International Journal of Mathematical Archive-6(6), 2015, 119-122
IJMA Available online through www.ijma.info ISSN 2229 – 5046

NUMERICAL STUDY OF UNSTEADY MHD HELE-SHAW FLOW AND HEAT TRANSFER THROUGH AN INCLINED CHANNEL WITH MOVING WALLS

DIPJYOTI SARMA*

Department of Mathematics, Dibrugarh University, Dibrugarh-786004, Assam, India.

(Received On: 06-05-15; Revised & Accepted On: 23-06-15)

ABSTRACT

A numerical solution for unsteady MHD Hele-Shaw flow of Rivlin-Ericksen fluid through an inclined channel having parallel walls moving in opposite direction has been discussed. The numerical solutions for velocity and temperature distribution have been obtained by using a central difference scheme and are presented graphically for various parametric conditions.

Keywords: *MHD flow, Heat transfer, Rivlin-Ericksen fluid, Hartmann number.*

1. INTRODUCTION

During the last few decades, much interest has been given by the investigators on the Rivlin-Ericksen fluid flows through channels, in presence of transverse magnetic field. These studies are mostly concerned with industrial importances.

Singh and Singh [1] studied the MHD flow of conducting second order Rivlin-Ericksen fluid in a porous channel having parallel walls inclined to the horizon under the action of uniform transverse magnetic field. The Saffman Taylor instability of a non-Newtonian fluid in a Hele-Shaw cell was studied by Kondic *et al.* [2]. Kelly and Hinch [3] numerically studied the Hele-Shaw flows driven by a quadruple by implementing a boundary integral algorithm. Bodosa and Borkakoti [4] studied MHD flow of Rivlin-Ericksen fluid along with heat transfer in an inclined channel with heat sources or sinks.

In a recent investigation Saxena and Dubey [5] carried out analytical study to investigate MHD Hele-Shaw flow of a Rivlin-Ericksen fluid through an inclined channel whose walls are moving in opposite direction under influence of magnetic field by applying perturbation technique.

In this paper numerical investigations have been made to study MHD Hele-Shaw flow of a Rivlin-Ericksen fluid through an inclined channel whose walls are moving in opposite direction under influence of magnetic field and compare the results with the analytical result obtained by Saxena and Dubey [5].

2. FORMULATION OF THE PROBLEM

In this problem, an unsteady motion of electrically conducting Rivlin - Ericksen fluid is considered in an inclined channel between two horizontal parallel walls separated by a distance of $2a$ under the action of an uniform external magnetic field of strength B_0. The walls of the channel are at $y = \pm a$ and are moving with same velocity U but in opposite direction.

Following assumptions are made in this study to solve the governing equations of the flow problem.

1) Flow is fully developed and
2) Viscous dissipation and Joule heat are neglected.
3) Velocities of both walls decrease exponentially with time.
4) At $t > 0$, temperature of the upper wall changes to $\frac{\partial T}{\partial y} = 0$ and the lower wall according to the relation $T = T_0 + (T_w - T_0)e^{-nt}$, where T_w and T_0 denotes the temperature of the walls.

*Corresponding Author: Dipjyoti Sarma**
Department of Mathematics, Dibrugarh University, Dibrugarh-786004, Assam, India.

The non-dimensionalized form of governing equation of the present channel flow problem are [5]

$$\frac{\partial V}{\partial x}=0 \tag{1.1}$$

$$\frac{\partial V}{\partial t}=-\frac{\partial p}{\partial x}+\frac{\partial^2 V}{\partial x^2}+R_c\frac{\partial^3 V}{\partial t\partial y^2}-MV+\frac{\operatorname{Sin}\theta}{F_r R_e}+G_r T \tag{1.2}$$

$$\frac{\partial T}{\partial t}=\frac{1}{P_r}\frac{\partial^2 T}{\partial y^2}+ST \tag{1.3}$$

where $R_c, M, F_r, R_e, G_r, P_r$ and S denotes elastic parameter, magnetic field parameter, Froude number, Reynolds number, Grashof number, Prandtl's number and source parameter respectively.

