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CHARACTERIZATIONS OF SOME GENERALIZED OPEN SETS

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ABSTRACT

We discuss the properties of δ– preopen sets and the relations of δ– preclosure and δ– preinterior operators with the closure and interior operators of some well known generalized topologies. Also, we characterize regular open sets, δ– open sets, semiopen sets, preopen sets, b– open sets and β– open sets.

Key words and Phrases: *δ– preopen, δ– open, semiopen, preopen, b– open and β– open sets, δ– preclosure and δ– preinterior.*

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1. INTRODUCTION AND PRELIMINARIES:

In 1993, Raychaudhuri and Mukherjee [11] introduced and studied δ– preopen sets in topological spaces. In 1997, *Á.Császár* [4] introduced and studied generalized open subsets of a set X defined in terms of monotonic functions from $\wp(X)$ to $\wp(X)$. δ– preopen set is a particular kind of generalized open set introduced by *Á.Császár* [4] but different from the well known generalized open sets, namely semiopen set, preopen set, b– open set and β–open set. In this paper, we further study these sets and discuss the relation between the interior and closure operators of these generalized open sets.

Let X be any nonempty set. Denote by Γ, the collection of all mappings $\gamma: \wp(X)\rightarrow\wp(X)$ such that $A \subset B$ implies $\gamma(A) \subset \gamma(B)$. As defined in [4], we mention here the following sub collections of Γ.

$\Gamma_0 = \{\gamma \in \Gamma \mid \gamma(\emptyset) = \emptyset \}$,
$\Gamma_1 = \{\gamma \in \Gamma \mid \gamma(X) = X \}$,
$\Gamma_2 = \{\gamma \in \Gamma \mid \gamma^2(A) = \gamma(A)$ for every subset A of $X \}$,
$\Gamma_- = \{\gamma \in \Gamma \mid \gamma(A) \subset A$ for every subset A of $X \}$ and
$\Gamma_+ = \{\gamma \in \Gamma \mid A \subset \gamma(A)$ for every subset A of $X\}$

Let $\gamma \in \Gamma$. A subset A of X is said to be γ – *open* [4] if $A \subset \gamma(A)$. B is said to be γ – *closed* [4] if its complement is γ – open. The smallest γ – closed set containing A is called the γ – *closure* of A [4] and is denoted by $c_\gamma(A)$. The largest γ – open set contained in A is called the γ – *interior* of A [4] and is denoted by $i_\gamma(A)$. If $\gamma_1, \gamma_2 \in \Gamma$, then we will denote $\gamma_1 \circ \gamma_2$ by $\gamma_1 \gamma_2$. For $\gamma \in \Gamma$, define $\gamma^*: \wp(X) \rightarrow \wp(X)$ by $\gamma^*(A) = X - \gamma(X - A)$ [4] for every subset A of X. Also, in Proposition 1.7 of [4], it is established that $(\gamma^*)^* = \gamma$, $(i_\gamma)^* = c_\gamma$ and $(c_\gamma)^* = i_\gamma$. We say that $\iota \in \Gamma$ is a *dual* of $\kappa \in \Gamma$ if $\iota^* = \kappa$ and clearly, if ι is a dual of κ, then κ is a dual of ι. If I is a collection of some of the symbols 0, 2, –, + and 1, then $\Gamma_I = \{ \gamma \in \Gamma \mid \gamma \in \Gamma_i$ for every $i \in I\}$. For example, $\gamma \in \Gamma_{012}$ means that $\gamma \in \Gamma_0$, $\gamma \in \Gamma_1$ and $\gamma \in \Gamma_2$. A subfamily $\mathcal{A}$ of $\wp(X)$ is called a *generalized topology* [6] if $\emptyset \in \mathcal{A}$ and $\mathcal{A}$ is closed under arbitrary union. By a space (X, τ), we will always mean the topological space (X, τ). If (X, τ) is a space, then $\Gamma_3(\tau) = \{ \gamma \in \Gamma \mid G \cap \gamma(A) \subset \gamma(G \cap A)$ for every $G \in \tau$ and $A \subset X \}$. A subset A of a space (X, τ) is said to be *regular open* if $A = int(cl(A))$ where *int* and *cl* are the *interior* and *closure* operators. The family of all regular open sets is a base for a topology τ_δ, coarser than τ, which is called the *semiregularization* of the topology τ. δint and δcl are the *interior* and *closure* operators in (X, τ_δ). A subset A of X is said to be α –*open* [9](resp. *semiopen* [7], *preopen* [8], b – *open* [2], β – *open* [3]) if $A \subset int(cl(int(A)))$ (resp. $A \subset cl(int(A))$, $A \subset int(cl(A))$, $A \subset int(cl(A)) \cup cl(int(A))$, $A \subset cl(int(cl(A)))$). A subset A of X is said to be α – closed (resp. *semiclosed, preclosed,* b – *closed,* β – *closed*) if $X - A$ is α – open (resp. *semiopen, preopen, b– open, β – open*). The family of all α– open (resp. semiopen, preopen, b – open, β – open) sets of a space will be denoted by $\alpha(X)$ (resp. $\sigma(X)$, $\pi(X)$, $b(X)$, $\beta(X)$). These families are generalized topologies whose closure and interior operators are denoted by c_α, i_α, c_σ, i_σ, c_π, i_π, c_b, i_b, c_β and i_β, respectively. We use the following lemmas without mentioning it explicitly.

