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SOMEWHAT *CLOSED FUNCTIONS [where * = r-; semi-; pre-; α-; β-; rα-; b-; γ-]

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ABSTRACT

*In this paper author tried to introduce a new variety of closed functions called Somewhat *closed functions and Almost somewhat *closed functions. Its basic properties are discussed.*

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Keywords: *Somewhat continuous functions and Somewhat closed functions, Almost somewhat closed functions.*

1. INTRODUCTION

b-open sets are introduced by Andrijevic in 1996. K. R. Gentry introduced somewhat continuous functions in the year 1971. V. K. Sharma and the present authors of this paper defined and studied basic properties of *v*-open sets and *v*-continuous functions in the year 2006 and 2010 respectively. T. Noiri and N. Rajesh introduced somewhat b-continuous functions in the year 2011. In the 2014 year author of this paper defined somewhat closed functions, following him in the same year S. Balasubramanian, C. Sandhya and P. A. S. Vyjayanthi introduced somewhat *v*-closed functions,. Inspired with these developments we introduce in this paper somewhat *-closed functions [where * = r-; semi-; pre-; α-; β-; rα-; b-; γ-] functions, and study their basic properties and interrelation with other type of such functions available in the literature. Throughout the paper (X, τ) and (Y, σ) (or simply X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned and for $A\subset(X; \tau)$, $cl(A)$ and A^o denote the closure of A and the interior of A in X, respectively.

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2. PRELIMINARIES

Definition 2.1: A function f is said to be

(i) somewhat continuous [resp: somewhat b-continuous] if for $U\in\sigma$ and $f^{-1}(U) \neq \varphi$, there exists an open [resp: b-open] set V in X such that $V\neq \varphi$ and $V\subset f^{-1}(U)$.

(ii) somewhat open [resp: somewhat b-open] provided that if $U\in\tau$ and $U\neq \varphi$, then there exists a open [resp: b-open] set V in Y such that $V \neq \varphi$ and $V\subset f(U)$.

(iii) somewhat closed[resp: somewhat *v*-closed; somewhat rg-closed; somewhat αg-closed] provided that if $U\in C(\tau)$ and $U\neq \varphi$, then there exists a non-empty closed[resp: *v*-closed; rg-closed; αg-closed] set V in Y such that $f(U) \subset V$.

Definition 2.2: If X is a set and τ and σ are topologies on X, then τ is said to be equivalent [resp: b-equivalent] to σ provided if $U\in\tau$ and $U \neq \varphi$, then there is an open[resp: b-open] set V in *X* such that $V \neq \varphi$ and $V \subset U$ and if $U\in\sigma$ and $U \neq \varphi$, then there is an open[resp: b-open] set V in (X, τ) such that $V\neq \varphi$ and $U \supset V$.

Definition 2.3: $A\subset X$ is said to be dense in X if there is no proper closed set C in X such that $M\subset C\subset X$.

Now, consider the identity function f and assume that τ and σ are *v*-equivalent. Then f and f^{-1} are somewhat continuous. Conversely, if the identity function f is somewhat continuous in both directions, then τ and σ are equivalent.

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3. SOMEWHAT * CLOSED FUNCTION [where * = r-; semi-; pre-; α-; β-; rα-; b-; γ-] CLOSED

Definition 3.1: A function f is said to be somewhat * closed provided that if $U\in C(\tau)$ and $U\neq \varphi$, then there exists a non-empty * closed set V in Y such that $f(U) \subset V$.

Example 1: Let X = {a, b, c}, τ = {φ, {a}, {a, b}, X} and σ = {φ, {a}, {b}, {a, b}, X}. The function f: (X, τ)→ (X, σ) defined by f(a) = b, f(b) = c and f(c) = a is somewhat * closed and somewhat closed.

Example 2: Let X = {a, b, c}, τ = {φ, {a}, {b}, {a, b}, X} and σ = {φ, {a}, {a, b}, X}. The function f: (X, τ)→ (X, σ) defined by f(a) = a, f(b) = c and f(c) = b is not somewhat * closed

Example 3: In Example 2, the function f: (X, τ)→ (X, σ) defined by f(a) = a, f(b) = c and f(c) = b is somewhat rα-closed but not somewhat r-closed; somewhat closed; somewhat semi-closed; somewhat pre-closed; somewhat α-closed; somewhat β-closed.

