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ADJACENT VERTEX SUM POLYNOMIAL

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ABSTRACT

Let G be a graph. The adjacent vertex sum polynomial is defined as $S(G, x) = \sum_{i=0}^{\Delta(G)} n_{\Delta(G)-i} \,.\, x^{\alpha_{\Delta(G)-i}}$ *where* $\Delta(G) = \max\{\deg v / v \in G\}$, $n_{\Delta(G)-i}$ *is the sum of the number of adjacent vertices of all the vertices of degree* $\Delta(G) - i$ *and* $\alpha_{\Delta(G)-i}$ *is the sum of the degree of adjacent vertices of all the vertices of degree* $\Delta(G) - i$. *In this paper I find the adjacent vertex sum polynomial of Cyclic graph, Complete graph, Generalized Peterson graph, Complete bipartite graph, Anti regular graph, Gear graph, Barbell graph and Book graph.*

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Key words: *Maximum degree, Adjacent vertices, Cyclic graph, Complete graph, Generalised Peterson graph, Complete bipartite graph , Anti regular graph, Gear graph, Barbell graph and Book graph.*

I define the adjacent vertex sum polynomial as follows.

Let G be a graph. The adjacent vertex sum polynomial is defined as $S(G, x) = \sum_{i=0}^{\Delta(G)} n_{\Delta(G)-i} .\, x^{\alpha_{\Delta(G)-i}}$ where $\Delta(G) = \max\{\deg v / v \in G\}$, $n_{\Delta(G)-i}$ is the sum of the number of adjacent vertices of all the vertices of degree $\Delta(G) - i$ and $\alpha_{\Delta(G)-i}$ is the sum of the degree of adjacent vertices of all the vertices of degree $\Delta(G) - i$.

Result: Adjacent vertex sum polynomial of Cyclic graph

For cyclic graph $C_n (n \geq 3)$ there are n vertices and each vertex is of degree 2 also for each vertex 2 vertices are adjacent vertices. Therefore the sum of the number of adjacent vertices of all the vertices of degree 2 is 2n and the sum of the degree of adjacent vertices of all the vertices of degree 2 is 4n.

Hence adjacent vertex sum polynomial of cyclic graph is $S(C_n, x) = 2n\, x^{4n}$ for all $n \geq 3$.

Result: Adjacent vertex sum polynomial of Complete graph

For complete graph $K_n (n \geq 1)$ there are n vertices and each vertex is of degree n – 1 also for each vertex n – 1 vertices are adjacent vertices. Therefore the sum of the number of adjacent vertices of all the vertices of degree n – 1 is n(n – 1) and the sum of the degree of adjacent vertices of all the vertices of degree n – 1 is $(n-1)^2 n$.

Hence adjacent vertex sum polynomial of complete graph K_n is $S(K_n, x) = n(n-1)\, x^{(n-1)^2 n}$ for all $n \geq 1$.

Result: Adjacent vertex sum polynomial of generalized Peterson graph

For generalized Peterson graph P(n, k) there are 2n vertices and 3n edges. Also for this graph each vertex has degree 3. For this graph each vertex has 3 vertices are adjacent. Therefore the sum of the number of adjacent vertices of all the vertices of degree 3 is 3 x 2n = 6n and the sum of the degree of adjacent vertices of all the vertices of degree 3 is 9 x 2n = 18n. Hence adjacent vertex sum polynomial of generalized Peterson graph P(n, k) is $S(P(n, k); x) = 6n x^{18n}$. $\forall n = 2, 3 \ldots$

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Result: Adjacent vertex sum polynomial of Complete bipartite graph $K_{m,n}$ ($m \neq n$)