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Lemma: 1.1 Let (X, τ) be a space, $A \subset X$ and $\mathscr{Q}$ ={int cl int, cl int, int cl, cl int cl }. If $\kappa \in \mathscr{Q}$, then the following hold [1, 2].

(a) $i_\kappa(A) = A \cap \kappa(A)$.
(b) $c_\kappa(A) = A \cup \kappa^*(A)$.
(c) $i_b(A) = i_\sigma(A) \cup i_\pi(A)$.
(d) $c_b(A) = c_\sigma(A) \cap c_\pi(A)$.

Lemma: 1.2 Let (X,τ) be a space and $A \subset X$. Then the following hold.

(a) If A is open, then $cl(A) = \delta cl(A)$ [12].
(b) If A is closed, then $int(A) = \delta int(A)$ [12].
(c) A is regular open if and only if $A = \delta int(\delta cl(A))$.

2. δ – PREOPEN SETS:

Let $\lambda = int\ \delta cl$, the composition of the operators *int* and *δcl*. Since $int \in \Gamma_{012-}$ and $\delta cl \in \Gamma_{012+}$, by Theorem 1.11 of [4], $\lambda \in \Gamma_{01}$ and by Corollary 1.14 of [4], $\lambda \in \Gamma_2$. If $\mu = \{A |\ A \subset \lambda(A)\}$, then μ is the family of all λ – open sets which is nothing but the family of all δ – *preopen sets* [11] and so λ – closed sets are nothing but δ – *preclosed sets*. Clearly, $\emptyset \in \mu$, $X \in \mu$ and by Proposition 2 of [11], arbitrary union of elements of μ is in μ. Therefore, μ is a generalized topology with $X \in \mu$. Moreover, one can easily prove that $RO(X) \subset \tau_\delta \subset \tau \subset \tau^\alpha \subset \pi(X) \subset \mu$, where $RO(X)$ (resp. τ^α) is the family of all regular open (resp.α– open) sets in (X, τ). The following Theorem 2.1 gives some properties of δ – preopen sets.

Theorem: 2.1 Let (X, τ) be a space and $\lambda = int\ \delta cl$. Then the following hold.

(a) $\lambda \in \Gamma_3(\tau_\delta)$, or equivalently, $G \cap int(\delta cl(A)) \subset int(\delta cl(G \cap A))$ for every δ– open set G and every subset A of X.
(b) $A \in \mu$ if and only if $A \cap V \in \mu$ for every δ–open (resp. *regularopen*) *set* V [11, Theorem 3].
(c) $\lambda^* \in \Gamma_{012}$ and $\lambda^* = cl\ \delta int$.