Example 4: Let X = {a, b, c}, τ = {φ, {a}, {a, b}, X} and σ = {φ, {a}, {b}, {a, b}, X}. The function f: (X, τ)→ (X, σ) defined by f(a) = c, f(b) = a and f(c) = b is somewhat β-closed but not somewhat r-closed; somewhat closed; somewhat semi-closed; somewhat pre-closed; somewhat α-closed; somewhat rα-closed.

Note 1: From definition 3.1 we have the following interrelations among the functions.

$$\begin{array}{lllllll}
\text{swt.r}\alpha\text{.closed} & & & & & & \\
\uparrow & & & & & & \\
\text{swt.r.closed} \rightarrow & \text{swt.}\pi\text{.closed} \rightarrow & \text{swt.closed} \rightarrow & \text{swt.}\alpha\text{.closed} \rightarrow & \text{swt.}s\text{.closed} \rightarrow & \text{swt.}\beta\text{.closed} \\
 & & \downarrow & & & \\
 & & \text{swt.p.closed} & & &
\end{array}$$

Theorem 3.1: Let f be a closed function and g somewhat-*-closed. Then $g\bullet f$ is somewhat-*-closed.

Proof: Let A be closed in X, then f(A) is closed in Y, Since g is somewhat-*-closed, there exists a *-closed set $V \neq \phi$ in Z such that $g(f(A))\subseteq V$. Thus for A closed in X and $A\neq \phi$, there exists a *-closed set $V \neq \phi$ in Z such that $g(f(A))\subseteq V$. Therefore $g\bullet f$ is almost somewhat-*-closed.

Theorem 3.2: For a bijective function f, the following are equivalent:

(i) f is somewhat * closed.

(ii) If $C\in\tau$, such that $f(C) \neq Y$, then there is a $D\in {}^*O(Y)$ such that $D \neq Y$ and $D\subset f(C)$.

Proof:

(i) ⇒(ii): Let $C\in\tau$ such that $f(C) \neq Y$. Then X-C is closed and X-C ≠ φ. Since f is somewhat * closed, there exists $V\neq \varphi\in {}^*C(Y)$ such that $f(X-C) \subset V$. Put D = Y-V. Clearly $D\in {}^*O(Y)$ and we claim D ≠ Y. If D = Y, then V = φ, which is a contradiction. Since $f(X-C) \subset V$, $D = Y-V \subset (Y- f(X-C)) = f(C)$.

(ii) ⇒(i): Let $U\neq\phi$ be any closed set in X. Then $C = X-U\in\tau$ and $f(X-U) = f(C) = Y- f(U)$ implies $f(C) \neq Y$. Therefore, by (ii), there is a $D\in {}^*O(Y)$ such that $D\neq Y$ and $D \subset f(C)$. Clearly $V = Y-D \neq \varphi\in {}^*C(Y)$. Also, $V = Y-D \supseteq Y- f(C) = Y- f(X-U) = f(U)$.

Theorem 3.3: The following statements are equivalent:

(i) f is somewhat * closed.

(ii) (ii)If A is a *-dense subset of Y, then $f^{-1}(A)$ is a *-dense subset of X.

Proof:

(i) ⇒(ii): Suppose A is a *-dense set in Y. If $f^{-1}(A)$ is not *-dense in X, then there exists a closed set B in X such that $f^{-1}(A) \subset B\subset X$. Since f is somewhat * closed and $X-B\in {}^*O(X)$, there exists a $C\neq \varphi\in {}^*O(Y)$ such that $C\subset f(X-B)$. Therefore, $C\subset f(X-B) \subset f(f^{-1}(Y-A)) \subset Y-A$. That is, $A\subset Y-C \subset Y$. Now, $Y-C\neq \varphi\in {}^*C(Y)$ and $A\subset Y-C \subset Y$. This implies that A is not a *-dense set in Y, which is a contradiction. Therefore, $f^{-1}(A)$ is a *-dense set in X.

(ii) ⇒(i): Suppose $A\neq \varphi\in {}^*O(X)$. We want to show that ${}^*(f(A))^o\neq \varphi$. Suppose ${}^*(f(A))^o = \varphi$. Then, ${}^*Cl(f(A)) = Y$. Therefore, by (ii), $f^{-1}(Y - f(A))$ is *-dense in X. But $f^{-1}(Y- f(A)) \subset X-A$. Now, $X-A\neq \varphi\in {}^*C(X)$. Therefore, $f^{-1}(Y- f(A)) \subset X-A$ gives $X = {}^*Cl(f^{-1}(Y- f(A))) \subset X-A$. This implies that A = φ, which is contrary to A≠ φ. Therefore, ${}^*(f(A))^o\neq \varphi$. Hence f is somewhat * closed.