The nondimensional boundary conditions are: (when $t>0$)

$$\left.\begin{array}{ll} V=-e^{-nt}, T=e^{-nt} & \text{at } y=-1 \\ V=e^{-nt}, \dfrac{\partial T}{\partial y}=0 & \text{at } y=-1 \end{array}\right\} \tag{1.4}$$

We use separation of variable technique to solve Eqs. (1.1) - (1.3). Let us consider

$$\begin{aligned} V&=f(y)e^{-nt} \\ T&=g(y)e^{-nt} \\ h&=h_0(y)e^{-nt} \end{aligned} \tag{1.5}$$

where n is the decay constant. Substituting (1.5) in Eqs. (1.1) - (1.3), we get

$$c_2\frac{d^2 f}{dy^2}+c_3 f=c_4+c_5 g=0 \tag{1.6}$$

$$\frac{d^2 g}{dy^2}+c_1 g=0 \tag{1.7}$$

Where,

$c_1=P_r(S+n), c_2=(1-nR_c), c_3=n-M,$

$c_4=-h_0-\dfrac{\operatorname{Sin}\theta e^{nt}}{F_r R_e}, c_5=-G_r$

and the boundary condition (1.4) becomes

$$\left.\begin{array}{l} f=-1, g=1 \text{ at } y=+1 \\ f=1, g'=0 \text{ at } y=-1 \end{array}\right\} \tag{1.8}$$

We express Eq.(1.6) and Eq.(1.7) in finite difference equations by utilizing central difference scheme with equal spacing and finally obtained following finite difference representations

$$f_i=s_1(f_{i+1}+f_{i-1})+s_2+s_3 g_i \tag{1.9}$$

$$g_i=s_4(g_{i+1}+g_{i-1}) \tag{1.10}$$

Where,

$$s_1=\frac{c_2}{2c_2-c_3h^2}, s_2=\frac{-c_4h^2}{2c_2-c_3h^2}, s_3=\frac{-c_5h^2}{2c_2-c_3h^2}, s_4=\frac{1}{2-c_1h^2}$$

The numerical boundary conditions for the present channel flow problem are:

$f_1=-1, f_{n+1}=1, g_1=1, g_{n+1}=g_{n-1}$

3. RESULTS AND DISCUSSION

In our numerical investigations, computation is carried out for different values of magnetic parameter M, source parameter S, elastic parameter R_c and Froude number F_r respectively and we have considered $M=1.5, S=0.05, R_c=0.1, F_r=3.0, h_0=1.0, n=1.0, t=1.0, \theta=30^0, P_r=0.5, G_r=5.0, R_e=1.0$ [5] unless otherwise stated in computation.

In figure 1, velocity profile for values of magnetic parameter is shown. It is observed that with the increasing value of M, fluid velocity decreases. Again from figure 2, we can conclude that velocity decreases whenever source parameter S increases.

In figure 3, velocity profile for values of elastic parameter R_c is shown. It is observed that with the increasing value of R_c, fluid velocity increases. Again from figure 4, we can conclude that velocity decreases whenever Froude number F_r increases.

In figure 5, temperature distribution is plotted against P_r, where it is observed that temperature distribution between the walls gradually increases along with the increasing values of P_r. Again from figure 6, we can conclude that temperature decreases whenever source parameter S increases.

The results reflected from these plots are in good agreement with the earlier analytical results obtained by Saxena and Dubey [5] for the inclined channel flow study with both walls moving in opposite direction except the result reflected in figure 2.