Proof: (a) In any space (X, τ), *cl* and *int* $\in \Gamma_3(\tau)$. Therefore, $int \in \Gamma_3(\tau)$ and $\delta cl \in \Gamma_3(\tau_\delta)$. Since $int \in \Gamma_3(\tau)$, $int \in \Gamma_3(\tau_\delta)$ and so by Proposition 2.1 of [4], $\lambda \in \Gamma_3(\tau_\delta)$.

(b) If $A \in \mu$, then $A \subset \lambda(A)$ and so $V \cap A \subset V \cap \lambda(A) \subset \lambda(V \cap A)$, by (a), for every δ – open set V. Therefore, $V \cap A \in \mu$. Converse follows from the fact that X is both regular open and δ – open.

(c) Since $\lambda \in \Gamma_{012}$, by Proposition 1.7 of [4], $\lambda^* \in \Gamma_{012}$. Since *int* and *cl* (resp. δ*int* and δ*cl*) are dual to each other, $\lambda^* = (int\ \delta cl)^* = cl\ \delta int$.

Theorem: 2.2 Let (X, τ) be a space. Then a subset A of X is a δ – preclosed set if and only if $\lambda^*(A) = cl(\delta int(A)) \subset A$.

Proof: A is δ – preclosed if and only if A is λ – closed if and only if $X - A \subset \lambda(X - A)$ if and only if $\lambda^*(A) = cl(\delta int(A)) \subset A$.

Theorem: 2.3 Let (X, τ) be a space. Then $\tau_\lambda = \{V \mid A \cap V \in \mu \text{ for every } A \in \mu\}$ is a topology and $\tau_\delta \subset \tau_\lambda \subset \mu$.

Proof: Since $\lambda \in \Gamma$, τ_λ is a topology by Theorem 2.16 of [4]. $\tau_\delta \subset \tau_\lambda$ follows from Theorem 2.1(b). That $\tau_\lambda \subset \mu$ is clear, since $X \in \mu$.

For the generalized topology $\mu = \{A \subset X \mid A \subset \lambda(A)\}$ and a subset A of X, define $i_\mu(A) = \cup\{U \in \mu \mid U \subset A\}$ and $c_\mu(A) = \cap\{X - U \mid U \in \mu, A \subset X - U\}$. Then i_μ = δ–*Pint* [10] and c_μ = δ–*Pcl* [10]. i_μ and c_μ will also be denoted by i_λ and c_λ, respectively. The following Theorem 2.4 and Corollary 2.5 give properties of the operators i_μ and c_μ. Also, note that Theorem 2.1(b) can be deduced from Theorem 2.4(c) as well as from Corollary 2.5(c).

Theorem: 2.4 Let (X, τ) be a space. Then the following hold.

(a) $c_\mu \in \Gamma_{012+}$ and $i_\mu \in \Gamma_{012-}$.
(b) $c_\mu \in \Gamma_3(\tau_\delta)$ [11, Lemma 1(a)].
(c) $i_\mu \in \Gamma_3(\tau_\delta)$.
(d) $i_\mu(A) = A \cap int(\delta cl(A))$ for every subset A of X [10, Theorem 8(d)].
(e) $c_\mu(A) = A \cup cl(\delta int(A))$ for every subset A of X [11, Theorem 2].
(f) If $A \in \tau_\delta$, then $c_\mu(A) = cl(A)$ [11, Theorem 4].
(g) If A is τ_δ – closed, then $i_\mu(A) = int(A)$.

Proof: (a) The proof follows from the definition.

(b) By Theorem 2.1(a), $\lambda \in \Gamma_3(\tau_\delta)$ and so by Corollary 2.6 of [4], $c_\mu \in \Gamma_3(\tau_\delta)$.

(c) By Theorem 2.1(a), $\lambda \in \Gamma_3(\tau_\delta)$ and so by Proposition 2.4 of [4], $i_\mu \in \Gamma_3(\tau_\delta)$.