Theorem 3.4: Let f be somewhat * closed and $A\in RC(X)$. Then $f_{/A}$ is somewhat * closed.

Proof: Let $U\neq \varphi$ be closed in τ_A. Since U is closed in A and A is closed in X, U is closed in X and since f is somewhat * closed, there exists a $V\in {}^*C(Y)$, such that $f(U) \subset V$. Thus, for any closed set $U\neq \varphi$ in τ_A, there exists a $V\in {}^*C(Y)$ such that $f(U) \subset V$ which implies $f_{/A}$ is somewhat * closed.

Theorem 3.5: Let f be a function and $X = A\cup B$, where $A,B\in RC(X)$. If the restriction functions $f_{/A}$ and $f_{/B}$ are somewhat * closed, then f is somewhat * closed.

Proof: Let $U\neq \varphi$ be closed in X. Since $X = A\cup B$, either $A\cap U \neq \varphi$ or $B\cap U \neq \varphi$ or both $A\cap U \neq \varphi$ and $B\cap U \neq \varphi$. Since U is closed in X, U is closed in both A and B.

Case-(i): If $A\cap U\neq\varphi$ is closed in A. Since $f_{/A}$ is somewhat * closed, there exists $V\in {}^*C(Y)$ such that $f(U\cap A) \subset f(U) \subset V$, which implies that f is somewhat * closed.

Case-(ii): If $B\cap U \neq \varphi$ is closed in B. Since $f_{/B}$ is somewhat * closed, there exists $V\in {}^*C(Y)$ such that $f(U\cap B) \subset f(U) \subset V$, which implies that f is somewhat * closed.

Case-(iii): If both $A\cap U \neq \varphi$ and $B\cap U \neq \varphi$. Then by case (i) and (ii) f is somewhat * closed.

Remark 1: Two topologies τ and σ for X are said to be *-equivalent if and only if the identity function f: $(X, \tau)\rightarrow (Y, \sigma)$ is somewhat * closed in both directions.

Theorem 3.6: Let f: $(X, \tau)\rightarrow (Y, \sigma)$ be somewhat * closed function. Let $\tau^\#$ and $\sigma^\#$ be topologies for X and Y, respectively such that $\tau^\#$ is $^{*-}$equivalent to τ and $\sigma^\#$ is *-equivalent to σ. Then f: $(X; \tau^\#)\rightarrow (Y; \sigma^\#)$ is somewhat * closed.

4. ALMOST SOMEWHAT * CLOSED FUNCTION [where * = r-; semi-; pre-; α-; β-; rα-; b-; γ-] CLOSED

Definition 4.1: A function f is said to be almost somewhat * closed provided that if $U\in RC(\tau)$ and $U\neq \varphi$, then there exists a non-empty * closed set V in Y such that $f(U) \subset V$.

Example 5: Let X = {a, b, c}, τ = {φ, {a}, {b, c}, X} and σ = {φ, {a}, {b}, {a, b}, X}. The function f: $(X, \tau)\rightarrow (X, \sigma)$ defined by $f(a) = b$, $f(b) = c$ and $f(c) = a$ is almost *closed and almost somewhat closed.

Example 6: Let X = {a, b, c}, τ = {φ, {a}, {b}, {a, b}, X} and σ = {φ, {a}, {a, b}, X}. The function f: $(X, \tau)\rightarrow (X, \sigma)$ defined by $f(a) = a$, $f(b) = c$ and $f(c) = b$ is not almost somewhat *closed

Note 2: From definition 4.1 we have the following interrelations among the functions.

al.swt.rα.closed
$\uparrow$
al.swt.r.closed $\rightarrow$ al.swt.π.closed $\rightarrow$ al.swt.closed $\rightarrow$ al.swt.α.closed $\rightarrow$ al.swt.*s*.closed $\rightarrow$ al.swt.β.closed
$\downarrow$
al.swt.p.closed

Theorem 4.1: Let f be an r-closed function and g somewhat * closed. Then $g\bullet f$ is almost somewhat * closed.

Proof: Let A be r-closed in X, then $f(A)$ is closed in Y, Since g is somewhat *-closed, there exists a *-closed set $V \neq \phi$ in Z such that $g(f(A))\subseteq V$. Thus for A r-closed in X and $A\neq \phi$, there exists a *-closed set $V \neq \phi$ in Z such that $g(f(A))\subseteq V$. Therefore $g\bullet f$ is almost somewhat -*-closed.