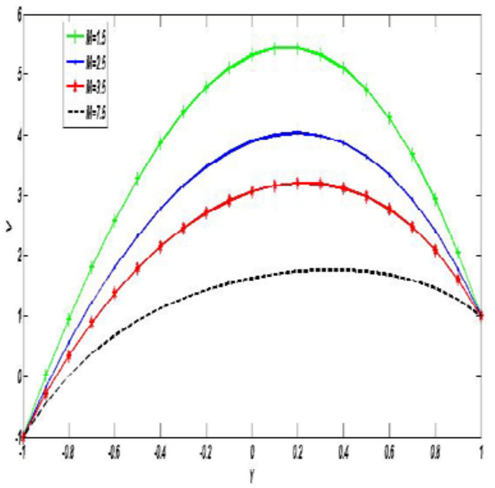

Fig. 1: Velocity profile for different values of M

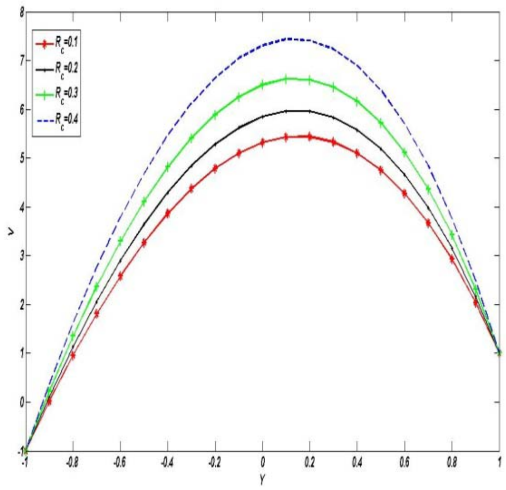

Fig. 3: Velocity profile for different values of R_e

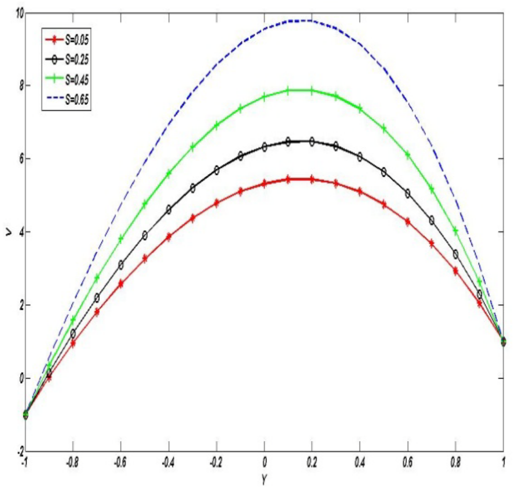

Fig. 2: Velocity profile for different values of S

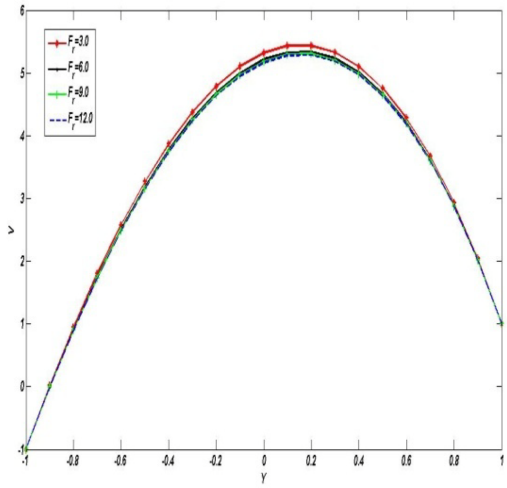

Fig. 4: Velocity profile for different values of F_r

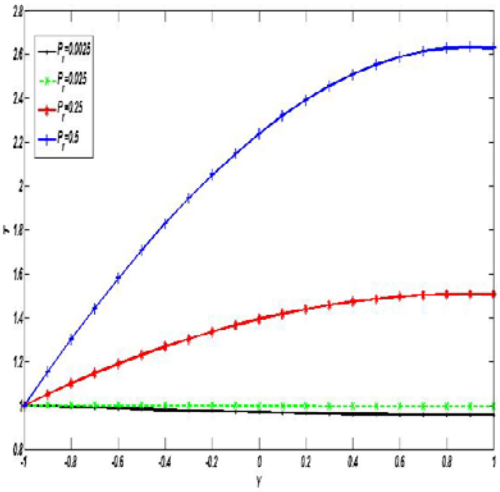

Fig. 5: Temperature profile for different values of P_r

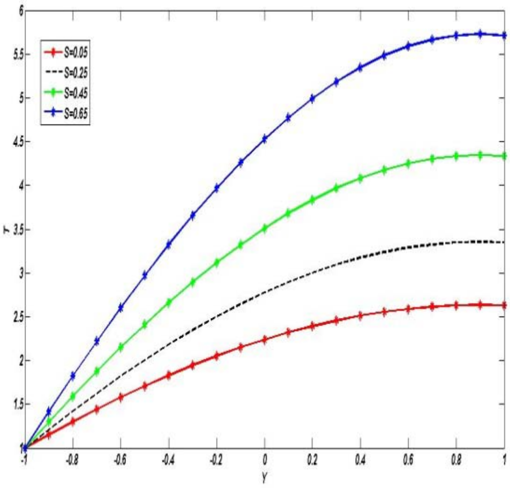

Fig. 6: Temperature profile for different values of S

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Source of support: Nil, Conflict of interest: None Declared