(d) Since *int* is a decreasing idempotent monotonic operator, δcl is an increasing monotonic operator and $\mu = int\, \delta cl$, by Theorem 1.3 of [5], $i_\mu(A) = A \cap int(\delta cl(A))$ for every subset A of X.

The proof of (e) follows from (d). The proof of (f) follows from (e). The proof of (g) follows from (d).

Corollary: 2.5 Let (X, τ) be a space. Then the following hold.

(a) $c_\mu(G \cap c_\mu(A)) = c_\mu(G \cap A)$ for every δ–open (resp. *regular open*) *set* G and every subset A of X.
(b) $c_\mu(G) = c_\mu(G \cap A)$ for every δ–open (resp. *regular open*) *set* G and every subset A of X such that $c_\mu(A) = X$.
(c) $G \cap i_\mu(A) \subset i_\mu(G \cap A)$ for every δ–open set G and every subset A of X.
(d) If $A \in \mu$ and G is δ–open, then $G \cap A \in \mu$.

Proof: (a) Let G be δ–open and $A \subset X$. By Theorem 2.4(b), $G \cap c_\mu(A) \subset c_\mu(G \cap A)$ and so $c_\mu(G \cap c_\mu(A)) \subset c_\mu(G \cap A)$. Clearly, $c_\mu(G \cap A) \subset c_\mu(G \cap c_\mu(A))$ and so the proof follows.

(b) The proof follows from (a).

(c) Since $i_\mu \in \Gamma_3(\tau_\delta)$, by Theorem 2.4(c), $G \cap i_\mu(A) \subset i_\mu(G \cap A)$ for every δ–open set G and every subset A of X.

(d) If $A \in \mu$, by (c), $G \cap A \subset i_\mu(G \cap A)$ for every δ–open set G and every subset A of X and so $G \cap A = i_\mu(G \cap A)$.

Therefore, $G \cap A \in \mu$.

3. IDENTITIES INVOLVING i_μ AND c_μ :

In the rest of the section, we discuss the relation between the operators δint, δcl, c_μ, i_μ, c_α, i_α, c_σ, i_σ, c_π, i_π, c_b, i_b, c_β and i_β. Throughout this section, the dual of an identity is denoted by the corresponding alphabet with the suffix “1 " and written almost in the same line.

Theorem: 3.1 *Let (X, τ) be a space and $A \subset X$. Then the following hold.*

(a) $\delta int(c_\sigma(A)) = \delta int(cl(A))$. ($a_1$) $\delta cl(i_\sigma(A)) = \delta cl(int(A))$.
(b) $\delta cl(c_\pi(A)) = \delta cl(c_\mu(A)) = \delta cl(c_\sigma(A)) = \delta cl(A)$.
(b_1) $\delta int(i_\pi(A)) = \delta int(i_\mu(A)) = \delta int(i_\sigma(A)) = \delta int(A)$.
(c) $c_\sigma(\delta int(A)) = int(\delta cl(\delta int(A)))$. ($c_1$) $i_\sigma(\delta cl(A)) = cl(\delta int(\delta cl(A)))$.
(d) $c_\pi(\delta int(A)) = \delta cl(\delta int(A)$. ($d_1$) $i_\pi(\delta cl(A)) = \delta int(\delta cl(A))$.

Proof:
(a) δint($c_\sigma(A)$) = δint($A \cup$ int(cl(A))), by Lemma 1.1(b) and so δint($c_\sigma(A)$) $\supset$ δint(int(cl(A))) = δint(cl(A)).Also, δint($c_\sigma(A)$) = δint($A \cup$ int(cl(A))) $\subset$ δint($A \cup$ cl(A)) = δint(cl(A)). Hence δint($c_\sigma(A)$) = δint(cl(A)).