Note that the Complete bipartite graph $K_{m,n}$ has m + n vertices and mn edges. Here m vertices have degree n and n vertices have degree m. Here for m vertices each has n vertices are adjacent. Therefore the sum of the number of adjacent vertices of all the vertices of degree n is mn and the sum of the degree of adjacent vertices of all the vertices of degree n is m^2n. Also here for n vertices each has m vertices are adjacent. Therefore the sum of the number of adjacent vertices of all the vertices of degree m is mn and the sum of the degree of adjacent vertices of all the vertices of degree m is mn^2. Hence adjacent vertex sum polynomial of complete bipartite graph $K_{m,n}$ ($m \neq n$) is

$$S(K_{m,n} ; x) = mn\, x^{m^2n} + mn\, x^{mn^2} \qquad \forall\ m \neq n$$

$$S(K_{m,n} ; x) = mn\, (x^{m^2n} + x^{mn^2}) \qquad \forall\ m \neq n$$

Result: Adjacent vertex sum polynomial of antiregular graph A_{2n}

Vertex polynomial of antiregular graph A_{2n} is

$$V(A_{2n}; x) = x^{2n-1} + x^{2n-2} + \ldots + x^{n+1} + 2x^n + x^{n-1} + \ldots + x^3 + x^2 + x \qquad \forall\quad n = 1, 2, 3 \ldots [1]$$

For A_{2n} the sum of the number of adjacent vertices of the vertex of degree 2n -1 is 2n – 1 and the sum of the degree of adjacent vertices of the above vertex is (2n – 2) + (2n – 3) + . . . + (n +1) + 2n + (n – 1) + . . . + 3 + 2 + 1. For A_{2n} the sum of the number of adjacent vertices of the vertex of degree 2n – 2 is 2n – 2 and the sum of the degree of adjacent vertices of the vertex of degree 2n – 2 is (2n – 1) + (2n – 3) + . . . + (n + 1) + 2n + (n – 1) + . . . + 3 + 2. For A_{2n} the sum of the number of adjacent vertices of the vertex of degree 2n – 3 is 2n – 3 and the sum of the degree of adjacent vertices of the vertex of degree 2n – 3 is (2n – 1) + (2n – 2) + (2n – 4) + . . . + (n +1) + 2n + (n – 1) + . . . + 4 + 3. For A_{2n} the sum of the number of adjacent vertices of the vertex of degree n +1 is n + 1 and the sum of the degree of adjacent vertices of the vertex of degree n + 1 is (2n – 1) + (2n – 2) + . . . + (n +3) + (n +2) + 2n + (n – 1). In A_{2n} there are two vertices of degree n. Therefore the sum of the number of adjacent vertices of the above two vertices of degree n is 2n and the sum of the degree of adjacent vertices of the above two vertices of degree n is 2(2n – 1) + 2(2n – 2) + 2(2n – 3) + . . . + 2(n + 1) + 2n. For A_{2n} the sum of the number of adjacent vertices of the vertex of degree n – 1 is n – 1 and the sum of the degree of adjacent vertices of the vertex of degree n – 1 is (2n – 1) + (2n – 2) + . . . + (n + 2) + (n +1). For A_{2n} the sum of the number of adjacent vertices of the vertex of degree 2 is 2 and the sum of the degree of adjacent vertices of the vertex of degree 2 is (2n – 1) + (2n – 2). For A_{2n} the number of adjacent vertex of the vertex of degree 1 is 1. In this case there is only one vertex of degree 2n – 1 is adjacent to 1.

$$\begin{aligned}
S(A_{2n} ; x) = {} & (2n-1)\, x^{(2n-2) + (2n-3) + \ldots + (n+1) + 2n + (n-1) + \ldots + 3 + 2 + 1} \\
& + (2n-2)\, x^{(2n-1) + (2n-3) + \ldots + (n+1) + 2n + (n-1) + \ldots + 3 + 2} \\
& + (2n-3)\, x^{(2n-1) + (2n-2) + (2n-4) + \ldots + (n+1) + 2n + (n-1) + \ldots + 4 + 3} \\
& + \ldots + (n+1)\, x^{(2n-1) + (2n-2) + \ldots + (n+3) + (n+2) + 2n + (n-1)} \\
& + 2n\, x^{2(2n-1) + 2(2n-2) + 2(2n-3) + \ldots + 2(n+1) + 2n} + (n-1)x^{(2n-1) + (2n-2) + \ldots + (n+1)} \\
& + \ldots + 2x^{(2n-1) + (2n-2)} + x^{2n-1}. \\
= {} & (2n-1)\, x^{2n^2 - 2n + 1} + (2n-2)\, x^{2n^2 - 2n + 1} + (2n-3)\, x^{2n^2 - 2n} \\
& + (2n-4)\, x^{2n^2 - 2n - 2} + (2n-5)\, x^{2n^2 - 2n - 5} + (2n-6)\, x^{2n^2 - 2n - 9} \\
& + (2n-7)\, x^{2n^2 - 2n - 14} + \ldots + (n+1)\, x^{2n^2 - 2n - \frac{n^2 - 5n + 4}{2}} + 2nx^{n(3n-1)} \\
& + (n-1)\, x^{(2n-1) + (2n-2) + (2n-3) + \ldots + (n+2) + (n+1)} \\
& + (n-2)\, x^{(2n-1) + (2n-2) + (2n-3) + \ldots + (n+3) + (n+2)} \\
& + (n-3)\, x^{(2n-1) + (2n-2) + (2n-3) + \ldots + (n+4) + (n+3)} \\
& + \ldots + 3x^{(2n-1) + (2n-2) + (2n-3)} + 2x^{(2n-1) + (2n-2)} + x^{(2n-1)}.
\end{aligned}$$

