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ON NANO GENERALISED CONTINUOUS FUNCTION IN NANO TOPOLOGICAL SPACE

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ABSTRACT

The purpose of this paper is to introduce a new class of continuous functions called Nano generalized continuous functions and to discuss some of its properties in terms of Ng –closed sets, Nano g-closure and Nano g- Interior.

Keywords: *Nano Topology, Ng-closed sets, Nano g-closure, Nano g-interior, Nano g- continuous Function, Nano g-closed map, Nano g- Homeomorphism.*

I. INTRODUCTION

Continuous functions is one of the main concepts of Topology. In 1991, Balachandran [1] *et.al*, introduced and studied the notions of generalized continuous functions. Different types of generalizations of continuous functions were studied by various author in the recent development of Topology. The notion of Nano topology was introduced by Lellis Thivagar [3] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined Nano closed sets, Nano-interior and Nano-closure. He has also defined Nano continuous functions, Nano open mapping, Nano closed mapping and Nano Homeomorphism. In [2] Bhuvaneswari *et.al*, introduced and studied some properties of Nano generalized closed sets in Nano topological spaces. In this paper we have introduced a new class of continuous functions called Nano generalized continuous functions and discuss some of its properties in terms of Ng –closed sets, Nano g-closure and Nano g- Interior.

II. PRELIMINARIES

Definition: 2.1[5] A subset A of (X, τ) is called a generalized closed set (briefly g-closed) if cl(A) ⊆ U whenever A ⊆ U and U is open in X.

Definition: 2.2 [1] A map f: (X, τ) → (Y, σ) is called g- continuous if $f^{-1}(V)$ is g- open in (X,τ) for every open set V in (Y, σ)

Definition:2.3 [3]Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X ⊆ U. Then,

(i) The lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and is denoted by $L_R(X)$. $L_R(X)=U\{R(x):R(x) \subseteq X, x\epsilon U\}$ where R(x) denotes the equivalence class determined by xϵU.
(ii) The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$. $U_R(X) = U\{R(x): R(x) \cap X\neq\Phi, x\epsilon U\}$
(iii) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not – X with respect to R and it is denoted by $B_R(X)$. $B_R(X) = U_R(X) - L_R(X)$.

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Property: 2.4 [3] If (U, R) is an approximation space and X, Y$\subseteq$U, then
1. $L_R(X) \subseteq X \subseteq U_R(X)$
2. $L_R(\Phi) = U_R(\Phi) = \Phi$
3. $L_R(U) = U_R(U) = U$
4. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
5. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
6. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
7. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
8. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
9. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
10. $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
11. $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$

Definition:2.5 [3] Let U be the universe , R be an equivalence relation on U and $\tau_R(X) = \{U, \Phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property 2.8, $\tau_R(X)$ satisfies the following axioms:

(i) U and $\Phi \in \tau_R(X)$.
(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. (U, $\tau_R(X)$) is called the Nano topological space. Elements of the Nano topology are known as Nano open sets in U. Elements of $[\tau_R(X)]^c$ are called Nano closed sets with $[\tau_R(X)]^c$ being called dual Nano topology of $\tau_R(X)$.

Remark:2.6 [3] If $\tau_R(X)$ is the Nano topology on U with respect to X, then the set $B = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition:2.7 [3]If (U, $\tau_R(X)$) is a Nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

(i) The Nano interior of the set A is defined as the union of all Nano open subsets contained in A and is denoted by NInt(A). NInt(A) is the largest Nano open subset of A.
(ii) The Nano closure of the set A is defined as the intersection of all Nano closed sets containing A and is denoted by NCl(A). NCl(A) is the smallest Nano closed set containing A.

Definition: 2.8 [2] A subset A of (U, $\tau_R(X)$) is called Nano generalized closed set (briefly Ng-closed) if $Ncl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in (U, $\tau_R(X)$).

Definition: 2.9 [4] Let (U, $\tau_R(X)$) and (V, $\tau_{R'}(Y)$) be two Nano topological spaces. Then a mapping f: (U, $\tau_R(X)$) $\rightarrow$ (V, $\tau_{R'}(Y)$) is Nano continuous on U if the inverse image of every Nano open set in V is Nano open in U.