Theorem 4.2: For a bijective function f, the following are equivalent:

(i) f is almost somewhat * closed.

(ii) If $C\in RO(X)$, such that $f(C) \neq Y$, then there is a $D\in {}^*O(Y)$ such that $D \neq Y$ and $D\subset f(C)$.

Proof:

(i) $\Rightarrow$(ii): Let $C\in RO(X)$ such that $f(C) \neq Y$. Then $X\text{-}C\in RC(X)$ and $X\text{-}C \neq \varphi$. Since f is almost somewhat * closed, there exists $V\neq \varphi\in {}^*C(Y)$ such that $f(X\text{-}C) \subset V$. Put $D = Y\text{-}V$. Clearly $D\in {}^*O(Y)$ and we claim $D \neq Y$. If $D = Y$, then $V = \varphi$, which is a contradiction. Since $f(X\text{-}C) \subset V$, $D = Y\text{-}V \subset (Y\text{-} f(X\text{-}C)) = f(C)$.

(ii) $\Rightarrow$(i): Let $U\neq\phi\in RC(X)$. Then $C = X\text{-}U\in RO(X)$ and $f(X\text{-}U) = f(C) = Y\text{-} f(U)$ implies $f(C) \neq Y$. Therefore, by (ii), there is a $D\in {}^*O(Y)$ such that $D\neq Y$ and $D \subset f(C)$. Clearly $V = Y\text{-}D \neq \varphi\in {}^*C(Y)$.

Also, V = Y-D $\supseteq$ Y- f(C) = Y- f(X-U) = f(U).

Theorem 4.3: The following statements are equivalent:

(i) f is almost somewhat * closed.

(ii) (ii)If A is a *-dense subset of Y, then f^{-1}(A) is a *-dense subset of X.

Proof:

(i) ⇒(ii): Suppose A is a *-dense set in Y. If f^{-1}(A) is not *-dense in X, then there exists a B$\in{}^*$C(X) such that f^{-1}(A) $\subset$ B$\subset$ X. Since f is almost somewhat * closed and X-B$\in{}^*$O(X), there exists a C$\neq\varphi\in{}^*$O(Y) such that C$\subset f$(X-B). Therefore, C$\subset f$(X-B) $\subset f(f^{-1}$(Y-A)) $\subset$ Y-A. That is, A$\subset$ Y-C $\subset$ Y.

Now, Y-C$\neq\varphi\in{}^*$C(Y) and A$\subset$ Y-C $\subset$ Y. This implies that A is not a *-dense set in Y, which is a contradiction. Therefore, f^{-1}(A) is a *-dense set in X.

(ii) ⇒(i): Suppose A$\neq\varphi\in{}^*$O(X). We want to show that $^*(f(A))^o\neq\varphi$. Suppose $^*(f(A))^o=\varphi$. Then, $^*Cl(f(A))$ = Y. Therefore, by (ii), f^{-1}(Y - f(A)) is *-dense in X. But f^{-1}(Y- f(A)) $\subset$ X-A. Now, X-A$\neq\varphi\in{}^*$C(X). Therefore, f^{-1}(Y- f(A)) $\subset$ X-A gives X = $^*Cl(f^{-1}$(Y- f(A))) $\subset$ X-A. This implies that A = φ, which is contrary to A$\neq\varphi$. Therefore, $^*(f(A))^o\neq\varphi$. Hence f is almost somewhat * closed.

Theorem 4.4: Let f be almost somewhat * closed and A$\in$RC(X). Then $f_{/A}$ is almost somewhat * closed.

Proof: Let U$\neq\varphi\in$RC($\tau_{/A}$). Since U$\in$RC(A) and A$\in$RC(X), U$\in$RC(X) and since f is almost somewhat * closed, there exists a V$\in{}^*$C(Y), such that f(U) $\subset$V. Thus, for any U$\neq\varphi\in$RC(τ_A), there exists a V$\in{}^*$C(Y) such that f(U) $\subset$V which implies $f_{/A}$ is almost somewhat * closed.

Theorem 4.5: Let f be a function and X = A$\cup$ B, where A,B$\in$RC(X). If the restriction functions $f_{/A}$ and $f_{/B}$ are almost somewhat * closed, then f is almost somewhat * closed.