(b) δcl($c_\pi(A)$) = δcl($A \cup$ cl(int(A))),by Lemma 1.1(b) and so δcl($c_\pi(A)$) = δcl(A) $\cup$ δcl(cl(int(A))) = δcl(A) $\cup$ δcl(δcl(int(A))), since cl(int(A)) = δcl(int(A)) and so δcl($c_\pi(A)$) = δcl(A). Again, δcl(A) $\subset$ δcl($c_\mu(A)$) $\subset$ δcl($c_\pi(A)$) = δcl(A) and so δcl(A) = δcl($c_\mu(A)$). δcl($c_\sigma(A)$) = δcl($A \cup$ int(cl(A))), by Lemma 1.1(b) and so δcl($c_\sigma(A)$) = δcl(A) $\cup$ δcl(int(cl(A))) = δcl(A). Therefore, δcl($c_\sigma(A)$) = δcl(A).

(c) c_σ(δint(A)) = δint(A) $\cup$ int(cl(δint(A))), by Lemma 1.1(b) and so c_σ(δint(A)) = int(δcl(δint(A))).

(d) c_π(δint(A)) = δint(A) $\cup$ cl(int(δint(A))), by Lemma 1.1(b) and so c_π(δint(A)) = cl(δint(A)).

Theorem: 3.2 *Let (X, τ) be a space and $A \subset X$. Then the following hold.*

(a) $cl(int(c_\mu(A))) = cl(c_\mu(int(A))) = i_\sigma(c_\mu(A)) = cl(int(A))$.
(a_1) $int(cl(i_\mu(A))) = int(i_\mu(cl(A))) = c_\sigma(i_\mu(A)) = int(cl(A))$.
(b) $i_\pi(i_\mu(A)) = i_\mu(i_\beta(A)) = i_\pi(A)$. ($b_1$) $c_\pi(c_\mu(A)) = c_\mu(c_\beta(A)) = c_\pi(A)$.
(c) $c_\mu(c_b(A)) = c_\pi(A)$. (c_1) $i_\mu(i_b(A)) = i_\pi(A)$.
(d) $c_\mu(c_\sigma(A)) = c_\alpha(A)$. ($d_1$) $i_\mu(i_\sigma(A)) = i_\alpha(A)$.

Proof:

(a) $cl(int(c_\mu(A))) = cl(int(A \cup cl(\delta int(A)))) \supset cl(int(A))$. Also, $cl(int(c_\mu(A))) = cl(int(A \cup cl(\delta int(A)))) \subset cl(int(A) \cup cl(\delta int(A))) = cl(int(A)) \cup cl(\delta int(A)) = cl(int(A))$. Therefore, $cl(int(c_\mu(A))) = cl(int(A))$. Also, $cl(c_\mu(int(A))) = cl(int(A) \cup cl(\delta int(A))) = cl(int(A)) \cup cl(\delta int(A)) = cl(int(A))$. $i_\sigma(c_\mu(A)) = c_\mu(A) \cap cl(int(c_\mu(A))) = c_\mu(A) \cap cl(int(A))$, by the above equality just proved and so $i_\sigma(c_\mu(A)) = (A \cap cl(int(A))) \cap cl(int(A)) = cl(int(A))$.

(b) $i_\pi(i_\mu(A)) = i_\mu(A) \cap int(cl(i_\mu(A))) = i_\mu(A) \cap int(cl(A))$, by Theorem 3.2($a_1$) and so $i_\pi(i_\mu(A)) = A \cap int(\delta cl(A)) \cap int(cl(A)) = A \cap int(cl(A)) = i_\pi(A)$.

(b) $i_\mu(i_\beta(A)) = i_\beta(A) \cap int(\delta cl(i_\beta(A))) = i_\beta(A) \cap int(\delta cl(A \cap cl(int(cl(A))))) \subset i_\beta(A) \cap int(\delta cl(cl(int(cl(A))))) \subset i_\beta(A) \cap int(cl(A))$. Also, $i_\mu(i_\beta(A)) = i_\beta(A) \cap int(\delta cl(A \cap cl(int(cl(A))))) \subset i_\beta(A) \cap int(cl(A \cap cl(int(cl(A))))) \subset i_\beta(A) \cap int(cl(A \cap int(cl(A)))) \supset i_\beta(A) \cap int(cl(A) \cap int(cl(A))) = i_\beta(A) \cap int(cl(A))$. Therefore, $i_\mu(i_\beta(A)) = i_\beta(A) \cap int(cl(A)) = A \cap cl(int(cl(A))) \cap int(cl(A)) = A \cap int(cl(A)) = i_\pi(A)$.