3. NANO GENERALIZED CONTINUOUS FUNCTION

Definition: 3.1 Let (U, $\tau_R(X)$) be a Nano Topological space and $A \subseteq U$. Then A is said to be Nano g-open if its complement is Nano g-closed.

Definition: 3.2 Let (U, $\tau_R(X)$) and (V, $\tau_{R'}(Y)$) be two Nano topological spaces. Then a mapping f: (U, $\tau_R(X)$) $\rightarrow$ (V, $\tau_{R'}(Y)$) is Nano g-continuous on U if the inverse image of every Nano open set in V is Nano g- open in U.

Example: 3.3 Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X = {a, b}.

Then $\tau_R(X)$ = {U, Φ, {a}, {a, b, d}, {b, d}} which are open sets.

The Nano closed sets = {U, Φ, {b, c, d}, {c}, {a, c}}.

The Nano generalized closed sets are { Φ,U,{c}, {a, c}, {b, c}, {c, d}, {a, b, c}, {b, c, d},{a, c, d}} Nano g-open sets are {Φ, U, {a, b, d}, {b, d}, {a, d}, {a, b}, {d}, {a},{b}}

Let V= {x, y, z, w} with V/R = {{x} ,{y, z},{w}} and Y={x, z}

Then $\tau_R(Y)$ = {V, Φ, {x}, {x, y, z}, {y, z}} which are open sets.

Define f: U→V as f(a) = x, f(b) = y, f(c) = w, f(d) = z. Then $f^{-1}(x)$ = {a}, $f^{-1}(x,y,z)$ = {a,b,d}, $f^{-1}(y,z)$ = {b,d} and $f^{-1}(V)$ = U.

That is the inverse image of every Nano open set in V is Nano g- open in U. Therefore f is Nano g- continuous.

Definition: 3.4 For every set A⊆U, we define the Ng- closure of A to be the intersection of all Ng- closed sets containing A

In symbols Ng – Cl(A) = ∩{B: B is Nano generalized closed set and A⊆B}

Definition: 3.5 For every set A⊆U, we define the Ng- Interior of A to be the union of all Ng- closed sets contained in A

In symbols Ng – Int(A) = ∪{B: B is Nano generalized closed set and B⊆A}

Proposition: 3.6 For any A⊆ U,

(i) Ng Cl(A) is the smallest Ng closed set containing A.
(ii) A is Ng closed if and only if Ng Cl(A) = A
(iii) A ⊆ Ng Cl(A) ⊆ Cl(A)

Proposition: 3.7 For any two subsets A and B of U,

(i) If A⊆B, then Ng Cl(A) ⊆ Ng Cl(B)
(ii) Ng Cl(A∩B) ⊆ Ng Cl(A) ∩ Ng Cl(B)

Theorem: 3.8 A function f: (U, τ_R(X)) → (V, $\tau_{R'}$(Y)) is Nano g-continuous iff the inverse image of every Nano closed set in V is Nano g-closed in U.

Proof: Let f be Nano g- continuous and F be Nano closed in V. That is V-F is Nano open in V. Since f is Nano g-continuous, $f^{-1}(V-F)$ is Nano g-open in U. That is U- $f^{-1}(V-F)$ is Nano g- closed in U. That is U- $f^{-1}(F)$ is Nano g-open in U. Therfore $f^{-1}(F)$ is Nano g-closed in U. Thus the inverse image of every Nano closed set in V is Nano g-closed in U, if f is Nano g-continuous on U.

Conversely, let the inverse image of every Nano closed set in V is Nano g-closed in U. Let G be Nano open in V. Then V-G is Nano closed in V. Then $f^{-1}(V-G)$ is Nano g-closed in U. That is U-$f^{-1}(G)$ is Nano g-closed in U. Therefore $f^{-1}(G)$ is Nano g-open in U. By definition, f is Nano g-continuous .

Theorem: 3.9 Let U and V be any two Nano topological spaces. Let f: (U, τ_R(X)) → (V, $\tau_{R'}$(Y)) be Nano continuous function. Then f is Nano g-continuous.