Proof: Let U$\neq\varphi\in$RC(X). Since X = A$\cup$ B, either A$\cap$ U $\neq\varphi$ or B$\cap$ U $\neq\varphi$ or both A$\cap$U $\neq\varphi$ and B$\cap$U $\neq\varphi$. Since U$\in$RC(X), U$\in$RC(A) and U$\in$RC(B).

Case (i): If A$\cap$U$\neq\varphi\in$RC(A). Since $f_{/A}$ is almost somewhat * closed, there exists V$\in{}^*$C(Y) such that f(U$\cap$A) $\subset f$(U) $\subset$ V, which implies that f is almost somewhat * closed.

Case (ii): If B$\cap$U $\neq\varphi\in$RC(B). Since $f_{/B}$ is almost somewhat * closed, there exists V$\in{}^*$C(Y) such that f(U$\cap$B) $\subset f$(U) $\subset$ V, which implies that f is almost somewhat * closed.

Case (iii): If both A$\cap$U $\neq\varphi$ and B$\cap$U $\neq\varphi$. Then by case (i) and (ii) f is almost somewhat * closed.

Theorem 4.6: Let f: (X, τ)$\rightarrow$ (Y, σ) be almost somewhat * closed function. Let $\tau^\#$ and $\sigma^\#$ be topologies for X and Y, respectively such that $\tau^\#$ is *-equivalent to τ and $\sigma^\#$ is *-equivalent to σ. Then f: (X; $\tau^\#$)$\rightarrow$ (Y; $\sigma^\#$) is almost somewhat * closed.

5. SOMEWHAT M-* CLOSED FUNCTION [where * = r-; semi-; pre-; α-; β-; rα-; b-; γ-] CLOSED

Definition 5.1: A function f is said to be somewhat M-* closed provided that if U$\in{}^*$C(τ) and U$\neq\varphi$, then there exists a non-empty * closed set V in Y such that f(U) $\subset$V.

Example 7: Let X = {a, b, c}, τ = {φ, {a}, X} and σ = {φ, {a}, {b, c}, X}. The function f: (X, τ)$\rightarrow$ (X, σ) defined by f(a) = a, f(b) = c and f(c) = b is somewhat M-* closed, somewhat * closed and somewhat closed.

Example 8: Let X = {a, b, c, d}, τ = {φ, {a}, {a, b, c}, X} = σ. The function f: (X, τ)$\rightarrow$ (X, σ) defined by f(a) = d, f(b) = c, f(c) = b and f(d) = a is not somewhat M-* closed.

Theorem 5.1: Let f be an M-*-closed function and g somewhat M-*-closed. Then g• f is somewhat M-*-closed.

Proof: Let A be *-closed in X, then f(A) is *-closed in Y, Since g is somewhat M-*-closed, there exists a *-closed set V $\neq\phi$ in Z such that $g(f(A))\subseteq$ V. Thus for A *-closed in X and A$\neq\phi$, there exists a *-closed set V $\neq\phi$ in Z such that $g(f(A))\subseteq$ V. Therefore g•f is somewhat M-*-closed.

Theorem 5.2: For a bijective function f, the following are equivalent:
(i) f is somewhat M-* closed.
(ii) If $C\in {}^*O(X)$, such that $f(C) \neq Y$, then there is a $D\in {}^*O(Y)$ such that $D \neq Y$ and $D\subset f(C)$.

Proof:
(i) ⇒(ii): Let $C\in {}^*O(X)$ such that $f(C) \neq Y$. Then $X-C\in {}^*C(X)$ and $X-C \neq \varphi$. Since f is somewhat M-* closed, there exists $V\neq \varphi\in {}^*C(Y)$ such that $f(X-C) \subset V$. Put $D = Y-V$. Clearly $D\in {}^*O(Y)$ and we claim $D \neq Y$. If $D = Y$, then $V = \varphi$, which is a contradiction. Since $f(X-C) \subset V$, $D = Y-V \subset (Y- f(X-C)) = f(C)$.

(ii) ⇒(i): Let $U\neq\phi\in {}^*C(X)$. Then $C = X-U\in {}^*O(X)$ and $f(X-U) = f(C) = Y- f(U)$ implies $f(C) \neq Y$. Therefore, by (ii), there is a $D\in {}^*O(Y)$ such that $D\neq Y$ and $D \subset f(C)$.

Clearly $V = Y-D \neq \varphi\in {}^*C(Y)$. Also, $V = Y-D \supseteq Y- f(C) = Y- f(X-U) = f(U)$.