(c) $c_\mu(c_b(A)) = c_\mu(c_\sigma(A) \cap c_\pi(A))$, by Lemma 1.1(d) and so $c_\mu(c_b(A)) = c_\mu((A \cup int(cl(A))) \cap (A \cup cl(int(A)))) = c_\mu(A \cup (int(cl(A)) \cap cl(int(A)))) \supset c_\mu(A) \cup c_\mu(int(cl(A)) \cap cl(int(A))) = c_\mu(A) \cup (int(cl(A)) \cap cl(int(A))) \cup cl(\delta int(int(cl(A)) \cap cl(int(A)))) = c_\mu(A) \cup (int(cl(A)) \cap cl(int(A))) \cup cl(\delta int(int(cl(A))) \cap \delta int(cl(int(A)))) = c_\mu(A) \cup (int(cl(A)) \cap cl(int(A))) \cup cl(\delta int(cl(A)) \cap \delta int(cl(int(A)))) = c_\mu(A) \cup (int(cl(A)) \cap cl(int(A))) \cup cl(\delta int(cl(int(A)))) = c_\mu(A) \cup (int(cl(A)) \cap cl(int(A))) \cup cl(int(A)) = c_\mu(A) \cup cl(int(A)) = A \cup cl(\delta int(A)) \cup cl(int(A)) = A \cup cl(int(A)) = c_\pi(A)$ and so $c_\mu(c_b(A)) \supset c_\pi(A)$. But, $c_\mu(c_b(A)) = c_\mu(c_\sigma(A) \cap c_\pi(A)) \subset c_\mu(c_\pi(A)) = c_\pi(A)$. Hence $c_\mu(c_b(A)) = c_\pi(A)$.

(d) $c_\mu(c_\sigma(A)) = c_\sigma(A) \cup cl(\delta int(c_\sigma(A))) = c_\sigma(A) \cup cl(\delta int(cl(A)))$, by Theorem 3.1(a) and so $c_\mu(c_\sigma(A)) = A \cup int(cl(A)) \cup cl(\delta int(cl(A))) = A \cup cl(\delta int(cl(A))) = A \cup cl(int(cl(A))) = c_\alpha(A)$.

4. CHARACTERIZATIONS OF GENERALIZED OPEN SETS:

In this section, we characterize regular open sets, δ– open sets, α– open sets, semiopen sets, preopen sets, β– open sets, δ– preopen sets and δ– semiopen sets in terms of the compositions of generalized interior and closure operators.

Theorem: 4.1 Let *(X, τ) be a space and $A \subset X$. Then the following are equivalent*.

(a) $A \in \tau_\delta$.
(b) $\delta int(i_\pi(A)) = A$.
(c) $\delta int(i_\mu(A)) = A$.
(d) $\delta int(i_\sigma(A)) = A$.

Proof: (a), (b), (c) and (d) are equivalent by Theorem 3.1(b_1).

Theorem: 4.2 *Let (X, τ) be a space and $A \subset X$. Then the following are equivalent*.

(a) *A is regular open*.
(b) $i_\pi(\delta cl(A)) = A$.
(c) $\delta int(c_\sigma(A)) = A$.
(d) $int(i_\mu(cl(A))) = A$.
(e) $int(cl(i_\mu(A))) = A$.
(f) $c_\sigma(i_\mu(A)) = A$.

Proof: (a) and (b) are equivalent by Theorem 3.1(d_1). (a) and (c) are equivalent by Theorem 3.1(a). (a), (d), (e) and (f) are equivalent by Theorem 3.2(a_1).