Proof: Let B be any Nano closed set in (V, $\tau_{R'}$(Y)). Since f is Nano continuous , $f^{-1}(B)$ is Nano closed in U. We know that every Nano closed set is Nano generalized closed set. So $f^{-1}(B)$ is Nano generalized closed in U. Therefore f is Nano g- continuous.

Remark: 3.10 The converse of the above theorem is true as seen from the example 3.
f^{-1}({y, z, w}) = {b, c, d}, f^{-1}({w}) = {c}, f^{-1}({x, w}) = {a, c} which are Nano closed sets in U.

Definition: 3.11 Let x be a point of (U, τ_R(X)) and P be a subset of U. Then P is called Ng- neighborhood of x in (U, τ_R(X)), if there exist a Ng open set Q of (U, τ_R(X)) such that x ϵ Q ⊆ P.

Theorem: 3.12 Let A be a subset of (U, τ_R(X)). Then xϵ Ng Cl(A) if and only if for any Ng-neighborhood W_x of x in (U, τ_R(X)), (A ∩ W_x) ≠ ∅.

Proof: Necessity: Assume that x ϵ Ng -Cl(A). Suppose that there exist a Ng – neighborhood W_x of x such that (A ∩ W_x) ≠ ∅ . Since W_x is a Ng- neighborhood of x, by definition there exist a Ng open set Q such that x ϵ Q ⊆ W_x.

Since W_x is such that (A ∩ W_x) = ∅, we have A∩Q = ∅ which implies A ⊆ Q^c. Since Q^c is a Ng closed set containing A, we have Ngcl(A) ⊆ Q^c and therefore x does not belong to Ng Cl(A), which is a contradiction.

Sufficiency: Assume that for each Ng neighborhood, W_x of x, (A ∩ W_x)≠ ∅. Suppose x does not belongs to Ng Cl(A). Then there exists a Ng closed set F of (U, τ_R(X)), such that A⊆ F and x does not belong to F. Thus x ϵ F^c and F^c is Ng open set in (U, τ_R(X)). But A ∩ F^c = ∅ which is a contradiction. Thus x ϵ Ng Cl(A).

Theorem: 3.13 Let f: (U, τ_R(X)) → (V, $\tau_{R'}$(Y)) be a function. Then the following are equivalent.

(i) The function is Ng – Continuous
(ii) The inverse of each Nano open set in (V, $\tau_{R'}$(Y)) is Ng-open in (U, τ_R(X)).
(iii) For each x in U, the inverse of every neighborhood of f(x) is a Ng neighborhood of x.
(iv) For each x in U and each neighborhood W of f(x), there is a Ng- neighborhood M of x such that f(M) ⊆ W.

(v) For each subset A of (U, $\tau_R(X)$), f(Ng Cl(A)) ⊆ NCl(f(A)).
(vi) For each subset B of (V,$\tau_{R'}(Y)$), NgCl($f^{-1}(B)$) ⊆ $f^{-1}(Ng\ Cl(B))$

Proof: (i) <=> (ii): Let f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Ng continuous and B be an Nano open set in (V, $\tau_{R'}(Y)$). Then B^C is Nano closed in V. Hence by the definition of Ng continuous function, $f^{-1}(B^C) = (f^{-1}(B))^C$ is Ng closed in U. Thus $f^{-1}(B)$ is Ng open in U.
Conversely, assume that $f^{-1}(B)$ is Ng open in U for each Nano open set B in V. Let F be a Nano closed set in (V, $\tau_{R'}(Y)$) .Then F^C is Nano open in (V, $\tau_{R'}(Y)$) and by assumption $f^{-1}(F^C)$ is Ng open in U. Also $f^{-1}(F^C) = (f^{-1}(F))^C$, we have $f^{-1}(F)$ is Nano g closed in U. So f is Ng continuous.