$$\begin{aligned}
\therefore S(A_{2n}; x) = {} & (2n-1)\, x^{2n^2 - 2n + 1} + (2n-2)x^{2n^2 - 2n + 1} + (2n-3)\, x^{2n^2 - 2n} \\
& + (2n-4)\, x^{2n^2 - 2n - 2} + (2n-5)x^{2n^2 - 2n - 5} + \\
& + (2n-6)\, x^{2n^2 - 2n - 9} + (2n-7)x^{2n^2 - 2n - 14} \\
& + (n+1)\, x^{2n^2 - 2n - \frac{n^2 - 5n + 4}{2}} + 2n\, x^{n(3n-1)} \\
& + (n-1)x^{\sum_{i=1}^{n-1}(2n-i)} + (n-2)\, x^{\sum_{i=1}^{n-2}(2n-i)} + (n-3)\, x^{\sum_{i=1}^{n-3}(2n-i)} \\
& + \ldots + 3x^{\sum_{i=1}^{3}(2n-i)} + 3x^{\sum_{i=1}^{2}(2n-i)} + x^{2n-1}
\end{aligned}$$

Result: Adjacent vertex sum polynomial of antiregular graph A_{2n+1}

Vertex polynomial of antiregular graph A_{2n+1} is

$$V(A_{2n+1}; x) = x^{2n} + x^{2n-1} + x^{2n-2} + \ldots + x^{n+1} + 2x^n + x^{n-1} + x^{n-2} + \ldots + x^3 + x^2 + x \qquad \forall\ n = 1, 2, 3 \ldots [1]$$

Similar to previous arguement adjacent vertex sum polynomial of A_{2n+1} is

$$S(A_{2n+1}; x) = 2nx^{(2n-1)+(2n-2)+\ldots+(n+1)+2n+(n-1)+\ldots+3+2+1}$$
$$+ (2n-1)\,x^{2n+(2n-2)+(2n-3)+\ldots+(n+1)+2n+(n-1)+(n-2)+\ldots+3+2}$$
$$+ (2n-2)\,x^{2n+(2n-1)+(2n-3)+\ldots+(n+1)+2n+(n-1)+(n-2)+\ldots+4+3}$$
$$+ (2n-3)\,x^{2n+(2n-1)+(2n-2)+(2n-4)+\ldots+(n+1)+2n+(n-1)+(n-2)+\ldots+5+4}$$
$$+ \ldots + (n+1)\,x^{2n+(2n-1)+(2n-2)+\ldots+(n+3)+(n+2)+2n}$$
$$+ 2n\,x^{2\times 2n+2(2n-1)+2(2n-2)+\ldots+2(n+2)+2(n+1)}$$
$$+(n-1)\,x^{2n+(2n-1)+(2n-2)+\ldots+(n+3)+(n+2)}$$
$$+ \ldots + 3x^{2n+(2n-1)+(2n-2)} + 2x^{2n+(2n-1)} + x^{2n}.$$