Theorem 5.3: The following statements are equivalent:
(i) f is somewhat M-* closed.
(ii) (ii)If A is a *-dense subset of Y, then $f^{-1}(A)$ is a *-dense subset of X.

Proof:
(i) ⇒(ii): Suppose A is a *-dense set in Y. If $f^{-1}(A)$ is not *-dense in X, then there exists a $B\in {}^*C(X)$ such that $f^{-1}(A) \subset B\subset X$. Since f is somewhat M-* closed and $X-B\in {}^*O(X)$, there exists a $C\neq \varphi\in {}^*O(Y)$ such that $C\subset f(X-B)$. Therefore, $C\subset f(X-B) \subset f(f^{-1}(Y-A)) \subset Y-A$. That is, $A\subset Y-C \subset Y$. Now, $Y-C\neq \varphi\in {}^*C(Y)$ and $A\subset Y-C \subset Y$. This implies that A is not a *-dense set in Y, which is a contradiction. Therefore, $f^{-1}(A)$ is a *-dense set in X.

(ii) ⇒(i): Suppose $A \neq \varphi\in {}^*O(X)$. We want to show that ${}^*(f(A))^o\neq \varphi$. Suppose ${}^*(f(A))^o = \varphi$. Then, ${}^*Cl(f(A)) = Y$. Therefore, by (ii), $f^{-1}(Y - f(A))$ is *-dense in X. But $f^{-1}(Y- f(A)) \subset X-A$. Now, $X-A\neq \varphi\in {}^*C(X)$. Therefore, $f^{-1}(Y- f(A)) \subset X-A$ gives $X = {}^*Cl(f^{-1}(Y- f(A))) \subset X-A$. This implies that $A = \varphi$, which is contrary to $A\neq \varphi$. Therefore, ${}^*(f(A))^o\neq \varphi$. Hence f is somewhat M-* closed.

Theorem 5.4: Let f be somewhat M-* closed and $A\in RC(X)$. Then $f_{/A}$ is somewhat M-* closed.

Proof: Let $U\neq \varphi\in {}^*C(\tau_{/A})$. Since $U\in {}^*C(A)$ and $A\in RC(X)$, $U\in {}^*C(X)$ and since f is somewhat M-* closed, there exists a $V\in {}^*C(Y)$, such that $f(U) \subset V$. Thus, for any $U\neq \varphi\in {}^*C(A)$, there exists a $V\in {}^*C(Y)$ such that $f(U) \subset V$ which implies $f_{/A}$ is somewhat M-* closed.

Theorem 5.5: Let f be a function and $X = A\cup B$, where $A, B\in RC(X)$. If the restriction functions $f_{/A}$ and $f_{/B}$ are somewhat M-* closed, then f is somewhat M-* closed.

Proof: Let $U\neq \varphi\in {}^*C(X)$. Since $X = A\cup B$, either $A\cap U \neq \varphi$ or $B\cap U \neq \varphi$ or both $A\cap U \neq \varphi$ and $B\cap U \neq \varphi$. Since $U\in {}^*C(X)$, $U\in {}^*C(A)$ and $U\in {}^*C(B)$.

Case (i): If $A\cap U\neq\varphi\in {}^*C(A)$. Since $f_{/A}$ is somewhat M-* closed, there exists $V\in {}^*C(Y)$ such that $f(U\cap A) \subset f(U) \subset V$, which implies that f is somewhat M-* closed.

Case (ii): If $B\cap U \neq \varphi\in {}^*C(B)$. Since $f_{/B}$ is somewhat M-* closed, there exists $V\in {}^*C(Y)$ such that $f(U\cap B) \subset f(U) \subset V$, which implies that f is somewhat M-* closed.

Case (iii): If both $A\cap U \neq \varphi$ and $B\cap U \neq \varphi$. Then by case (i) and (ii) f is somewhat M-* closed.

Theorem 5.6: Let f: $(X, \tau)\rightarrow (Y, \sigma)$ be somewhat M-* closed function. Let $\tau^\#$ and $\sigma^\#$ be topologies for X and Y, respectively such that $\tau^\#$ is $^{*-}$equivalent to τ and $\sigma^\#$ is *-equivalent to σ. Then f: $(X; \tau^\#)\rightarrow (Y; \sigma^\#)$ is somewhat M-* closed.

Conclusion: In this paper author defined somewhat * closed functions, almost somewhat * closed functions and somewhat M-* closed functions [where * = r-; semi-; pre-; α-; β-; rα-; b-; γ-], studied its properties.

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