(ii) => (iii): For x ϵ (U, $\tau_R(X)$), let W be the neighborhood of f(x). Then there exist an Nano open set B in (V, $\tau_{R'}(Y)$) such that f(x) ϵ B⊆ W. Consequently, $f^{-1}(B)$ is Ng open set in (U, $\tau_R(X)$) and x ϵ $f^{-1}(B)$ ⊆ $f^{-1}(W)$. Thus $f^{-1}(W)$ is Ng neighborhood in (U, $\tau_R(X)$).

(iii) =>(iv): Let x ϵ U and W be the neighborhood of f(x). Then by assumption, M= $f^{-1}(W)$ is a Ng neighborhood of x and f(M) = f($f^{-1}(W)$) ⊆ W.

(iv) <=> (v): Suppose (iv) holds and let y ϵ f(NgCl(A)) and let W be the neighborhood of y. Then there exists x ϵ U and a Ng neighborhood M of x such that f(x) = y, x ϵ M, x ϵ Ng Cl (A) and f(M) ⊆ W. Since x ϵ Ng Cl(A), by previous theorem, (M ∩ A) ≠ ∅ and hence (f(A) ∩ W) ≠ ∅ . Hence y= f(x) ϵ NCl(f(A)). That is f(NgCl(A)) ⊆ NCl(f(A).

Conversely, suppose that (v) holds and let xϵU and W be the neighborhood of f(x). Let A = $f^{-1}(V/W)$. Since f(NgCl(A)) ⊆ NCl(f(A) ⊆ V/W, We have NgCl(A) = A. Since x does not belong to NgCl(A), there exist a Ng neighborhood M of x such that (M ∩ A) = ∅ and hence f(M) ⊆ f(U/A) ⊆ W.

(v) <=> (vi) Suppose that (v) holds and B be any subset of (V, $\tau_{R'}(Y)$) . Then replacing A by $f^{-1}(B)$ in (v), we have f(NgCl($f^{-1}(B)$)) ⊆ NCl(f($f^{-1}(B)$)) ⊆ NCl(B). That is NgCl($f^{-1}(B)$) ⊆ f^{-1}(NCl(B)).

Conversely suppose that (vi) holds and let B=f(A) where A is the subset of (U, $\tau_R(X)$). Then we have
NgCl(A) ⊆ NgCl($f^{-1}(B)$) ⊆ f^{-1}(NCl(B)) ⊆ f^{-1}(NCl(f(A)) which implies f(NgCl(A)) ⊆ NCl(f(A)).

This completes the proof of the theorem.

Theorem:3.14 For each x in (U, $\tau_R(X)$) and for each neighborhood B of f(x), if there is a Ng neighborhood M of x such that f(M) ⊆ B, then for each point x in (U, $\tau_R(X)$) and each Nano open set B in (V, $\tau_{R'}(Y)$) with f(x) ∈ B, there is a Ng open set A in U such that x ∈ A, f(A) ⊆ B.

Proof: For x in (U, $\tau_R(X)$), let B be a Nano open set containing f(x). Then B is a neighborhood of f(x). So by assumption, there exists Ng neighborhood M of x, such that f(M) ⊆ B. Hence there exists a Ng open set A in U such that x ∈ A ⊆ M . So f(A) ⊆ f(M) ⊆B **.** That is f(A) ⊆ B.

Theorem: 3.15 For each subset A of (U, $\tau_R(X)$), f(NgCl(A)) ⊆ NCl(f(A)) if and only if the inverse of each Nano closed set in V is N g-closed set in U.

Proof: Suppose the inverse of each Nano closed set is N g-closed set. Let A be a subset of U. Since A ⊆ f^{-1}(f(A)), we have A ⊆ f^{-1}(NCl(f(A))). Since NCl(f(A) is Nano closed in V, by assumption f^{-1}(NCl(f(A) is Ng closed set containing A. Also NgCl(A) ⊆ f^{-1}(NCl(f(A))). Thus f(NgCl(A)) ⊆ f[f^{-1}(NCl(f(A)))] ⊆ NCl(f(A)).

Conversely, suppose the inverse of each Nano closed set in Nano g closed. Let B be a closed subset of (V, $\tau_{R'}(Y)$). Then by assumption, f(NgCl($f^{-1}(B)$) ⊆ NCl(f($f^{-1}(B)$) ⊆ NCl(B) ⊆B. That is NgCl($f^{-1}(B)$) ⊆ $f^{-1}(B)$ which implies $f^{-1}(B)$ is Ng closed.

