \) be a space and \( A \subseteq X \). Then the following are equivalent.

(a) \( A \in t_\delta \).
(b) \( \partial int(i_1(A)) = A \).
(c) \( int(i_1(A)) = A \).
(d) \( \partial int(i_1(A)) = A \).

Proof: (a), (b), (c) and (d) are equivalent by Theorem 3.1(b).

Theorem 4.2 Let \( (X, t) \) be a space and \( A \subseteq X \). Then the following are equivalent.

(a) \( A \) is regular open.
(b) \( i_1(\partial cl(A)) = A \).
(c) \( \partial int(c(A)) = A \).
(d) \( \partial int(cl(i_1(A))) = A \).
(e) \( int(cl(i_1(A))) = A \).
(f) \( c_i(\partial cl(A)) = A \).

Proof: (a) and (b) are equivalent by Theorem 3.1(d). (a) and (c) are equivalent by Theorem 3.1(a). (a), (d), (e) and (f) are equivalent by Theorem 3.2(a).

Theorem 4.3 Let \( (X, t) \) be a space and \( A \subseteq X \). Then the following are equivalent.

(a) \( A \) is \( \alpha \)-open.
(b) \( A \subseteq int(cl(c(A))) \).
(c) \( A \subseteq int(cl(i_1(A))) \).
(d) \( A \subseteq int(cl(c(A))) \).
(e) \( i_1(A) = A \).
(f) \( \partial int(c_i(A)) = A \).

Proof: (a), (b), (c), (d) and (f) are equivalent by Theorem 3.2(a). (a) and (c) are equivalent by Theorem 3.1(a). (a) and (e) are equivalent by Theorem 3.2(d).
(a) $A$ is semiopen.
(b) $A \subseteq \text{cl}(\text{i}_{d}(\text{int}(A)))$.
(c) $A \subseteq \text{dcl}(\text{i}_{d}(A))$.
(d) $\text{cl}(A) = \text{cl}(\text{int}(c_{d}(A)))$.
(e) $A \subseteq c_{d}(\text{int}(A))$.

**Proof:** (a), (b), (d) and (e) are equivalent by Theorem 3.2(a). (a) and (c) are equivalent by Theorem 3.1(a).

**Theorem: 4.5** Let $(X, \tau)$ be a space and $A \subseteq X$. Then the following are equivalent.

(a) $A$ is preopen.
(b) $A \subseteq \text{int}(\text{i}_{d}(\text{cl}(A)))$.
(c) $A \subseteq \text{int}(\text{cl}(\text{i}_{d}(A)))$.
(d) $\text{i}_{d}(\text{i}_{d}(A)) = A$.
(e) $\text{i}_{d}(\text{i}_{d}(A)) = A$.
(f) $\text{i}_{d}(\text{i}_{d}(A)) = A$.
(g) $A \subseteq c_{d}(\text{int}(A))$.
(h) $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$.

**Proof:** (a), (b), (c) and (g) are equivalent by Theorem 3.2(a). (a), (d) and (f) are equivalent by Theorem 3.2(b). (a) and (e) are equivalent by Theorem 3.2(c). (a) and (h) are equivalent by Theorem 3.1(a).

**Theorem: 4.6** Let $(X, \tau)$ be a space and $A \subseteq X$. Then the following are equivalent.

(a) $A$ is $\beta$– open.
(b) $A \subseteq \text{cl}(\text{int}(\text{c}_{d}(A)))$.
(c) $A \subseteq \text{cl}(\text{int}(\text{cl}(\text{i}_{d}(A))))$.
(d) $A \subseteq \text{cl}(\text{c}_{d}(\text{i}_{d}(A)))$.
(e) $A \subseteq \text{cl}(\text{int}(\text{cl}(\text{i}_{d}(A))))$.

**Proof:** (a), (c), (d) and (e) are equivalent by Theorem 3.2(a). (a) and (b) are equivalent by Theorem 3.1(a).

**Theorem: 4.7** Let $(X, \tau)$ be a space and $A \subseteq X$. Then the following are equivalent.

(a) $A$ is $\delta$ – preopen.
(b) $A \subseteq \text{dint}(\text{dcl}(\text{c}_{d}(A)))$.
(c) $A \subseteq \text{dint}(\text{dcl}(\text{c}_{d}(A)))$.
(d) $A \subseteq \text{i}_{d}(\text{dcl}(A))$.
(e) $A \subseteq \text{dint}(\text{dcl}(\text{c}_{d}(A)))$.

**Proof:** (a), (b), (c) and (e) are equivalent by Theorem 3.1(b). (a) and (d) are equivalent by Theorem 3.1(d).

**Theorem: 4.8** Let $(X, \tau)$ be a space and $A \subseteq X$. Then the following are equivalent.

(a) $A$ is $\delta$– semiopen.
(b) $A \subseteq \text{cl}(\text{dint}(\text{i}_{d}(A)))$.
(c) $A \subseteq \text{cl}(\text{dint}(\text{i}_{d}(A)))$.
(d) $A \subseteq \text{cl}(\text{dint}(\text{i}_{d}(A)))$.
(e) $A \subseteq \text{c}_{d}(\text{dint}(A))$.

**Proof:** (a), (b), (c) and (d) are equivalent by Theorem 3.1(b). (a) and (e) are equivalent by Theorem 3.1(d).

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