$$\therefore\ S(A_{2n+1}; x) = 2n\,x^{2n^2} + (2n-1)\,x^{2n^2} + (2n-2)\,x^{2n^2-1} + (2n-3)\,x^{2n^2-3}$$
$$+ (2n-4)\,x^{2n^2-6} + (2n-5)x^{2n^2-10} + (2n-6)x^{2n^2-15}$$
$$+ (n+1)\,x^{2n^2-\frac{n^2-3n+2}{2}} + 2n\,x^{n(3n+1)}$$
$$+ (n-1)\,x^{\sum_{i=0}^{n-2}(2n-i)} + (n-2)\,x^{\sum_{i=0}^{n-3}(2n-i)} + (n-3)\,x^{\sum_{i=0}^{n-4}(2n-i)}$$
$$+ \ldots + 3\,x^{\sum_{i=0}^{2}(2n-i)} + 2\,x^{\sum_{i=0}^{1}(2n-i)} + x^{2n}.$$

Result: Adjacent vertex sum polynomial of Gear graph

Vertex polynomial of Gear graph G_n is

$$V(G_n ; x) = x^{n-1} + (n-1)\,x^3 + (n-1)\,x^2 \qquad \forall\ n = 2, 3, \ldots \text{ [1]}$$

For Gear graph G_n has $2n - 1$ vertices and $3(n - 1)$ edges. In this graph one vertex has $n - 1$ vertices are adjacent. Therefore the sum of the number of adjacent vertices of the above vertex is $n - 1$ and the sum of the degree of adjacent vertices of the above vertex is $3(n - 1)$. For G_n the sum of the number of adjacent vertices of all the vertices of degree 3 is $3(n - 1)$ and the sum of the degree of adjacent vertices of all the vertices of degree 3 is $(n - 1)(n + 3)$. For G_n the sum of the number of adjacent vertices of all the vertices of degree 2 is $2(n - 1)$ and the sum of the degree of adjacent vertices of all the vertices of degree 2 is $6(n - 1)$. Hence adjacent vertex sum polynomial of gear graph is $S(G_n, x) = (n-1)\,x^{3(n-1)} + 3(n-1)\,x^{(n-1)(n+3)} + 2(n-1)\,x^{6(n-1)} \quad \forall\ n = 2, 3, \ldots$

Result: Adjacent vertex sum polynomial of Barbell graph

Vertex polynomial of Barbell graph B_n is $V(B_n ; x) = 2[x^n + (n-1)\,x^{n-1}] \quad \forall\ n = 1, 2, 3, \ldots$ [1]

For Barbell graph B_n has $2n$ vertices and $n^2 - n + 1$ edges. In this graph each vertex of degree n is adjacent to n vertices. Therefore the sum of the number of adjacent vertices of all the vertices of degree n is 2n and the sum of the degree of adjacent vertices of all the vertices of the degree n is $2(n^2 - n + 1)$. For B_n the sum of the number of adjacent vertices of all the vertices of degree $n - 1$ is $2(n-1)^2$ and the sum of the degree of adjacent vertices of all the vertices of degree $n - 1$ is $2(n - 1)[n + (n - 1)(n - 2)]$. Hence adjacent vertex sum polynomial of Barbell graph is

$$S(B_n, x) = 2nx^{2(n^2-n+1)} + 2(n-1)^2\,x^{2(n-1)[n+(n-1)(n-2)]} \qquad \forall\ n = 1, 2, 3, \ldots$$

Result: Adjacent vertex sum polynomial of Book graph.

Vertex polynomial of Book graph B'_n is

$$V(B'_n ; x) = 2x^{n+1} + 2nx^2 \quad \forall\ n = 1, 2, 3, \ldots \text{ [1]}$$

For Book graph B'_n has $2(n+1)$ vertices and $3n + 1$ edges. In this graph 2 vertices have degree $n + 1$. Therefore the sum of the number of adjacent vertices of the above two vertices is $2(n+1)$ and the sum of the degree of adjacent vertices of the above two vertices is $2(3n+1)$. For B'_n the sum of the number of adjacent vertices of all the vertices of degree 2 is 4n and the sum of the degree of adjacent vertices of all the vertices of degree 2 is $2n(n+3)$. Hence adjacent vertex sum polynomial of Book graph is

$$S(B'_n ; x) = 2(n+1)\,x^{2(3n+1)} + 4nx^{2n(n+3)} \qquad \forall\ n = 2, 3, 4, \ldots$$

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