4. NANO g-CLOSED MAPS AND NANO g- HOMEOMORPHISM

Definition: 4.1 A map f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is said to be Ng-closed map if the image of every Nano closed set in U is Ng-closed in V.

Theorem: 4.2 A mapping f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Ng-closed map iff NgCl(f(A)) ⊆ f(NCl(A)) for every subset A of U.

Proof: Suppose that f is Ng-closed and A⊆U. Then f(NCl(A)) is Ng-closed in V as NCl(A) is Ng-closed in U. Since A⊆NCl(A), f(A) ⊆ f(NCl(A)). By the proposition [3.6] and [3.7] NgCl(f(A)) ⊆ NgCl(f(NCl(A))) ⊆ f(NCl(A)).

Conversely, let A be any Nanoclosed set in U. Then A= NCl(A) which implies f(A) = f(NCl(A)). By hypothesis NgCl(f(A)) ⊆ f(NCl(A)). That is NgCl(f(A)) ⊆ f(A). By the propositions, f(A) ⊆ NgCl(f(A). Therefore f(A) = NgCl(f(A). That is f(A) is Ng-closed and hence f is Ng-closed.

Theorem: **4.3** If f: $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is Ng-closed map, then for each subset A of U, NCl(NInt(f(A)))⊆ f(NCl(A)).

Proof: Let f be a Ng –closed map and A⊆ U. Since NCl(A) is a Nano closed set in U, we have f(NCl(A)) is Ng –closed. We know that every Ng-closed set is Nano preclosed set. So f(NCl(A)) is Nano preclosed set. That is NCl(NInt(f(NCl(A)))) ⊆ f(NCl(A)). (i.e)NCl(NInt(f(A))) ⊆ f(NCl(A)).

Definition: 4.4 A function f: $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is said to be Nano g-homeomorphism if

1. f is 1-1 and onto
2. f is Ng- continuous
3. F is Ng-open

Theorem: 4.5 Let f: $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ be a one to one onto mapping. Then f is a Ng-homeomorphism iff f is Ng-closed and Ng-continuous.

Proof: Let f be a Ng-homeomorphism. Then f is Ng-continuous. Let B be an arbitrary Nano closed set in U. Then B is Ng-closed set in U. Then U-B is Ng-open. Since f is Ng-open, f(U-B) is Ng-open in V. That is V- f(B) is Ng-open in V. Therefore f(B) is Ng-closed in V. Thus the image of every Nano closed set in U is Nano g-closed in V.

Conversely, let f be N g-closed and N g-continuous. Let B be Nano open in U. B is Ng-open in U. Then U-B is N g-closed in U. Since f is Ng-closed, f(U-B) = V-f(B) is Ng-closed in V. Therfore f(B) is Ng-open in V. Thus f is Ng-open and hence f is Ng- homeomorphism.

REFERENCES

1. K.Balachandran, P.Sundaram and H.Maki, On generalized continuous maps in topological spaces, Mem. Fac. Sci. Kochi Univ.math.,12(1991), 5-13.
2. K.Bhuvaneswari, and K.Mythili Gnanapriya, Nano Generalized Closed sets in Nano topological spaces, International Journal of scientific and Research publication, July 2014.
3. Lellis Thivagar, M and Carmel Richard, On Nano forms of Weakly open sets, International Journal of Mathematics and Statistics Invention, Volume 1, Issue 1, August 2013, PP- 31-37.
4. LellisThivagar, M and Carmel Richard, On Nano continuity, Mathematical Theory and Modeling, Vol3, No.7, 2013.
5. Levine, N. (1963), Generalised Closed sets in Topology, Rend.Cire.Math.Palermo, 19(2), 89-96.
6. S.Pious Missier and Vijilius Helena Raj, Gs À Continuous function in topological space, ISRN Geometry, Vol 2013(2013).
7. Z. Pawlak (1982) “Rough Sets”, International Journal of Information and Computer Sciences, 11(1982), 341-356.

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