Theorem: 4.3 *Let (X, τ) be a space and $A \subset X$. Then the following are equivalent*.

(a) *A is α– open*.
(b) $A \subset int(cl(c_\mu(int(A))))$.
(c) $A \subset int(\delta cl(i_\sigma(A)))$.
(d) $A \subset int(cl(int(c_\mu(A))))$.
(e) $i_\mu(i_\sigma(A)) = A$.
(f) $A \subset int(i_\sigma(c_\mu(A)))$.

Proof: (a), (b), (c), (d) and (f) are equivalent by Theorem 3.2(a). (a) and (c) are equivalent by Theorem 3.1(a_1). (a) and (e) are equivalent by Theorem 3.2(d_1).

Theorem: 4.4 *Let (X, τ) be a space and $A \subset X$. Then the following are equivalent*.

(a) *A is semiopen*.
(b) $A \subset cl(c_\mu(int(A)))$.
(c) $A \subset \delta cl(i_\sigma(A))$.
(d) $cl(A) = cl(int(c_\mu(A)))$.
(e) $A \subset i_\sigma(c_\mu(A))$.

Proof: (a), (b), (d) and (e) are equivalent by Theorem 3.2(a). (a) and (c) are equivalent by Theorem 3.1(a_1).

Theorem: 4.5 Let *(X, τ) be a space and $A \subset X$. Then the following are equivalent*.

(a) *A is preopen.*
(b) $A \subset int(i_\mu(cl(A)))$.
(c) $A \subset int(cl(i_\mu(A)))$.
(d) $i_\pi(i_\mu(A)) = A$.
(e) $i_\mu(i_b(A)) = A$.
(f) $i_\mu(i_\beta(A)) = A$.
(g) $A \subset c_\sigma(i_\mu(A))$.
(h) $A \subset \delta int(c_\sigma(A))$.

Proof: (a), (b), (c) and (g) are equivalent by Theorem 3.2(a_1). (a), (d) and (f) are equivalent by Theorem 3.2(b). (a) and (e) are equivalent by Theorem 3.2(c_1). (a) and (h) are equivalent by Theorem 3.1(a).

Theorem: 4.6 Let *(X, τ) be a space and $A \subset X$. Then the following are equivalent*.

(a) *A is* β– *open*.
(b) $A \subset cl(\delta int(c_\sigma(A)))$.
(c) $A \subset cl(int(cl(i_\mu(A))))$.
(d) $A \subset cl(c_\sigma(i_\mu(A)))$.
(e) $A \subset cl(int(i_\mu(cl(A))))$.

Proof: (a), (c), (d) and (e) are equivalent by Theorem 3.2(a_1). (a) and (b) are equivalent by Theorem 3.1(a).

Theorem: 4.7 *Let (X, τ) be a space and $A \subset X$. Then the following are equivalent.*

(a) *A is δ – preopen*.
(b) $A \subset \delta int(\delta cl(c_\pi(A)))$.
(c) $A \subset \delta int(\delta cl(c_\mu(A)))$.
(d) $A \subset i_\pi(\delta cl(A))$.
(e) $A \subset \delta int(\delta cl(c_\sigma(A)))$.

Proof: (a), (b), (c) and (e) are equivalent by Theorem 3.1(b). (a) and (d) are equivalent by Theorem 3.1(d_1).

Theorem: 4.8 *Let (X, τ) be a space and $A \subset X$. Then the following are equivalent*.

(a) *A is δ– semiopen*.
(b) $A \subset cl(\delta int(i_\pi(A)))$.
(c) $A \subset cl(\delta int(i_\mu(A)))$.
(d) $A \subset cl(\delta int(i_\sigma(A))$.
(e) $A \subset c_\pi(\delta int(A)))$.

Proof: (a), (b), (c) and (d) are equivalent by Theorem 3.1(b_1). (a) and (e) are equivalent by Theorem 3.1(d).

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